Formulation of the quantitative influence of local and overall initial deflections on load-bearing capacity of unstiffened welded square box section columns under axial compression

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Abstract

Although it is well known that initial deflections have a significant influence on load-bearing capacity of unstiffened square box section columns under pure compression, their quantitative influence is not clear. In this study, 378 finite element models were analyzed to reveal the quantitative influence of local and overall initial deflections on load-bearing capacity. In the parametric analyses, nine combinations of the amplitude of the local and overall initial deflections were considered. In addition, normalized width-thickness and slenderness ratios were varied with ranges from 0.3 to 1.8 and from 0.3 to 1.5, respectively. As a result, the initial deflection influence coefficient was proposed to describe the quantitative influence on load-bearing capacity and formulated as functions of the local and overall initial deflections, normalized width-thickness and slenderness ratios. The accuracy of the proposed formula was verified through the comparison with the FEA results including additional 108 FEA results with normalized slenderness ratio of 0.9 and 1.2.

Keywords: coupled buckling; load-bearing capacity; initial deflections; unstiffened square welded box section; axial compression; parametric formulae.

23 1. Introduction

24 Thin-walled compression members suffer from buckling instability due to high slenderness and width-
25 thickness ratios [1,2,3,4]. It is also well known that initial deflections have a significant influence on
26 buckling instability behavior [5,6]. Initial deflections, which consist of the local initial deflection on the
27 plate and the overall initial deflection on the column, appear during the manufacturing and assembling. In
28 addition, the amplitude is uncertain.

29 Many researchers had measured the amplitude of initial deflections of box section columns. In Usami's
30 experiment [7], amplitudes of overall initial deflection on 19 welded box section columns were measured.
31 The maximum was $L/1590$ and average was $L/3850$, where L was the column length. In Rasmussen's
32 research [8] with six welded box section columns measured, their maximum and average amplitudes were
33 $L/546$ and $L/1172$, respectively. Somodi and Kövesdi [9] had measured initial deflections on 49 welded
34 steel columns made of various steel grades. The results show that a total of 23 columns had overall initial
35 deflection amplitudes larger than $L/1000$ and the average value of total 49 columns is $L/945$. In Degée's
36 research [10], the maximum and average were $L/2500$ and $L/3560$ for the overall initial deflection,
37 respectively; the maximum value was $b/250$ for the local initial deflection where b was the width of the
38 plate. In Shi's experiment [11], amplitudes of local initial deflection on four welded stub columns were
39 measured. The maximum and average were $b/341$ and $b/512$, respectively. However, these values are much
40 smaller than those allowed in some specifications such as $L/1000$ of Chinese [12], American [13], Japanese
41 [14], specifications and $b/150$ of Japanese [14].

42 Many numerical studies aimed at investigating the influence of the initial deflection have been
43 contributed. Kwon [15] investigated the effect of local initial deflection on load-bearing capacity of welded

44 RHS columns undergoing coupled buckling. The increase of the local initial deflection from $b/2000$ to
45 $b/100$ resulted in the reduction of the load-bearing capacity about 28.5%. In Inose's study [16], the increase
46 of the initial deflection from $b/450$ to $b/150$ led to the reduction of load-bearing capacity of welded box
47 section columns up to 11.4%. Lu Yang [17] emphasized that the local initial deflection had the more
48 obvious influence on the structure with high normalized width-thickness ratio.

49 Ban [18] studied the influence of the overall initial deflection on the overall buckling behavior of
50 welded box section steel columns. The increase of overall initial deflection from $L/2500$ to $L/1000$ resulted
51 in a decrease of load-bearing capacity by about 7% on average. Coelho [19] conducted a series of numerical
52 analyses to investigate the sensitivity of the load-bearing capacity to imperfection. Models with four
53 different amplitude of the overall initial deflection, which are $L/1000$, $L/500$, $L/250$ and $L/125$, were
54 analyzed. The increase of the overall initial deflection resulted in the 40% decrease of load-bearing capacity.
55 Kang [20] studied the effect of overall initial deflection on load-bearing capacity of welded steel box
56 columns by selecting two different values i.e. $L/1000$ and $L/500$ with the normalized slenderness ratios
57 from 0.8 to 1.8. The results revealed that with the increase of the overall initial deflection, the average and
58 maximum of reduction on load-bearing capacity were about 7.2% and 9.4% respectively. Moreover, the
59 influence of the overall initial deflection showed to be related to the normalized slenderness ratio. The
60 maximum reduction appeared at the normalized slenderness ratio of 1.25.

