1 A fractal model for characterizing fluid flow in fractured rock
2 masses based on randomly distributed rock fracture networks
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22 **Abstract:**

23 A fractal model that represents the geometric characteristics of rock fracture networks is
24 proposed to link the fractal characteristics with the equivalent permeability of the fracture networks.
25 The fracture networks are generated using the Monte Carlo method and have a power law size
26 distribution. The fractal dimension D_T is utilized to represent the tortuosity of the fluid flow, and
27 another fractal dimension D_f is utilized to represent the geometric distribution of fractures in the
28 networks. The results indicate that the equivalent permeability of a fracture network can be
29 significantly influenced by the tortuosity of the fluid flow, the aperture of the fractures and a
30 random number used to generate the fractal length distribution of the fractures in the network. The
31 correlation of fracture number and fracture length agrees well with the results of previous studies,
32 and the calculated fractal dimensions D_f are consistent with their theoretical values, which confirms
33 the reliability of the proposed fractal length distribution and the stochastically generated fracture
34 network models. The optimal hydraulic path can be identified in the longer fractures along the fluid
35 flow direction. Using the proposed fractal model, a mathematical expression between the equivalent
36 permeability K and the fractal dimension D_f is proposed for models with large values of D_f . The
37 differences in the calculated flow volumes between the models that consider and those that do not
38 consider the influence of fluid flow tortuosity are as high as 17.64% – 19.51%, which emphasizes
39 that the effects of tortuosity should not be neglected and should be included in the fractal model to
40 accurately estimate the hydraulic behavior of fracture networks.

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42 **Keywords:** Fracture network; Permeability; Fractal dimension; Geometric distribution; Tortuosity

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45 1. Introduction

46 Permeability is a crucial hydro-mechanical property of rock masses and is important in many areas of
47 geosciences and geoengineering, including dam foundations and petroleum reservoirs. The permeability of a rock
48 mass is mainly governed by rock fractures that separate intact rock blocks with negligible matrix permeability
49 (e.g., granite and basalt) [1-2]. A tremendous amount of effort has been exerted to understand the behavior of fluid
50 flow in rock masses in recent decades [3-7]. However, accurately estimating the permeability of rock masses is
51 still challenging because of the complexities of fracture distributions at the macro-structural (i.e., geometry of the
52 fracture network) and micro-structural (i.e., geometry of the void spaces within single fractures) levels [8-11]. One
53 key difficulty is that rock fractures typically have rough surfaces between which fluid flows non-uniformly. A
54 particle will travel a longer distance along a tortuous path through a rough-walled fracture than through a
55 parallel-walled fracture. Another difficulty is mathematically describing the geometric distributions of rock
56 fractures in fracture networks, which usually contain several sets of fractures with different orientations, lengths
57 and apertures. Fortunately, the distribution of fractures in fracture networks have been found to exhibit fractal
58 characteristics [12-16], which provides a possible approach for describing the geometric characteristics of fracture
59 networks while considering both the macro-scale and micro-scale properties of the fractures.

60 A few predictive fractal models have been developed to calculate the permeability of stochastic rock fracture
61 networks. The purpose of their models and the outcome of their studies are summarized in Table 1.

62 Based on the fractal models proposed for the porous media [10] and regular tree networks [21, 27] to
63 calculate the permeability of rock masses, the present study focused on extending this fractal model to the

fractured media consisted of randomly generated stochastic discrete fracture networks (DFNs) using the Monte Carlo method. A probability density function was derived to depict the trace length l of each rock fracture between a minimum and maximum trace length, and the apertures of the fractures were correlated with their trace lengths. Flow simulations of models with various fractal dimensions were conducted, and the relations between the fractal dimensions and the equivalent permeability were estimated.

2. Fractal characteristics of rock fractures

Mandelbrot (1982) [28] verified that the cumulative size distribution of islands on the surface of the earth followed the power law

$$N'(A' > a') \sim a'^{(-D/2)} \quad (1)$$

where N' is the total number of islands with an area A' greater than a constant a' and D is the fractal dimension that represents the size distribution of the islands.

Based on this theory, *Majumdar* and *Bhushan* [29] developed an equivalent equation to describe the distribution of islands by regarding a'_{max} as the largest island:

$$N'(A' \geq a') = \left(\frac{a'_{max}}{a'} \right)^{D/2} \quad (2)$$

Eq. (2) shows that there is only one largest island on the earth, which is true in the physical world. *Xu et al.* [21, 27] used Eq. (2) to describe the geometric distribution of pores in porous media that are embedded with randomly distributed 2-D fractal-like tree networks, where $a'_{max} = g\lambda^2$, $a' = g\lambda^2$, λ is the diameter of a pore, and g is a geometric factor. The distribution of fractures in 2-D rock masses is considered to be analogous to that of islands on the surface of the earth and that of pores in porous media, which yields

$$N(L \geq l) = \left(\frac{l_{max}}{l} \right)^{D_f/2} \quad (3)$$

where N is the total number of fractures with a length L greater than a constant fracture length l ; D_f is in the range of $[1, 2]$ for 2-D fracture networks; and l_{max} is the maximum trace length of fractures in a rock mass.

Differentiating Eq. (3) with respect to l leads to

$$-dN = \frac{D_f}{2} l_{max}^{D_f/2} l^{-(D_f/2+1)} dl \quad (4)$$

The negative sign on the left side of Eq. (4) implies that the number of fractures decreases with increasing trace length. This equation gives the correlation of fracture number and trace length. The total number of fractures N_t can be calculated by setting $l = l_{min}$, which yields

$$N_t(L \geq l_{min}) = \left(\frac{l_{max}}{l_{min}} \right)^{D_f/2} \quad (5)$$

where l_{min} is the minimum trace length of the fractures in a rock mass. Dividing Eq. (4) by Eq. (5) gives

$$-\frac{dN}{N_t} = \frac{D_f}{2} l_{min}^{D_f/2} l^{-(D_f/2+1)} dl = f(l) dl \quad (6)$$

where $f(l) = \frac{D_f}{2} l_{min}^{D_f/2} l^{-(D_f/2+1)}$ is the probability density function; according to probability theory, this function satisfies the following equation:

$$\int_{-\infty}^{+\infty} f(l) dl = \int_{l_{min}}^{l_{max}} f(l) dl = 1 - \left(\frac{l_{min}}{l_{max}} \right)^{D_f/2} \equiv 1 \quad (7)$$

