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Fatigue Strength of Notched Specimen under Combined Stress*

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From the fatigue test of notched specimens of isotropic 70:30 brass, under bending, torsion, and combined stress state of them, a simple criterion for the slip-band crack initiation has been obtained. In spite of the complexity of the triaxial stress state below the notch root, only two stress components, the maximum shearing stress $\tau_{\max}$ and the normal stress σ_n acting on a particular slip plane, are responsible for initiating slip-band cracks. Using the averaged stress values over the half-grain-size depth below the notch root, linear sum of them, $\tau_{\max} + a \cdot \sigma_n$, acts just as the torsional stress in unnotched specimens. At the fatigue limit, therefore, the value for any combined stress state coincides with the torsional one. This relation is more general than the one which has been obtained before for unnotched specimens because of including the case for the unnotched as an extremely shallow notch.

1. Introduction

Most machine parts or structural elements are usually subject to complex loadings. But, fatigue experiments under complex loadings have seldom been performed because of the cumbersomeness and the necessity of a special loading apparatus and hence few experimental results have been obtained. Vast data on fatigue tests up to the present have been obtained under simple loadings such as direct, bending, or torsional stresses⁽¹⁾. Especially for notched specimens, very few fatigue experiments have been done under combined stress states.

sss Fatigue experiments under combined stress states were performed actively in two periods: The first period was when fatigue strengths under simple loadings were extensively investigated. As a part of those investigations, fatigue strengths under combined stresses were studied, too. It has been proposed that the fatigue strength under combined stresses be estimated using the empirical formula with fatigue strengths under pure bending and pure torsion⁽²⁾⁻⁽⁴⁾. Second period followed recently. Several criteria of fatigue failure under combined stress states have been demonstrated being based on the intrinsic behavior of fatigue process⁽⁵⁾⁻⁽¹⁰⁾: Mataka⁽⁵⁾⁻⁽⁸⁾ has shown that critical shear stress acting on the plane where a fatigue fracture occurs is influenced by normal stress acting on the same plane. The same authors⁽⁹⁾ have also investigated heat generation associated with the fatigue under combined stresses.

Tanaka et al.⁽¹⁰⁾ have presented the idea that fatigue strengths are to be rearranged in terms of the equivalent stress associated with Von Mises' plastic flow criterion.

Notched specimens were mainly investigated during the first period. Specimens with special grooves^{(11),(12)}, key ways⁽¹¹⁾⁻⁽¹²⁾, and side holes⁽¹³⁾ were tested and their fatigue strengths were simply compared with those of smoothed specimens. Systematical investigations for various notch geometries and stress combinations have not been performed yet.

It is surprising to notice that no attention was paid to the orientation of materials used in most experiments up to the present. Most experiments have been performed using rolled metals, the strength of which varies widely with directions. The plane where the maximum shear acts varies with the combination of stresses: The fatigue strength depends on the slip resistance on that plane. Thus, measured strength inevitably involves the effect of the material anisotropy. One fatigue criterion induced from some experiments may not be appropriate to another material which is rolled differently from the original material. Therefore, to investigate the intrinsic effect of multi-axial stresses, it is essential that isotropic materials be used.

One of the authors has already performed fatigue experiments of unnotched specimens made of many kinds of fairly isotropic materials under combined stresses and presented the results⁽¹⁴⁾ that the maximum shear stress amplitude at the fatigue limit is affected by the normal stress acting on the same plane on which the shear stress acts no matter how the combination of bending and torsion varies. The main object of this paper is to confirm the validity of this criterion to notched specimens and to understand the mechanism of fatigue crack initiation. Since some notch geometry shows a differ-

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ent stress concentration in bending and torsion, the resultant stress concentration factor under the combined stress should be defined previously. The factor will be defined, in this paper, simply as the ratio of maximum shear stress theoretically calculated for notched specimen to the one for unnotched specimen. Therefore, to get the same concentration factor under different combinations of bending and torsion, notch geometry has to be changed corresponding to the combination. Triaxial stress state produced at the notch root sets up three different principal shear stresses. Slipped planes associated with them should be considered in relation to slip bands formation.

In this experiment were employed two kinds of stress concentration factors and five kinds of stress states: pure bending, pure torsion, and three different combinations of bending and torsion. Obtained results for notched specimens of 70:30 brass were discussed in conjunction with the results of unnotched specimens.