61 Referring to the previous works, it can be concluded that the local initial deflection shows significant
62 influence on the load-bearing capacity of the structure with high normalized width-thickness ratio, while
63 the overall initial deflection shows more effects on the slender compression members. In addition, such a
64 structure tends to be sensitive to initial deflection i.e. a tiny changing of the amplitude of the initial

deflection will lead to a significant influence on load-bearing capacity. More importantly, the sensitivity of the structure to the initial deflection is related with the normalized width-thickness and slenderness ratios.

Although many experiments and numerical analysis have been conducted, the quantitative influence of the initial deflections of such structure is still not clear. In this study, unstiffened welded square cross-section columns with S355 steel under axial compression were chosen as a target. Nonlinear finite element models were developed to analyze their load-bearing capacity. A variety of the normalized width-thickness and slenderness ratios were considered to cover the possible diverse range of columns. Various combinations of amplitude of the local and overall initial deflections were prepared for FE models. Based on the results of parametric analyses, initial deflection influence coefficient is proposed as a function of the initial deflections and the normalized width-thickness and slenderness ratios to describe the quantitative influence caused by the difference in amplitude of the local and overall initial deflections on the load-bearing capacity.

2. Validation of FE modelling

2.1 Existing experiment on unstiffened box section columns

Degée et al. conducted the experiments of six columns (S-series) [10]. Two specimens (W-series) were tested by Pavlovčič et al. [21]. Somodi and Kövesdi [9] also carried out twelve column tests; ten of them (W3-series) were introduced in this paper except two columns due to unknown residual stress value. All of the above-mentioned specimens were made of S355 steel. To further cover wider ranges of width-thickness and slenderness ratios, eight specimens (R-series) made of HT80 steel having high width-thickness and slenderness ratios tested by Usami [7] were also considered in FE model validation. Cross-section is presented in Fig.1 and dimensions of all specimens are listed in Table.1. Two specimens were prepared for

each dimension except Pavlovčič's test. It should be noted that in Somodi's test, plate thickness of some A-specimen is a little different from B-specimen, the upper value is for the A-specimen and the other is for the B-specimen. The parameters λ and R are defined as Eq. (1) and (2), respectively,

$$\lambda = \frac{1}{\pi} \sqrt{\frac{\sigma_y}{E} \frac{L}{r}} \quad (1)$$

$$R = \frac{b}{t} \sqrt{\frac{12(1-\nu^2)}{k\pi^2} \frac{\sigma_y}{E}} \quad (2)$$

where λ is the normalized slenderness ratio, R is the normalized width-thickness ratio; σ_y is the nominal yield stress; E is Young's Modulus; L is the length of the column; r is the radius of gyration; b is the width of the plate; t is the thickness of the plate; ν is the Poisson's ratio; and k is the buckling coefficient.

The initial deflection was measured prior to the experiment as listed in Table.2. The maximum measured local initial deflection on the plate was $b/250$ in Degée's test, $b/520$ in Pavlovčič's test while local initial deflection was not measured for W3-series and R-series specimens. The overall initial deflection for W3-series specimens listed in Table.2 is the sum of out-of-straightness deflection and load eccentricity. The columns were tested under the pin-ended condition which can rotate only around one axis.

2.2 FE modelling

The general-purpose FEA software MSC.Marc was applied to the numerical analyses of load-bearing capacity of unstiffened box section columns. Thick shell element (No.75) [22] was used in the model. Since the meshing size and the shape of the elements have an influence on the accuracy of the calculation [23], the shape of the elements was made to be as square as possible and meshing size was made to be small enough. Moreover, considering the convenience of residual stress setting, plates were divided into twenty elements for parametric analysis as shown in Fig.2 (a). The von Mises yield criterion was adopted in the analysis. Bilinear model with strain hardening coefficient of $E/100$ was used to describe the material

property of S355, and trilinear model based on measured material property [7] was used for R-series specimens. The Poisson ratio was set to 0.3 and 0.24 for S355 and HT80, respectively.