By eliminating the constant value 1, Eq. (7) becomes

$$\left(\frac{l_{min}}{l_{max}} \right)^{D_f/2} \equiv 0 \quad (8)$$

Eq. (8) implies that $l_{min} < l_{max}$ should be satisfied for Eq. (7) to hold. Eq. (8) is therefore a necessary condition

for a fracture distribution to exhibit fractal characteristics. In this study, $l_{min}/l_{max} \leq 10^{-3}$ is used as a threshold to

enable fluid flow in 2-D fracture networks to be effectively represented using a fractal model. For all of the

fractures with trace lengths in the range of $[l_{min}, l_{max}]$, the cumulative probability (R) can be integrated as

$$R(l) = \int_{l_{min}}^l f(l) dl = \int_{l_{min}}^l \frac{D_f}{2} l_{min}^{D_f/2} l^{-(D_f/2+1)} dl = 1 - \left(\frac{l_{min}}{l} \right)^{D_f/2} \quad (9)$$

Eq. (9) implies that when $l \rightarrow l_{min}$, $R = 0$, and when $l \rightarrow l_{max}$, $R = 1$. As long as l is in the range of $[l_{min}$,

$l_{max}]$, the value of R is in the range of $[0, 1]$. Therefore, by assigning random numbers between 0 and 1 to R , the

correlated trace length l can be back-calculated by

$$l = \frac{l_{min}}{(1-R)^{2/D_f}} = \left(\frac{l_{min}}{l_{max}} \right) \frac{l_{max}}{(1-R)^{2/D_f}} \quad (10)$$

To facilitate the calculation, each fracture is labeled with an integer from 0 to N_t . For the i th fracture, the

trace length l_i can be calculated using a random number R_i as follows:

$$l_i = \frac{l_{min}}{(1-R_i)^{2/D_f}} = \left(\frac{l_{min}}{l_{max}} \right) \frac{l_{max}}{(1-R_i)^{2/D_f}} \quad (11)$$

where $i = 1, 2, 3, \dots, N_t$ and N_t is the total number of fractures in the network.

Eq. (11) represents the fractal length distributions in 2-D rock fracture networks and is significantly

correlated with the minimum trace length l_{min} , the random number R , the fractal dimension D_f and the total number

of fractures N_t . The validity of Eq. (11) will be verified in Sections 4.2 and 4.3 by comparing the correlation of the

fracture number and fracture length with the results of other studies and by comparing the fractal dimension D_f of

DFN models generated using Eq. (11) with their theoretical values.

117 Previous studies of fracture sizes from centimeters to meters at different sites have found that the aperture
 118 and the trace length of fractures have positive linear correlations [30-31] or power law correlations [32-33].
 119 Longer fractures usually have higher permeabilities and larger apertures than shorter fractures. In this study,
 120 assuming that the DFN models are composed of a large number of small fractures with small hydraulic apertures
 121 and a much smaller number of longer fractures with larger apertures, the trace lengths of the fractures are
 122 randomly generated according to Eq. (11) and are correlated with the fracture aperture as follows [4, 34]:

$$\frac{l_i^{-D_f} - l_{min}^{-D_f}}{l_{max}^{-D_f} - l_{min}^{-D_f}} = \frac{g(e_i) - g(e_{min})}{g(e_{max}) - g(e_{min})} \quad (12)$$

$$g(e_i) = erf\left[(\ln e_i - \bar{e}_{log}) / \sqrt{2b}\right] \quad (13)$$

125 where erf is an error function; $\bar{e}_{log}$ and b are the first and second moments of the log-normal distribution of the
 126 apertures, respectively; e_{min} and e_{max} indicate the minimum and maximum apertures, respectively; e_i is the
 127 hydraulic aperture of the i th fracture; and l_i can be calculated from Eq. (11).

128 Finally, the aperture of the i th fracture can be obtained by forcing $\bar{e}_{log} = 0$ and $b = 1$ as follows [4, 34]:

$$e_i = exp\left\langle \sqrt{2}erfinv\left\{ \frac{l_i^{-D_f} - l_{min}^{-D_f}}{l_{max}^{-D_f} - l_{min}^{-D_f}} \left[erf\left(\frac{\ln e_{max}}{\sqrt{2}}\right) - erf\left(\frac{\ln e_{min}}{\sqrt{2}}\right) \right] + erf\left(\frac{\ln e_{min}}{\sqrt{2}}\right) \right\} \right\rangle \quad (14)$$

130 where $erfinv$ is the inverse error function.

131 Rough surfaces of fractures render the streamlines of fluid flow nonlinear, which can increase the end-to-end
 132 distances required for fluid flow through fractures and therefore reduce their equivalent permeability. Here, as
 133 shown in Fig. 1, l is the straight length of the fracture pathways in the flow direction, l_t is the tortuous length along
 134 the fracture profile, and e is the hydraulic aperture of a single fracture [35]. Generally, $l_t > l$, except for fractures

135 with flat surfaces (parallel plate model), where $l_t = l$. The tortuosity is defined as a parameter to depict the ratio of
 136 l_t to l , in order to describe their difference induced by fracture surface roughness in 2-D rock fractures [36]. The
 137 correlation of the tortuous length l_t and the straight length l of fractures is also considered to be analogous to that
 138 of pores in porous media [10, 37], as expressed by

$$139 \quad l_t = e^{1-D_T} l^{D_T} \quad (15)$$

140 where D_T is the fractal dimension of the non-linear streamline of fluid flow, with a value in the range of [1, 2] for
 141 a single fracture in a 2-D rock fracture network. When $D_T = 1$, the streamline is linear, resulting in $l_t = l$,
 142 corresponding to a 2-D fracture. As a consequence, the value of D_T can depict the non-linearity of the streamline, as
 143 well as the effect of tortuosity of fluid flow.