2. Specimens and testing apparatus

The material used was a 23 mm thick plate of 70:30 brass, which was specially manufactured so as not to introduce severe orientation by Kobe Steel Works Ltd. Photomicrographs of grain configuration on longitudinal sections, parallel and perpendicular to the rolled surface and on a

transverse section are shown in Fig. 1. On these sections, average grain sizes were estimated according to ASTM method, in three perpendicular directions: longitudinal, transverse, and normal and were found to be the same in all directions, 48 μ m. Chemical composition and mechanical properties in longitudinal and transverse directions are shown in Tables 1 and 2, respectively. Judging from the results of above examinations, the material can be assumed to be almost isotropic.

Specimens for bending, torsion, and combined state of the two were machined to geometries shown in Fig. 2, taking care that the neutral plane of bending in specimens coincided with the central plane of the rolled plates. Circumferential V-grooves introduced correspond to two different stress concentration factors: $\alpha = 1.7$ and $\alpha = 2.8$. These factors are defined as the ratio of the actual maximum shear stress to the nominal shear stress when the applied bending and torsional stress states are simply superposed under the condition of the plane stress. Therefore, to maintain the same factor in any combin-

Table 1 Chemical composition (wt. %)

Cu	Pb	Fe	Zn
69.2	Tr.	Tr.	Bal.

Table 2 Mechanical properties

	Young's modulus kg/mm ²	Yield strength kg/mm ²	Tensile strength kg/mm ²	Elongation %	Reduction of area %
Longitudinal	11400	10.3	32.7	63.9	82.7
Transverse	11200	10.9	32.4	60.2	83.0

Fig. 1 Microstructure of 70:30 brass used: (A) longitudinal section parallel to rolled surface, (B) longitudinal section perpendicular to rolled surface, (C) transverse section.

Fig. 2 Specimen geometries, sizes in mm. (a) torsion; (b) combined; (c) bending.

Table 3 Standard values of notch root radii, in mm.

Combined angle θ	Stress concentration factor	
	$\alpha = 1.7$	$\alpha = 2.8$
0°	0.49	0.11
10°	0.48	0.11
22.5°	0.46	0.10
41°	0.41	0.10
90°	0.34	0.10

ed stress, the notch root radius ρ must be varied appropriately. Standard values of ρ are shown in Table 3 for five kinds of combined angles, θ . The angle θ is taken between the specimen axis and the torque axis of the testing machine: $\theta = 0^\circ$ corresponds to pure torsion; $\theta = 90^\circ$ to pure bending; $\theta = 10^\circ$, 22.5° , and 41° are combined stress states. All specimens were machined, polished with emery paper, and then annealed at 400°C in a vacuum for an hour. As the preparatory micro-Vickers hardness measurement showed that a surface layer about $100\text{ }\mu\text{m}$ thick would be work-hardened during machining, the surface layer of the specimen was removed by electro-polishing so the amount removed together with the amount previously removed by emery paper exceeded $100\text{ }\mu\text{m}$. During the polishing process, notch geometry was adjusted to final dimensions. For stress calculations, these final dimensions of notches were used. Calculated stress concentration factor, therefore, varied a little in each specimen.

Schenk type bending and torsional fatigue machine, 4 kg·m capacity, was used. In the course of the fatigue testing, the machine was stopped to detect the crack initiation. At that time, the amplitude of a twisting angle was adjusted so that the stress amplitude might be constant. The stress was completely reversed at 1800 cpm. Special grips as reported previously⁽¹⁴⁾ were used in order to generate a combined stress state, where bending and torsional stresses are combined in the phase. One example is shown in Fig. 3. Since inertia of those grips is fairly large, the torque detected at the torque bar which is placed behind the chuck of the machine is usually

Fig. 3 Grips to generate combined stress state, $\theta = 10^\circ$.

larger than what the specimen is actually subjected to. Hence, a conversion diagram between the torque measured at the torque bar and the actual one was previously prepared for use. From the converted value of the torque, the stress amplitude was calculated.

3. Results

Fig. 4-(a) and -(b) show S-N diagrams obtained in this experiment. Ordinates are plotted with maximum shear stress amplitude on a specimen surface. Fatigue life was defined as the stress repetition until 0.1 mm slip-band cracks initiated, as shown in Fig. 5 for example. Fatigue limit corresponds to the threshold stress below which a 0.1 mm slip-band crack does not initiate after 2×10^7 repetitions. From fatigue strengths obtained in this experiment and the fatigue strengths of unnotched specimens, the fatigue notch factor β was calculated to be 5 % less than the stress concentration factor $\alpha = 1.7$ for both bending and torsion but 2.3 and 2.5 for bending and torsion corresponding to $\alpha = 2.8$.

(a)

(b)

Fig. 4 S-N diagrams.
(a) $\alpha = 1.7$; (b) $\alpha = 2.8$.