In order to set the boundary conditions, two nodes were set at the midpoint of the top and bottom cross-sections. The central node was set as the main node and the other nodes in the cross-section as subordinate nodes. Then the “RBE2” function [22] was used to connect the central node (master node) with the whole nodes at the cross-section (slave nodes). The RBE2 function can make a rigid link between the master node and a list of slave nodes. The nodes were located away from the column in order to take the dimension of the test equipment into consideration as shown in Fig.2 (b). Considering that the columns were tested with pin-ended supports around the weak axis “y”, the rotation Y was free at both top and the bottom node as shown in Table.3. At the top node, displacement Z was set free so that the displacement loading can be applied to the top main node.

Residual stress is also one of the most important factors to the stability behavior of compression members. It was assumed to exist in every plate with compression in the middle area and tension at both sides of the plate as shown in Fig.3 to be in self-equilibrium condition. Values of tensile and compressive residual stresses for each specimen were set as listed in Table.4. Since less of conclusive residual stress value in Degée’s research [10], a calibrated distribution (i.e. Model 6 in [10]), which resulted in the closest load-bearing capacity to experimental results, was used for specimens S1, S2 and S3. For the specimens tested by Somodi [9] (W3 series), the tensile and compressive residual stresses were determined referring to the values measured for specimens with the same cross section by Somodi [24]. For W-series specimens, measured compressive residual stress [21] of 130MPa ($0.35\sigma_y$) and 160MPa ($0.43\sigma_y$) for flange and web, respectively, were used. For R-series specimens, based on measurement, 0.6 and 0.1 of yield strength was

128 arranged for tensile and compressive residual stress, respectively. Although it is known that residual stresses
 129 are b/t ratio dependent values, one residual stress distribution is assumed in the parametric analysis in order
 130 to independently evaluate the influence of initial deflections on load-bearing capacity. Referring to
 131 Fukumoto's statistics [26], assumed compressive stress was set to $0.25\sigma_y$, which is the average value of 216
 132 cases. The residual stress distribution input in every plate assumed for FE model in parametric analysis is
 133 shown in Fig.4.

134 Images of overall and local initial deflection shapes are shown in Fig.5. Half-sinusoidal wave shape
 135 expressed by Eq. (3) was assumed for the overall initial deflection. The double trigonometric function
 136 shown in Eq. (4) was assumed for the local initial deflection. The number of half-wave m were determined
 137 by the aspect ratio of the plate to give the minimum buckling strength according to the elastic buckling
 138 theory. Schematic illustration of initial deflections in the case of the specimen with $m=3$ is shown in Fig.6
 139 as an example. The measured amplitude of the initial deflections is used for the FE modelling. For W3-
 140 series and R-series specimens, the local initial deflection was assumed to be $b/200$ according to Eurocode
 141 3 [25] since it had not been measured.

142

$$143 \quad w_g = w_{g0} \sin \frac{\pi z}{L} \quad (3)$$

$$144 \quad \begin{cases} w_l = w_{l0} \sin \frac{m\pi z}{L} \cos \frac{\pi x}{b} \\ w_l = w_{l0} \sin \frac{m\pi z}{L} \cos \frac{\pi y}{b} \end{cases} \quad (4)$$

145

146 where w_{g0} is the amplitude of overall initial deflection; w_{l0} is the amplitude of local initial deflection; m is
 147 the number of half-sinusoidal wave giving the minimum buckling strength determined by the aspect ratio

148 of the plate.