144 3. A fractal model for permeability estimation

145 3.1 Fluid flow in a single fracture

146 Although real rock fractures have rough walls and variable apertures, fluid flow through rock fractures is
 147 usually described by the cubic law [5], which assumes that fractures consist of two smooth parallel walls. Under
 148 these conditions and substituting Eq. (11) and Eq. (15) into the cubic law, the flow rate of the i th fracture can be
 149 written as

$$150 \quad q(i) = \frac{e_i^3}{12\mu} \frac{\Delta P_i}{l_t(i)} = \frac{e_i^{2+D_T}}{12\mu} \frac{\Delta P_i}{l_i^{D_T}} = \frac{e_i^{2+D_T}}{12\mu} \frac{\Delta P_i}{l_{min}^{D_T}} (1 - R_i)^{2D_T / D_f} \quad (16)$$

151 where $q(i)$ is the fluid volume through the i th fracture, ΔP_i is the local hydraulic pressure difference applied
 152 between the tips of the fracture, and μ is the viscosity of the fluid.

153 If a model consists of a single fracture that satisfies the parallel plate model, then $D_T = 1.0$ and $D_f = 2.0$. Eq.

154 (16) reduces to

155

$$q_{pp}(i) = \frac{e_i^3}{12\mu l_{min}} \Delta P_i (1 - R_i) = \frac{e_i^3}{12\mu l_i} \Delta P_i \quad (17)$$

156 where $q_{pp}(i)$ is the fluid volume through the i th fracture based on the parallel plate model. Eq. (17) is the
157 standard form of the cubic law, which is the most simplified case of Eq. (16). The fractal length distribution
158 presented in Eq. (11) is similar to the power law length distribution in 2-D random fracture networks [17-18].

159 3.2 Fluid flow in a 2-D fracture network

160 Algorithms for DFN generation and fluid flow through fracture networks were extensively described in the
161 manual of UDEC [38], and only a few principal features are presented here. Three aspects should be addressed to
162 generate a DFN. The first involves the regularization of DFN models. Because fluid flows within connected
163 fracture networks, the fracture elements outside the model, the isolated fracture elements, and the “dead-end”
164 fracture elements do not contribute to the fluid flow and should therefore be deleted. Second, parameters (i.e.,
165 trace length, hydraulic aperture, and orientation) should be assigned to each fracture, and the mass continuity
166 equations at each fracture intersection should be established. The third aspect addresses the iteration scheme of
167 these equations for given boundary conditions. Steady-state fluid flow was adopted in this study for calculating
168 the equivalent permeability of DFNs, and a generic hydraulic boundary condition with a constant hydraulic
169 gradient in the x -direction was assumed, indicating that the directivity of the equivalent permeability is horizontal
170 (see Fig. 2).

In a 2-D fracture network, fractures are line segments, and fracture intersection points are denoted as nodes.

The summation of the flow rate of each fracture connected to a node is zero or equals the added source term.

Similarly, the summation of the total flow rate of the entire fracture network equals zero or the summation of the

source terms added at each node. Fig. 2(b) shows a region composed of a node m and several fractures from the

DFN model shown in Fig. 2(a). Taking into account the balance of fluid flow, the equilibrium equation of node m

can be written as

$$\left(\sum_{n=1}^M q_n \right)_m + Q_m = 0, (m = 1, 2, \dots, N_{node}) \quad (18)$$

where q_n is the fluid volume through the fracture element n , M is the total number of fracture elements connected

to node m , Q_m is a source term at node m , and N_{node} is the total number of nodes in the entire fracture network.

Generally, under steady-state flow without any source terms, $Q_m = 0$ and $M \leq 4$ (two fractures intersect at one

node) given that the probability that more than two fractures intersect at the same node is nearly zero. Steady-state

flow was assumed; therefore, the fluid volume that flows into a fracture network is equal to that flowing out of the

network. The fluid rate at the outlet boundary was then utilized to estimate the equivalent permeability of the DFN

model.

Using Darcy's law as shown in Eq. (19), the equivalent permeability K is obtained in Eq. (20):

$$Q = A \frac{K}{\mu} \frac{\Delta P'}{L'} \quad (19)$$

$$K = \frac{\mu L'}{A \Delta P'} Q(e, D_T, l_{min}, D_f, R, \Delta P') \quad (20)$$

where A is the cross-sectional area perpendicular to the fluid flow direction, L' is the length of the fracture

network in the fluid flow direction, Q is the total fluid volume through the fracture network per second, $\Delta P'$ is

190 the hydraulic pressure difference of the DFN model between the inlet boundary and the outlet boundary, and K is
191 the equivalent permeability of the fracture network.

192 Eqs. (16) and (20) are the fundamental equations of the proposed fractal model for calculating the equivalent
193 permeability of fracture networks and are a function of e , D_T , l_{min} , D_f , R and $\Delta P'$. In this model, D_f can be
194 calculated by applying the box-counting method to fracture networks [39] and D_T is a given input parameter based
195 on in situ geological survey data. No empirical constants are involved in this fractal model, and all of the
196 parameters have clear physical meanings.

197 3.3 Basic parameters

198 *Baghbanan and Jing* [4] studied the hydraulic properties of DFNs based on field mapping results of a real
199 rock mass. For simplification, we used the values of most of the parameters from their study. The maximum trace
200 length l_{max} and minimum trace length l_{min} were 500 m and 0.5 m, respectively, with a ratio l_{min}/l_{max} of 0.001, which
201 satisfied the condition of Eq. (8). The range of D_f was 1.3 – 1.8, below which the connectivity of the fracture
202 networks becomes very poor and above which the rock masses are so fragmented that they rarely exist in the
203 nature. Wakabayashi et al. [40], based on the 10 classical single fractures suggested by Barton [41], studied the
204 relation of D_T and JRC (joint roughness coefficient) and derived a regression equation, where the JRC is a
205 commonly used parameter for describing the surface roughness of rock fractures [42]. Their results show that the
206 range of D_T was 1.000 – 1.018, which roughly corresponds to JRC = 0 – 20. The maximum and minimum
207 apertures were 100 μm and 1 μm , respectively. These parameters were utilized to generate the fracture networks
208 with the Monte Carlo method using a fractal probability density function for the trace length distribution

209 according to Eq. (11). The fracture orientation and fracture center point distribution were assumed to be uniformly
 210 distributed because this study is only focused on the validity of the proposed fractal model and its influence on the
 211 equivalent permeability of 2-D fracture networks.