4. Discussion

4.1 Stress states on a surface layer of a specimen subjected to combined stresses

The stress state in notched specimens is distinguished from that in unnotched ones because the triaxial stress state is produced near the notch root. Beneath the notch root, circumferential and radial normal stresses exist together with the axial one. In Fig. 6, Mohr's stress circles well represent these situations. Full line circles correspond to the state where slipping produces no steps on a specimen surface as shown schematically in Fig. 7-(b) and broken line circles to the state

Fig. 5 Slip bands of both A and B types and slip-band cracks.

Fig. 6 Mohr's stress circles to represent the combined stress state. ξ is the depth from the surface in mm.

Fig. 7 Schematic representation of two types of slip.

where slipping produces considerable steps as in Fig. 7-(a). Two types of slip will be called according to Fig. 7, A and B types, hereafter.

In unnotched specimens B type shear stress always becomes the first principal shear stress and pure bending is an extreme case where A and B type stresses coincide. On the contrary, in notched specimens, A-type shear stress becomes possibly larger than B type one even in the combined stress state. As shown in Fig. 6, as an example, at combined angles below 22.5° , B type shear becomes dominant at the notch root, whereas, above 41° , A-type shear becomes the first principal shear stress. In general, which type of the shear becomes the maximum depends on the notch geometry and the combined angle. Fig. 8 shows the region where each principal shear stress becomes dominant on a diagram of notch geometry ρ/a vs. combined angle θ . In the diagram, notches were assumed to be deep hyperbolic for the convenience of calculation. An increase in notch sharpness and an increase in the bending component alter the slip plane of the maximum shear from B type to A. However, in real polycrystalline metals, this tendency can be strongly influenced by grain configuration and intrinsic crystallographic slip planes. Slip bands corresponding to A type shear may be observed on a specimen surface as bands perpendicular to the first principal direction whereas slip bands for B type may incline to about 45° in that direction. In Fig. 5, both type slip bands can be observed where the direction of σ_1 was theoretically calculated for the notch geometry of this specimen.

Fig. 8 Notch geometry ρ/a and combined angle determine which type of principal shear stress becomes dominant on the specimen surface. ρ is a notch root radius and a , radius of the notched section.

4.2 Criterion of the fatigue crack initiation

For notched specimens, stress concentrates at the notch root with steep gradient. As the slip behavior is related to finite geometrical dimensions, say one grain size or so, it is reasonable to consider that not the stress value at a point but the representative value over some region is responsible for initiating slip-band cracks. Following the idea that when investigating the fatigue strength of a stress raiser which generates steep stress gradient inwards, the combination of the maximum stress on the surface and the gradient⁽¹⁵⁾ or the mean value of the stress over a certain region⁽¹⁶⁾ becomes a parameter, it is assumed in this paper that slip-band cracks initiate when the mean value over a certain region ξ of m 'th power of a linear combination of the maximum shear stress $\tau_{\max}$ and the related normal stress σ_n acting on the same plane attains the critical, the material constant. At the fatigue limit, then, this can be formulated as:

$$\frac{1}{\xi} \int_0^{\xi} (\tau_{\max} + a \cdot \sigma_n)^m d\xi = \tau_0 (\text{const.}) \quad (1)$$

where a is a positive constant. For simplicity, herein, assuming $m = 1$ and expressing the mean values over ξ region as:

$$\bar{\tau}_{\max} = \frac{1}{\xi} \int_0^{\xi} \tau_{\max} d\xi$$

$$\bar{\sigma}_n = \frac{1}{\xi} \int_0^{\xi} \sigma_n d\xi$$

Equation (1) becomes:

$$\bar{\tau}_{\max} = \tau_0 - a \bar{\sigma}_n \quad (2)$$

where τ_0 and a are constants. Equation (2) must become a criterion for unnotched specimens as a special case, too. However, alternative expression for unnotched specimens is possible as:

$$\tau_{\max} = \tau_0 - a \cdot \sigma_n \quad (3)$$

Because the unnotched specimen has a stress distribution with very gentle gradient and the stress value on the surface can stand for the mean value over ξ region. Equation (3) is just the same as already obtained for unnotched specimens⁽¹⁴⁾. Constant τ_0 in Eqs. (2) and (3) can be explained as the torsional fatigue strength of the unnotched specimen because, in pure torsion, $\sigma_n = 0$.