149 2.3 Validation of the modelling

150 All the 26 columns listed in Table 1 were modeled, and their ultimate strengths were obtained. The
151 comparison between experiment and FEA results is shown in Fig.7 and Table 5. It can be seen that most of
152 the FEA results for Degée's test (S-series) were a little lower than experimental results. The possible reason
153 is that local initial deflection used in FE models is the measured maximum, which is greater than the real
154 value for most of the specimens. For Pavlovčič test (W-series), the FEA results shows good agreement with
155 test results. For Smodi's test (W3-series), numerical analysis provides good prediction on load-bearing
156 capacity with a little larger deviation than that of Pavlovčič test. It can be observed that the difference
157 between FEA and test results of W3-80-5-2000A reached up to 11.89% while difference on W3-80-5-2000B
158 was small. Possible reason is that the same residual stresses and assumed local initial deflection were
159 introduced for these two specimens, while their values may be different in tested specimens which resulted
160 in 15.6% difference on load-bearing capacity (0.74 and 0.64) between them. For R-series specimens with
161 high width-thickness ratios, it can be observed that FE results are slightly lower than test results due to the
162 relatively large local initial deflection assumed. The average and standard deviation of errors for all
163 specimens are 2.50% and 4.07%, respectively. Based on the above discussion, it was concluded that the
164 model could be used in the following parametric analysis with sufficient accuracy.

165 3. Parametric analysis

166 3.1 Combination of the parameters

167 To evaluate the quantitative influence of the initial deflections on the load-bearing capacity, 378
168 models with different normalized slenderness ratio, normalized width-thickness ratio, and amplitude of

169 local and overall initial deflections were prepared and analyzed. The dimension of cross-section, the
170 normalized slenderness ratio λ and normalized width-thickness ratio R are listed in Table 6. Seven different
171 normalized slenderness and six width-thickness ratios are adopted with the range from 0.3 to 1.8 and from
172 0.3 to 1.5, respectively, to cover a main part of the practice. Table 7 lists the combinations of the normalized
173 slenderness and width-thickness ratios.

174 For evaluating the quantitative influence of the initial deflections on the load-bearing capacity, nine
175 combinations of the amplitude of the local and overall initial deflections listed in Table 8 were considered.
176 The Combi5 corresponds to the combination of the allowable initial deflections in the Japanese
177 specification [14].

178 *3.2 Influence of the local initial deflection*

179 Results of models with the normalized width-thickness ratio R from 0.3 to 1.5 and constant normalized
180 slenderness ratio λ of 1.0 are set as examples to show the influence of the local initial deflection. The
181 comparison of load-bearing capacities for different local initial deflection are shown in Fig.8. In the range
182 of $R \leq 0.5$, the reduction on load-bearing capacity caused by increasing local initial deflection is less than
183 5%. In the range of R from 0.7 to 1.5, it reaches up to 19.2% at most with the increase of the amplitude of
184 local initial deflection from $b/450$ to $b/75$.

185 In this study, initial deflection influence coefficient (IDIC) is introduced to describe the quantitative
186 influence of the initial deflections and defined as a ratio of the load-bearing capacity of a model with each
187 Combi to that of the Combi5. The calculation of $IDIC_{FEA}$ based on FEA results can be performed as follows.

$$188 \quad IDIC_{FEA} = \frac{F_{u,Combi-i}}{F_{u,Combi-5}} \quad (5)$$

189 where the F_u is the load-bearing capacity of the specimen.

Relationship between the IDIC and the amplitude of the local initial deflection normalized by the standard amplitude i.e. $b/150$ is shown in Fig.9. It can be seen that IDIC decreases approximately linearly as the amplitude of local initial deflection increases. The decreasing slope is nearly the same for different amplitudes of the overall initial deflection. Therefore, linear regression analysis is conducted on the data when the amplitude of the overall initial deflection is $L/1000$.