212 4. Results and analyses

213 4.1 Determination of the model size

214 The size of a DFN model should be determined prior to performing the fluid flow simulation. In Eqs. (3), (16)
 215 and (20), the model size was unknown. It is well known that D_f is not correlated with the model size, which is a
 216 fundamental property of fractal models. For fracture networks, the fractal dimension is positively correlated with
 217 the mass density (total length per square meter) of the fractures. To obtain the mathematical relation between D_f
 218 and the mass density d_m , a series of DFN models with different values of d_m were generated, and their fractal
 219 dimensions D_f were calculated using the box-counting method. The results are shown in Fig. 3, in which a
 220 regression function could be obtained as follows:

$$221 \quad D_f = a'' + b'' \ln(d_m + c'') \quad (21)$$

222 where a'' , b'' , and c'' are three regression parameters equal to 0.4594, 0.2797 and 8.4968, respectively.

223 According to the definition of the mass density of fractures, d_m can be expressed as follows:

$$224 \quad d_m = \frac{\sum_{i=1}^{N_t} l_i}{L_t^2} = \sum_{i=1}^{N_t} l_i \cdot (L_t)^{-2} \quad (22)$$

225 where L_t is the theoretical model length.

226 Substituting Eq. (22) into Eq. (21), L_t can be calculated as follows:

227

$$L_t = \left\{ \sum_{i=1}^{N_t} l_i / \left[\exp\left(\frac{D_f - a''}{b''}\right) - c'' \right] \right\}^{0.5} \quad (23)$$

228

Using the calculated value of L_t in Eq. (23), the number density of fractures d_n , which defines the total

229

number of fractures in a given area of the fracture network, is obtained as follows:

230

$$d_n = \frac{N_t}{L_t^2} \quad (24)$$

231

Consequently, for a constant fractal dimension D_f , the mass density d_m and the number density d_n are also

232

constants. For an arbitrary model length $L_n > L_t$, the new fracture number N_t' can be calculated based on Eqs.

233

(23) and (24) as follows:

234

$$N_t' = \frac{N_t L_n^2}{L_t^2} = N_t L_n^2 \left[\exp\left(\frac{D_f - a''}{b''}\right) - c'' \right] / \sum_{i=1}^{N_t} l_i \quad (25)$$

235

Considering previous studies on the representative elementary volume (REV) of fracture networks [4], we

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chose a model size L_n of 5.0 m for the calculations. The mass density d_m , the theoretical fracture number N_t , the

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theoretical model size L_t , and the fracture number N_t' of any model can be calculated with Eqs. (22), (5), (23)

238

and (25), respectively, as shown in Table 2.

239

4.2 Validity of the fractal length distribution

240

The length of each fracture was generated using Eq. (11) until the index i reached N_t' for different fractal

241

dimensions D_f from 1.3 to 1.8 (Table 2). Higher values of the fractal dimension D_f lead to higher probabilities of

242

generating longer fractures. The fracture number $n(l, 0.5)$ represents the number of fractures with lengths in the

243

range of $(l - 0.5, l + 0.5)$. Fig. 4 and Table 3 show the statistical results of the relation between fracture length and

244

fracture number using the parameters shown in Table 2. For each D_f , three sets of random number seeds were

utilized to generate different fracture length distributions using Eq. (11). Each fracture length-fracture number curve was fitted using a power law function in the form of

$$n(l, 0.5) = \alpha l^{-a} \quad (26)$$

where $n(l, 0.5)$ is the number of fractures with lengths in the range of $(l - 0.5, l + 0.5)$, α is the coefficient of proportionality, and a is the power law exponent.

With increasing D_f , more fractures with relatively long lengths appear, and the length of the longest fracture increases, as shown in Fig. 4. Table 3 and Fig. 5 show that the average power law exponent a ranges from 2.29 to 1.70, corresponding to fractal dimensions D_f of 1.3 to 1.8. These parameters have a linear relationship that can be expressed by

$$a = -D_f + 3.49 \quad (27)$$

For 2-D fracture networks, D_f ranges from 1 to 2, which results in values of the power law exponent a from 2.49 to 1.49 according to Eq. (27). These results reveal that the fracture length also follows a power law length distribution with the power law exponent a linearly correlated with the fractal dimension D_f . The theoretical value of the power law exponent a ranges from 1.49 to 2.49, which agrees well with the values reported in the literature from in situ measurements and theoretical analyses (Table 4) [17-18, 43-45]. Therefore, the proposed fractal length distribution approach is reliable and is capable of describing the characteristics of fracture distributions using a fractal dimension method.

4.3 Fractal evaluation methods

263 The stochastic DFN modeling technique was utilized to generate and characterize fracture networks.
264 Measuring length density is difficult because each model contains a large number of fractures. The geometric
265 distribution of rock fractures in a fracture network has fractal characteristics [25]. Therefore, the fractal dimension
266 may serve as an effective approach for assessing the geometric characteristics of rock fractures.

267 Because it is a simple and precise approach, the box-counting method was used and improved in this study to
268 calculate the fractal dimensions of the DFN models. First, the original image was converted to monochrome (Fig.
269 6(a)), and a binary image (Fig. 6(b)) was then obtained based on the adaptive threshold of the grayscale. The
270 example shown in Fig. 6(b) is 280 pixels in length and 280 pixels in width, and each pixel has a value of either 1
271 or 0 (null). Second, the image was covered by square boxes with different dimensions from 280x280 pixels (1 box)
272 to 1x1 pixel (280x280 boxes). If the pixels in a box were not null, the box was given a value of 1; otherwise, it
273 was given a value of 0. Finally, a log-log plot of the box count (N_c) vs. the number (N_b) of total boxes (Fig. 6(c))
274 was drawn, and the slope represents the fractal dimension. To verify the validity of this program, a series of
275 well-known fractal graphs were generated, and the results were compared with the theoretical values. The
276 deviations between the calculated and theoretical values are less than 2%; therefore, the program is considered to
277 reliably estimate the fractal dimensions D_f .