Assuming notched specimens illustrated in Fig. 2 to be circular cylinders with deep hyperbolic circumferential grooves, regardless of differences of notch depth and notch angle, stress distributions were obtained by superposing Neuber's solutions for pure bending and pure torsion⁽¹⁷⁾. Principal stresses, principal shear stresses, and principal directions were calculated in the radial direction below the notch root. As principal direction rotates with depth, the plane on which the maximum shear stress acts becomes wavy. However, the slipped plane was assumed to be plain over one grain size since the variation of the direction over mean-grain size interval was as small as less than one degree. Mean values of the principal shear stress $\bar{\tau}_{\max}$ and the related normal stress $\bar{\sigma}_n$ at the fatigue limit were obtained by numerical integration, with ξ being chosen to be one half of the mean grain size, 0.024 mm. These mean values are also obtained from the stress concentration factors and the stress gradients with little errors, where the triangular

Fig. 9 $\bar{\tau}_{\max} - \bar{\sigma}_n$ diagram

Data on B type lying near $\bar{\sigma}_n = 0, 1, 3, 5, 8$ correspond to combined angles; $\theta = 0^\circ, 10^\circ, 22.5^\circ, 41^\circ$, and 90° , respectively.

stress distribution is assumed to have the same surface stress and the same gradient as the actual one. Estimated mean values $\bar{\tau}_{\max}$ are plotted against $\bar{\sigma}_n$ corresponding to both A and B type slips, in Fig. 9. The shear stress for A type slip at 41° becomes comparable to the stress for B type after averaged, whereas on the surface, A type shear stress is larger than B type. At an angle less than 41° , B type shear is still larger than A type after averaged. At the fatigue limit of pure bending, $\theta = 90^\circ$, principal stresses exceeded the 0.2 % offset stress of this material. Hence, the pure bending case should be investigated separately. Experimental data on B type slip are well correlated by Eq. (2) as long as the surface part of the specimen does not yield. But, data on A type are not. A-type slip-band cracks are seldom observed in the experiment. A-type slip may be difficult to occur in spite of the stress calculation. The straight line in Fig. 9 corresponds to the constants:

$$\begin{aligned}\tau_0 &= 4.50 \text{ kg/mm}^2 \\ a &= 0.237\end{aligned}\quad (4)$$

and gives the interpretation that about one-quarter level of the normal stress acting on the slip plane contributes to the maximum shear stress for the initiation of the slip-band cracks. Square plots on lines $\bar{\tau}_{\max} = 0$ and $\bar{\tau}_{\max} = \bar{\sigma}_n$ represent fatigue strengths of pure torsion and pure bending, respectively, for unnotched specimens of this material. The fact that fatigue strengths of unnotched specimens lie just on the line for the notched ones indicated that Eq. (3) is a special expression of Eq. (2).

Fatigue strength of a specimen with a certain notch geometry can vary with the combination of bending and torsion. Using values of constants obtained in Expression (4), variation of fatigue strength of the notched specimen against the combined angle can be calculated as shown in Fig. 10. Ordinate is a ratio of nominal maximum

Fig. 10 Variations of fatigue strength with the combination of bending and torsion.

shear stress

$$\tau_{\text{nominal}} = \frac{16 M}{\pi d^3}$$

at the notch root to the torsional fatigue strength τ_0 . So long as the notch sharpness is not severe, torsional strength is the highest of all combined states for the same notch geometry. Increase of bending component reduces the strength. But, for very sharp notches, as seen for $\rho/a = 10^{-3}$ in the diagram, variation of the strength becomes small.

5. Conclusions

From experimental investigation of fatigue strength, which was defined to be the stress below which a 0.1 mm slip-band crack does not initiate, under the combined stresses of bending and torsion on isotropic 70:30 brass notched specimens, the following is concluded:

- (1) Slip bands observed on the specimen surface may consist of two types: one makes steps on the surface and the other makes no steps.
- (2) The maximum shear stress corresponding to the latter slip type mentioned above is equivalent to the principal shear stress in the surface layer which is assumed to be in the plane stress state. After being averaged over one half of a mean-grain-size depth, the shear stress of this type is considered to be responsible for initiation of slip-band cracks as long as the principal stress does not exceed the yield strength of the material.
- (3) At fatigue limit, the maximum shear stress and the normal stress acting on the same plane are only necessary parameters to formulate a criterion for initiation of slip-band cracks. The ratio of the contribution of the former to the latter stress is about 4:1 in this material.
- (4) Criterion for the fatigue strength of unnotched specimens can be derived from the one for notched specimens. Hence, the values of constants involved can be evaluated from pure torsion and pure bending fatigue strengths of unnotched specimens.
- (5) Using the obtained criterion, variation of the fatigue strength with the combination of bending and torsion is estimated for any notch geometry.

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