The decreasing slope α is related to normalized width-thickness ratios as shown in Fig.10, in which seven series of data with different normalized slenderness ratios are plotted. It can be seen that the decreasing slope increases first and then decreases with the increase of the normalized width-thickness ratio. Since the decreasing slope represents the reduction of load-bearing capacity by local initial deflection, the peak points show the width-thickness ratio where the reduction reaches the maximum. In addition, the peak point of the decreasing slope α tends to move to larger width-thickness ratio with the increase of normalized slenderness ratios. When the normalized width-thickness ratio R is small (i.e., $R \leq 0.5$), the influence of the local initial deflection seems to be slight. In the range of R from 0.7 to 1.5, column strength becomes comparatively sensitive to the local initial deflection. When normalized slenderness ratio λ is small (i.e., $\lambda \leq 0.5$), the influence reaches the maximum with the decreasing slope about 0.1 at $R=0.7$. It corresponds to the 17.2% decrease of the load-bearing capacity. When λ is large (i.e., $\lambda \geq 1.0$), the load-bearing capacity of columns with higher R -value becomes sensitive to the local initial deflection. With R and λ equal to 1.2 and 1.8, respectively, the influence of local initial deflection reaches the maximum with the decreasing slope of 0.1375 corresponding to the 20.7% decrease of load-bearing capacity.

Failure modes of all FE models have been checked. They are listed in Table.9. The symbols of “O” and “C” in the table represent overall buckling and coupled buckling, respectively. The typical buckling

failure modes are demonstrated in Fig.11 taking two models as examples. The deformation of the columns is detailed at two points. One is at the point of load-bearing capacity, and the other is at the 85% of the load-bearing capacity after peak value. It can be recognized that coupled buckling or overall buckling occurs for 12 combinations of width-thickness and slenderness ratios indicated by “C/O” depending on the amplitudes of initial deflections. For these combinations, the failure modes are detailed in Table 10. The failure mode of R05λ05-07 and R07λ14-18 changes from overall buckling to coupled buckling with the increase of local initial deflection from $b/450$ to $b/150$. Failure mode change of R05λ09-18 occurs when increasing initial deflection from $b/150$ to $b/75$. This indicates that the amplitude of local initial deflection could change the failure mode in the normalized width-thickness ratio range from 0.5 to 0.7. The failure mode change by the local initial deflection could result in the larger reduction of load-bearing capacity by it. Therefore, it could be thought that this change is a cause inducing the movement of peak point of decreasing slope.

3.3 Influence of the overall initial deflection

Results of models with the normalized slenderness ratios λ from 0.3 to 1.8 and constant normalized width-thickness ratio R of 0.9 are set as examples to show the influence of the overall initial deflection. The comparison of load-bearing capacities for different overall initial deflection are shown in Fig.12. In the case of $\lambda=0.3$, the reduction on load-bearing capacity by increasing overall initial deflection is less than 5%. As the slenderness ratio increases, reduction on the load-bearing capacity reaches up to 15.1% at most with the amplitude of overall initial deflection increases from $L/3000$ to $L/500$.

Relationship between the IDIC and the amplitude of the overall initial deflection normalized by the standard amplitude i.e. $L/1000$ is shown in Fig.13. The similar tendency to Fig. 9 can be seen, namely, the IDIC decreases approximately linearly as the amplitude of overall initial deflection increases, and the

232 decreasing slope is nearly the same for different amplitudes of the local initial deflection.

233 Linear regression analysis is conducted on the data when the amplitude of the local initial deflection
234 is $b/150$. The absolute values of the decreasing slope β is related to the normalized slenderness ratio λ as
235 shown in Fig.14 in which six series of data with different normalized width-thickness ratio R are plotted. It
236 can be seen that the decreasing slope β increases first and then decreases with the increase of the λ -value.
237 This tendency is similar to Fig. 10. However, being different from Fig. 10, the peak value of the decreasing
238 slope appears at almost the same λ -value of 1.4. As shown in Table 10, the amplitude of overall deflection
239 does not change the failure mode. This may be a cause of the difference.

240 With λ from 0.7 to 1.8, column load-bearing capacity becomes comparatively sensitive to overall initial
241 deflection. The maximum decreasing slope ranges from 0.081 to 0.1166. In addition, the influence of overall
242 initial deflection on load-bearing capacity is related to normalized width-thickness ratio R . With the
243 decrease of R , the influence of overall initial deflection increases. This is because column strength is mainly
244 dominated by overall buckling when R is small. With R and λ equal to 0.3 and 1.4, respectively, the influence
245 reaches the maximum with the decreasing slope of 0.1166 corresponding to the 17.9% decrease of load-
246 bearing capacity.