278 For each fractal dimension D_f , 10 sets of DFN models (numbered from 1 to 10) were developed using the
279 fractal length distribution of Eq. (11). The fractal dimensions (D_f) of these DFN models were then calculated using
280 the box-counting method and compared with their theoretical values (Fig. 7). When D_f was small (e.g., 1.3 and
281 1.4), the calculated fractal dimensions D_f were underestimated compared to their theoretical values because many

282 isolated fractures that did not connect with other fractures were deleted from the models. At higher values of D_f
283 (1.5 and 1.6), the calculated and theoretical values agreed with each other, and the discrepancies decreased
284 because the fractures became denser and the connectivity of the models improved. These results are robust
285 evidence of the validity of applying Eq. (3) from porous media to fractured rock masses and the validity of the
286 fractal length distribution presented in Eq. (11).

287 *4.4 Characteristics of the flow patterns*

288 Several examples of DFN models with side lengths of 5.0 m and D_f values from 1.3 to 1.6 are shown in the
289 left column of Fig. 8. These models were generated using the Monte Carlo method based on the parameters
290 presented in Section 3. In these models, the upper and lower boundaries were assumed to be impermeable, and
291 fluid flowed from left to right (see Fig. 2) with a hydraulic head of 5 m. When the inflow volume was equal to that
292 of the outflow, the fluid flow was regarded as a steady-state flow. The flow rate distributions at these conditions
293 are shown in the central column of Fig. 8. The right column of Fig. 8 shows corresponding remarks. In Figs. 8(b)
294 and 8(e), preferential flow paths exist along the relatively long fractures that are subparallel to the flow direction
295 (horizontal) and particularly in those that intersect the inlet and outlet boundaries. With an increase of D_f (see Figs.
296 8(h) and 8(k)), more short fractures achieve large flow rates and the flow rate distributions within the networks
297 become more homogeneous. These observations were also reported by *De Dreuzy et al.* [17-19], who found that
298 the connectivity and permeability of 2-D fracture networks are controlled by the largest fracture in the system
299 when only a few fractures exist and controlled by the fractures that are smaller than average when a large number
300 of fractures exist.

301 4.5 Influences of D_f and D_T on the equivalent permeability

302 As shown in Eq. (16), the proposed model includes a random number R_i that introduces randomness to the
303 fracture networks. Ten random numbers corresponding to each fractal dimension D_f were generated and applied to
304 the DFN modeling. The relation between the equivalent permeability K and the value of D_f calculated utilizing
305 these random numbers is shown in Fig. 9. Due to the randomness of the trace length and the location and
306 orientation of rock fractures induced by the random numbers, DFN models with the same D_f value can have large
307 differences in their geometric distributions, which results in variations in the calculated equivalent permeabilities
308 (see Fig. 9) [19]. When D_f is small (e.g., 1.3), the equivalent permeability varies by approximately 5 orders of
309 magnitude. The range of variation decreases to less than 2 orders of magnitude when D_f becomes large (e.g.,
310 1.7-1.8). With increasing D_f , the influence of the random number decreases, which is reasonable because the
311 permeability of a model with only a few fractures can be affected more by the randomness of the trace length,
312 location and orientation of the fractures than models that contain many fractures (see Figs. 8(a) and 8(j)). By
313 taking the mean value of the results for each D_f , we obtained an approximate curve that indicates that the changes
314 in equivalent permeability with D_f follows an exponential law.

315 Fig. 10 shows the relation between the equivalent permeability and D_T with trace length ratios l_{min}/l_{max} of
316 2.0×10^{-4} , 1.0×10^{-4} and 0.5×10^{-4} , which were obtained using a constant $l_{min} = 0.05$ m and l_{max} values of 250 m, 500
317 m and 1000 m, respectively. With an increase of D_T , the equivalent permeability decreases because higher values
318 of D_T represent greater tortuosity and thus greater resistance to fluid flow in the fractures, which results in a lower
319 equivalent permeability. Fig. 10 also shows that as the trace length ratio l_{min}/l_{max} increases, the equivalent

320 permeability decreases. The increase of l_{min}/l_{max} will decrease the total number of fractures N_t for a given D_f of a
 321 DFN model. The smaller total number of fractures N_t results in worse connectivity of the network and decreases
 322 the equivalent permeability in turn. The three regression functions in Fig. 10 show that the changes in equivalent
 323 permeability with D_T follow exponential laws, which are affected by the trace length ratio.

324 The cubic law in the form of Eq. (17) is usually utilized for fluid flow in fractures by assuming that each
 325 fracture consists of two parallel walls. According to Eq. (16), the modified cubic law that considers the effect of
 326 D_T can be rewritten as

$$327 \quad q(i) = \frac{e_i^{2+D_T}}{12\mu} \frac{\Delta P}{l_i^{D_T}} \quad (28)$$

328 D_T is the source of the difference between Eq. (17) and Eq. (28), and its effects on DFN models can be
 329 estimated by the equivalent relative error $\bar{\delta}$:

$$330 \quad \bar{\delta} = \frac{Q_{cubic} - Q_{tortuosity}}{Q_{cubic}} \quad (29)$$

331 where Q_{cubic} is the fluid volume through the DFN model based on the parallel plate model (see Eq. (17)) and
 332 $Q_{tortuosity}$ is the fluid volume considering the tortuosity of the flow (see Eq. (28)).

333 Fig. 11 shows the relation between $\bar{\delta}$ and D_T at different values of D_f . The equivalent relative error $\bar{\delta}$
 334 increases with an increase in D_T and follows an exponential law. The values of $\bar{\delta}$ vary little with different
 335 random numbers at a given D_f , which indicates that the random number mainly represents the macro-scale
 336 properties of the fracture network. The four figures in Fig. 11 show that the values of $\bar{\delta}$ vary little among cases
 337 with different values of D_f , which shows that D_f is another macro-scale parameter that, along with the random
 338 number, determines the geometric characteristics of the fracture networks. In the cases with $D_T = 1.018$, which

339 corresponds to a JRC value of 20, the maximum values of $\bar{\delta}$ range from 17.64% to 19.51%, which indicates that
340 the effect of tortuosity is not negligible and should be included in fractal models to accurately estimate the
341 hydraulic behavior of fracture networks.

342 4 Conclusions

343 In this study, a fractal model was established to assess the equivalent permeability of 2-D rock fracture
344 networks. The fractal dimension D_T and the fractal dimension D_f were used in the model to represent the effects of
345 the tortuosity of fluid flow in the fractures (micro-scale) and the geometric characteristics of the fracture
346 distributions (macro-scale), respectively. Fluid flow was simulated in the generated fracture network models, and
347 the relation between the fractal dimension and the equivalent permeability was analyzed.