247 **4. Formulation of the initial deflection influence coefficient**

248 *4.1 Proposed formulae*

249 In the previous chapter, the quantitative influence caused by the initial deflections on the load-bearing
250 capacity was observed with a wide range of the normalized slenderness and width-thickness ratios taken
251 into consideration. The value of IDIC tends to decrease linearly as the local or overall initial deflections
252 increase. The decreasing slope caused by the local/overall initial deflection is nearly invariable regardless

of the amplitude of the overall/local initial deflection. This fact may suggest that there is no significant coupling effect on IDIC between two initial deflections. Therefore, the IDIC can be calculated as,

255

$$\text{IDIC} = f(R, \lambda) * (I_p - 1) + g(\lambda, R) * (I_c - 1) + 1 \quad (6)$$

257

In the formula, $f(R, \lambda)$ is the decreasing slope α , $g(R, \lambda)$ is the decreasing slope β ; I_p is the local initial deflection normalized by the standard local initial deflection ($b/150$); I_c is the overall initial deflection normalized by the standard overall initial deflection ($L/1000$). The terms, I_p, I_c minus 1, are set to satisfy the condition that the IDIC is equal to 1 when the local and overall initial deflections are $b/150$ and $L/1000$, respectively.

263 4.2 Decreasing slope α

From the numerical results presented in Fig.10, it can be seen that the influence of local initial deflection is related with both normalized width-thickness ratio and slenderness ratio. As the width-thickness ratio increases, the decreasing slope increases first then decreases after the peak value. In addition, the peak value of the decreasing slope α appears at different normalized slenderness ratio. Referring to Fig. 10, the decreasing slope α is assumed as Eq. (7),

269

$$f(R, \lambda) = p_1 * R^{(p_2 + p_3 * R)} \quad (7)$$

271

where R and λ is normalized width-thickness and slenderness ratios, respectively.

Coefficients p_1, p_2, p_3 and their determination coefficient are obtained as listed in Table 11 through the

274 regression analysis of the above numerical results with Levenberg-Marquardt method. It is understood that
 275 the coefficients p_1, p_2, p_3 can be associated with the normalized slenderness ratio. Relationship between the
 276 coefficients and the normalized slenderness ratio is plotted in Fig.15. The authors tried several functions
 277 including linear, polynomial, logarithmic, exponential, trigonometric functions for the regression analysis.
 278 As a result, the polynomial regression was adopted since it provided comparatively accurate prediction and
 279 the formula format is simpler than others. The obtained formulae to estimate each coefficient are shown in
 280 Eq. (8).

281

$$282 \quad \begin{cases} p_1 = -0.0384\lambda^4 + 0.2037\lambda^3 - 0.3362\lambda^2 + 0.1511\lambda - 0.0958 \\ p_2 = -5.8278\lambda^5 + 35.7294\lambda^4 - 80.1948\lambda^3 + 78.3656\lambda^2 - 30.3199\lambda + 7.0455 \\ p_3 = -6.1618\lambda^4 + 24.2455\lambda^3 - 30.0485\lambda^2 + 14.1142\lambda - 7.6727 \end{cases} \quad (8)$$

283

284 4.3 Decreasing slope β

285 From the numerical results presented in Fig.14, it can be seen that the influence of overall initial
 286 deflection is also related to both normalized width-thickness ratio and slenderness ratio. As the slenderness
 287 ratio increases, the decreasing slope increases first, then decreases after the peak value. In addition, the peak
 288 value of the decreasing slope appears at almost the same λ -value of 1.4. The cubic function is assumed for
 289 the decreasing slope β as Eq. (9),

290

$$291 \quad g(\lambda, R) = q_1(\lambda - 1.4)^3 + q_2(\lambda - 1.4)^2 + q_3 \quad (9)$$

292

293 where R and λ are normalized width-thickness and slenderness ratios, respectively.

294 Coefficients q_1, q_2, q_3 and determination coefficient are obtained as listed in Table 12 through the

295 regression analysis of the above numerical results. It is understood that the coefficients q_1, q_2, q_3 can be
 296 associated with the normalized width-thickness ratio. Relationship between the coefficients and the
 297 normalized width-thickness ratio is plotted in Fig.16. By means of the polynomial regression, the formulae
 298 to estimate each coefficient are obtained as follows.