348 The results showed that the correlation of fracture number and fracture length based on the proposed fractal
349 length distribution in this study agrees well with reported values from the literature, which confirmed the
350 reliability of the proposed length distribution approach. Comparisons of the fractal dimension D_f between the
351 values calculated using the box-counting method and the theoretical values agree with each other, which verified
352 the validity of the fractal DFN models that were developed using the proposed fractal length distribution. When D_f
353 is small (e.g., less than 1.5), fluid flow mainly occurs in a few long fractures that are subparallel to the flow
354 direction and particularly in the fractures that intersect the inlet and outlet boundaries of the models. When D_f
355 exceeds a certain value (e.g., 1.5), the flow rate distribution becomes more homogeneous, and shorter
356 non-persistent fractures dominate the preferential flow paths. The equivalent permeabilities of models generated
357 using different random numbers vary significantly with changes of D_f when D_f is small (e.g., less than 1.5), and

358 they become more stable when D_f is relatively large (e.g., greater than 1.6). This behavior is consistent with the
359 observations of the flow paths, which show that the models become more homogeneous at larger values of D_f .
360 Therefore, a mathematical expression between the equivalent permeability K and the fractal dimension D_f (e.g.,
361 the exponential relationship presented in this study) can be expected to be applicable for models with large values
362 of D_f . For models with small values of D_f , other parameters, such as connectivity, should be taken into account to
363 improve the accuracy of the predictions. Compared with the parallel plate model, the maximum deviation of the
364 calculated flow volume that considers the effect of tortuosity (D_T) can be as high as 19.51% when $D_T = 1.018$,
365 which corresponds to a JRC value of 20. These results show that both the geometric characteristics of the fracture
366 distributions and the geometric characteristics (surface roughness) of single rock fractures (the source of
367 tortuosity) have significant influence on the hydraulic behavior of fracture networks. Further development of the
368 proposed model is required to estimate its scaling effects, which might have important impacts on the equivalent
369 permeability and were not considered in this study.

370

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375 **References:**

376 [1] Barenblatt GI, Zheltov YP, Kochina IK. Basic concepts in the theory of seepage of homogenous liquids in fissured rocks. Sov.
377 Appl. Math. Mech. 1960(24):852-864 (English Translation).

378 [2] Barenblatt GI, Zheltov YP. Fundamental equations of filtration of homogenous liquids in fissured rocks. Sov. Dokl. Akad. Nauk.
379 1960(13):545-548 (English Translation).

380 [3] National Research Council (US). Committee on Fracture Characterization, Fluid Flow. Rock fractures and fluid flow:
381 contemporary understanding and applications. Natl Academy Pr, 1996.

382 [4] Baghbanan A, Jing L. Hydraulic properties of fracture rock masses with correlated fracture length and aperture. Int J Rock Mech
383 Min Sci 2007;44(5):704-719.

384 [5] Koyama T, Neretnieks I, Jing L. A numerical study on differences in using Navier-Stokes and Reynolds equations for modeling
385 the fluid flow and particle transport in single rock fractures with shear. Int J Rock Mech Min Sci 2008;45:1082-1101.

386 [6] Zhao Z, Jing L, Neretnieks I, et al. Numerical modeling of stress effects on solute transport in fractured rocks. Computers and
387 Geotechnics, 2011, 38(2): 113-126.

388 [7] Liu R, Jiang Y, Li B, et al. Numerical calculation of directivity of equivalent permeability of fractured rock masses network. Rock
389 and Soil Mechanics, 2014, 35(8): 2394-2400. (in Chinese)

390 [8] Oliveira R, Graca JC. In situ testing of rocks. In: Bell FG, ed. Ground Engineer's Reference Book, London:
391 Butterworth-Heinemann 1987; 26/1-26/28.

392 [9] Hudson JA, Harrison JP. Engineering Rock Mechanics: An introduction to the Principles. Amsterdam: Elsevier; 1997.

393 [10] Yu B, Cheng P. A fractal permeability model for bi-dispersed porous media. International Journal of Heat and Mass Transfer.
394 2002(45):2983-2993.

395 [11] Zhang L. Engineering Properties of Rocks. Volume 4 of Elsevier Geo-Engineering Book Series. Amsterdam: Elsevier; 2005.

396 [12] Barton CC, Larsen E. Fractal geometry of two-dimensional fracture networks at Yucca Mountain, south-western Nevada. In:
397 Proceedings of the International Symposium on Fundamentals of Rock Joints. Bjorkliden, Sweden, 1985: 77-84.

398 [13] Barton CC, Hsieh PA. Physical and hydrologic-flow properties of fractures. Field Trip Guidebook, T385. 28th International
399 Geological Congress, Washington DC; 1989.

400 [14] La Pointe PR. A method to characterize fracture density and connectivity through fractal geometry. Int. J. Rock Mech. Min. Sci.
401 Geomech. Abstr. 1988(25):421-429.

402 [15] Kulatilake P, Shou G, Huang T H, et al. New peak shear strength criteria for anisotropic rock joints. International journal of rock
403 mechanics and mining sciences & geomechanics abstracts. Pergamon, 1995, 32(7): 673-697.

404 [16] Kulatilake P, Um J. Requirements for accurate quantification of self-affine roughness using the roughness-length method.
405 International Journal of Rock Mechanics and Mining Sciences, 1999, 36(1): 5-18.

406 [17] De Dreuzy J R, Davy P, Bour O. Hydraulic properties of two-dimensional random fracture networks following a power law
407 length distribution: 2. Permeability of networks based on lognormal distribution of apertures. Water Resources Research, 2001;37(8):
408 2079-2095.

409 [18] De Dreuzy J R, Davy P, Bour O. Hydraulic properties of two-dimensional random fracture networks following a power law
410 length distribution: 1. Effective connectivity. Water Resources Research, 2001;37(8): 2065-2078.

411 [19] De Dreuzy J R, Davy P, Bour O. Hydraulic properties of two-dimensional random fracture networks following power law
412 distributions of length and aperture. Water Resources Research, 2002;38(12): 1276.

413 [20] Yu B, Zou M and Feng Y. Permeability of fractal porous media by Monte Carlo simulations. International Journal of Heat and
414 Mass Transfer. 2005(48):2787-2794.

415 [21] Xu P, Yu B, Feng Y and Liu Y. Analysis of permeability for the fractal-like tree network by parallel and series models. *Physica A*.
416 2006(369):884-894.

417 [22] Bejan A, Lorente S. The constructal law and the evolution of design in nature. *Physics of Life Reviews*, 2011;8(3): 209-240.

418 [23] Bejan A, Zane J P. Design in nature: how the constructal law governs evolution in biology, physics, technology, and social
419 organization. Random House Digital, Inc.; 2013.

420 [24] Zou M, Yu B, Feng Y and Xu P. A Monte Carlo method for simulating fractal surfaces. *Physical A*. 2007(386):176-186.

421 [25] Jafari A and Babadagli T. Estimation of equivalent fracture network permeability using fractal and statistical network properties.
422 *Journal of Petroleum Science and Engineering*. 2012(92-93):110-123.

423 [26] Zheng Q, Yu B. A fractal permeability model for gas flow through dual-porosity media. *Journal of Applied Physics*.
424 2012:024316.

425 [27] Xu P, Yu B, Qiu S and Cai J. An analysis of the radial flow in the heterogeneous porous media based on fractal and constructal
426 tree networks. *Physica A*. 2008(387):6471-6483.

427 [28] Mandelbrot BB. *The Fractal Geometry of Nature*. Freeman, San Francisco. 1982: 15, 23-57, 117-119.

428 [29] Majumdar AA, Bhushan B. Role of fractal geometry in roughness characterization and contact mechanics of surfaces. *J. Tribol*;
429 1990;112(2):205-216.

430 [30] Stone D. Sub-surface fracture maps predicted from borehole data: an example from the Eye-Dashwa Pluton, Atikolan, Canada.
431 *Int J Rock Mech Min Sci Geomech Abstr* 1984;21(4):183-194.

432 [31] Vermilye JM, Scholz CH. Relation between vein length and aperture. *J Struct Geol* 1995;17(3):423-434.

433 [32] Hatton CG, Main IG, Meredith PG. Non-universal of fracture length and opening displacement. *Nature* 1994;367:160-162.

434 [33] Renshaw CE, Park JC. Effect of mechanical interactions on the scaling of fracture length and aperture Nature
435 1997;386:482-484.

436 [34] Baghbanan A, Jing L. Stress effects on permeability in a fractured rock mass with correlated fracture length and aperture. Int J
437 Rock Mech Min Sci 2008;45:1320-1334.

438 [35] Olsson R, Barton N. An improved model for hydromechanical coupling during shearing of rock joints. International Journal of
439 Rock Mechanics and Mining Sciences, 2001;38(3): 317-329.

440 [36] Cai J, Yu B, Zou M, et al. Fractal analysis of invasion depth of extraneous fluids in porous media. Chemical Engineering Science,
441 2010, 65(18): 5178-5186.

442 [37] Yu B, Lee L and Cao H. A fractal in-plane permeability model for fabrics, Polym. Compos. 2002(23):201-221.

443 [38] Itasca Consulting Group Inc. UDEC User's guide, ver 4.0, Minneapolis, Minnesota, 2004.

444 [39] Jiang Y, Tanabashi Y, Li B, Xiao J. Influence of geometrical distribution of rock joints on deformation behavior of underground
445 opening. Tunnelling and Underground Space Technology. 2006;21:485-491.

446 [40] Wakabayashi N, Fukushige I. Experimental study on the relation between fractal dimension and shear strength. In: Myer LR,
447 Cook NGW, Goodman RE, Tsang CF (eds) Fractured and jointed rock masses. Balkema, Rotterdam, 1995: 125–131.

448 [41] Barton N, Choubey V. The shear strength of rock joints in theory and practice. Rock mechanics, 1977;10(1-2):1-54.

449 [42] Jang B A, Jang H S, Park H J. A new method for determination of joint roughness coefficient. Proceedings of the IAEG;2006.

450 [43] Dverstorp B, Andersson J. Application of the discrete fracture network concept with field data: Possibilities of model calibration
451 and validation. Water Resources Research, 1989, 25(3): 540-550.

- 452 [44] Tsang Y W, Tsang C F, Hale F V, et al. Tracer transport in a stochastic continuum model of fractured media. *Water Resources*
453 *Research*, 1996, 32(10): 3077-3092.
- 454 [45] Bour O, Davy P. Connectivity of random fault networks following a power law fault length distribution. *Water Resources*
455 *Research*, 1997, 33(7): 1567-1583.

Fig. 1 Schematic view of fluid flow through a rough fracture. Tortuosity induced by fracture surface roughness makes the actual traveling length l_t of fluid flow larger than the straight length l .

Fig. 2 (a) Hydraulic boundary conditions applied to a fracture network. The quadrilaterals represent hydraulic pressures, and the hydraulic gradient was fixed at 1 kPa/m to generate horizontal fluid flow. (b) Fluid flow equilibrium of a node in a fracture network.

Fig. 3 The relation between the fractal dimension D_f and the mass density of fractures.

Fig. 4 Correlation of fracture number and fracture length with fractal dimensions D_f ranging from 1.3 to 1.8.

Fig. 5 Linear relation between the power law exponent a and the fractal dimension

D_f .

Fig. 6 Image processing and calculation of the fractal dimension.

Fig. 7 Comparisons of fractal dimension D_f between calculated values and theoretical values using 10 sets of DFN models for each D_f .

(a) DFN model ($D_f=1.3$)

(b) Flow rate ($D_f=1.3$)

Flow rates
Max flow rate = $9.739E-11$
Thickness of each line
= $3.000E-12$

(c) Remarks ($D_f=1.3$)

(d) DFN model ($D_f=1.4$)

(e) Flow rate ($D_f=1.4$)

Flow rates
Max flow rate = $2.430E-11$
Thickness of each line
= $1.000E-12$

(f) Remarks ($D_f=1.4$)

(g) DFN model ($D_f=1.5$)

(h) Flow rate ($D_f=1.5$)

Flow rates
Max flow rate = $6.006E-10$
Thickness of each line
= $2.500E-11$

(i) Remarks ($D_f=1.5$)

(j) DFN model ($D_f=1.6$)

(k) Flow rate ($D_f=1.6$)

Flow rates
Max flow rate = $1.110E-06$
Thickness of each line
= $5.000E-08$

(l) Remark ($D_f=1.6$)

Fig. 8 Geometric distributions of DFN models (a, d, g, j; left column), correlated flow rate distributions and flow paths (b, e, h, k; central column) and remarks (c, f, i, l; right column) with varying fractal dimensions, D_f , from 1.3 to 1.6.

Fig. 9 Equivalent permeabilities of models with D_f values from 1.3 to 1.8 generated using 10 sets of random numbers.

Fig. 10 Relations between equivalent permeability with the fractal dimensions D_T and the trace length ratios $l_{\min}/l_{\max}$.

(a) $D_f = 1.3$

(b) $D_f = 1.4$

(c) $D_f = 1.5$

(d) $D_f = 1.6$

Fig. 11 Relative errors of DFN models with D_T values from 1.000 to 1.018 and D_f values from 1.3 to 1.6.

Table 1 Review of fractal DFN models used to calculate the permeability of rock masses.

Authors	Year	Purpose of the model	Outcome of the study
<i>De Dreuzey et al.</i> [17-19]	2001a; 2001b; 2002	Investigation of the hydraulic properties of 2-D fracture networks with random fracture geometries that follow a power law length distribution.	They analyzed the influence of the power law exponent a in their models and found that if a was greater than 3, the classical percolation model based on a population of small fractures was applicable, and the fluid flow appeared to be relatively homogeneous in the flow direction. In contrast, if a was less than 2, the applicable model was made up of the largest fractures of the network, and the main flow paths were composed of a few large fractures. Between the two limits ($2 < a < 3$), relatively uniform fluid flow occurred in all of the fractures.
<i>Yu et al.</i> [10]	2002	Development of a fractal model to calculate the equivalent permeability of bi-dispersed porous media.	They extensively evaluated the influences of the fractal dimension D_f (which represents the fracture distribution) and D_T (which represents tortuosity) on the equivalent permeability. They found that the equivalent permeability was a function of the tortuous fractal dimension, pore area fractal dimension, sizes of particles and clusters, micro-porosity inside clusters, and the effective porosity of a medium.
<i>Yu et al.</i> [20]	2005	Establishment of a 2-D fractal model to calculate the permeability of a porous media model generated by the Monte Carlo method.	Their model can predict the transport properties (i.e., permeability, thermal conductivity, dispersion coefficient and electrical conductivity) of saturated or unsaturated fractal porous media.
<i>Xu et al.</i> [21]	2006	Development of a fractal model for fluid flow in porous media that are	They found that the permeability of the model that incorporated the flow tortuosity was

		embedded with randomly distributed fractal-like tree networks using the constructal theory proposed by Bejan and Lorente [22] and Bejan and Zane [23].	approximately 20% lower than the model that did not consider the tortuosity.
<i>Zou et al.</i> [24]	2007	Establishment of a fractal model to analyze the 3-D surfaces of rock fractures.	Their results indicated that larger values of the fractal dimension D of the profile of a rough surface or smaller values of the scaling constant G signify a smoother surface topography.
<i>Jafari and Babadagli</i> [25]	2012	Estimation of the equivalent permeability of fracture networks using numerical simulations	They derived a nonlinear multivariable regression to address the equivalent permeability by calculating three parameters: $X1$ (the connectivity index), $X2$ (the box-counting fractal dimension of the fracture lines) and $X3$ (the hydraulic conductivity).
<i>Zheng et al.</i> [26]	2012	Development of a fractal model for fluid flow in porous media that are embedded with randomly distributed fractal-like tree networks using the constructal theory proposed by Bejan and Lorente [22] and Bejan and Zane [23].	They derived an analytical expression for the gas permeability in dual-porosity media based on the pore size of the matrix and the diameter of the mother channel of embedded fractal-like tree networks. They found that for a certain fracture network, the dimensionless permeability K^+ , which is defined as the ratio of the porous matrix permeability K_m to the fracture network permeability K_f , increases with increasing matrix porosity ε .

Table 2 Parameters associated with a model size of 5.0 m×5.0 m.

D_f	d_m (m/m ²)	L_t (m)	N_t	L_n (m)	N_t'
1.3	11.6905	2.8276	89	5.0	278
1.4	20.3656	2.6331	125	5.0	450
1.5	32.7688	2.7023	177	5.0	606
1.6	50.5020	2.6907	251	5.0	866
1.7	75.8559	2.7149	354	5.0	1200
1.8	112.1052	2.8814	501	5.0	1508

Table 3 Calculation results based on the proposed fracture length distribution (Eq. (11)).

D_f	Case	Value (α)	Average (α)	Value (a)	Average (a)	Value (R^2)	Average (R^2)
1.3	Case_1	155.41		-2.3231		0.8962	
	Case_2	157.12	145.25	-2.3820	-2.2888	0.9176	0.8851
	Case_3	123.22		-2.1613		0.8414	
1.4	Case_1	129.65		-1.8506		0.7713	
	Case_2	127.05	138.49	-1.9661	-1.9681	0.7851	0.8012
	Case_3	158.78		-2.0876		0.8471	
1.5	Case_1	207.02		-1.8478		0.8135	
	Case_2	197.12	224.34	-1.9599	-1.9792	0.8128	0.8445
	Case_3	268.89		-2.1298		0.9073	
1.6	Case_1	263.01		-1.9245		0.8093	
	Case_2	299.36	265.80	-1.9525	-1.8626	0.8774	0.8308
	Case_3	235.04		-1.7109		0.8057	
1.7	Case_1	243.88		-1.7332		0.7176	
	Case_2	432.56	334.92	-1.9410	-1.8163	0.8958	0.7949
	Case_3	328.32		-1.7746		0.7713	
1.8	Case_1	457.16		-1.7395		0.8026	
	Case_2	315.56	357.35	-1.6771	-1.7016	0.7285	0.7557
	Case_3	299.33		-1.6881		0.7360	

Table 4 Comparisons between the values of the power law exponent a in this and other studies.

Authors	Year	Value of the power law exponent a
Dverstop and Anderson	1989	1.7
Tsang et al.	1996	3.0
Bour and Davy	1997	1.0 – 3.0
De Dreuzy	2001	0.0 – 3.5
This study		1.49 – 2.49