299

$$300 \quad \begin{cases} q_1 = 0.037R^3 - 0.0716R^2 - 0.0126R + 0.0635 \\ q_2 = 0.0626R^3 - 0.16R^2 + 0.0584R + 0.1264 \\ q_3 = -0.0252R^3 + 0.0659R^2 - 0.0228R - 0.1108 \end{cases} \quad (10)$$

301

302 **5. Verification of the proposed formulae**

303 *5.1 Comparison between formulae and FEA results*

304 The IDICs obtained by the proposed formulae were compared with those by FEA for 378 models to
 305 verify the accuracy of the formulae. The comparisons are divided into seven figures by the normalized
 306 slenderness ratios as shown in Fig.17 (a)-(g). Additional 108 FEA results with normalized slenderness ratio
 307 of 0.9 and 1.2, which are not used in formula fitting analysis, are considered in formula verification
 308 procedure as shown in Fig.18. The absolute value of maximum errors and average errors between formulae
 309 results and FEA results are shown in Table 13 and Table 14, respectively. It can be seen that most of the
 310 formulae results show good agreement with FEA results with maximum errors less than 5%. Only in four
 311 cases, the formulae results are found with errors more than 5%, but less than 7%. These relatively large
 312 errors are caused by the fact that the influence tendency of initial deflections is not strictly linear. Although
 313 the accuracy can be improved by considering the quadratic function on the influence tendency, the authors
 314 do not think that it is necessary due to accurate prediction results in most cases and more complicated

315 calculation by using quadratic function.

316 Therefore, it can be concluded that the proposed IDIC formulae have sufficient accuracy and reliability
317 for estimating the quantitative influence of the initial deflections on the load-bearing capacity of the
318 unstiffened welded box section columns.

319 5.2 Comparison between formulae and experimental results

320 In Kishi's study [26], unstiffened box section members with a variety of the normalized slenderness
321 and width-thickness ratios were focused as a target. The standard initial deflections i.e. $b/150$ for the local
322 initial deflection and $L/1000$ for the overall initial deflection were applied. Based on numerical analysis, a
323 simplified formula Eq. (11) to predict the load-bearing capacity was proposed.

324

$$325 \quad \sigma_{cr}/\sigma_u = 1 - (0.0502\lambda^2 - 0.2485\lambda + 0.6077)(R - 0.5) \quad (11)$$

$$326 \quad \sigma_u/\sigma_y = \begin{cases} 1 & \lambda \leq 0.2 \\ 1.059 - 0.258\lambda - 0.190\lambda^2 & 0.2 < \lambda \leq 1.0 \\ 1.427 - 1.039\lambda + 0.223\lambda^2 & \lambda > 1.0 \end{cases} \quad (12)$$

327

328 where σ_{cr} is the load-bearing capacity; σ_u is the overall buckling strength of the column with the normalized
329 slenderness ratio of λ specified in the specification [14]; and R is the normalized width-thickness ratio.

330 In this study, formulae to estimate the quantitative influence caused by the initial deflection are
331 proposed. The product between the proposed formulae in this study and the existing simplified formula
332 from Kishi will be the load-bearing capacity of the columns with actual amplitude of initial deflections.
333 The comparison between current formulae and experimental results [10, 21] is shown in Table 15. Since
334 the local initial deflection was not measured in Somodi's test [9], the test results of W3-series specimens
335 were not used in this part. It can be seen that the existing formula considering initial deflections of $b/150$

and $L/1000$ comparatively underestimates the load-bearing capacity for all specimens. The product between existing formula and IDICs provides better prediction on the load-bearing capacity.

6. Influence of residual stress on initial deflection influence coefficient

For the welded columns, residual stress would have significant influence on their load-bearing capacity. According to previous statistics on residual stress of 216 normal strength steel columns by Fukumoto [26], compressive residual stress varied from 0 to 0.6. In this section, to investigate the influence of residual stress on the initial deflection influence coefficient, specimens with short, medium and high columns length were chosen for FE analysis with compressive residual stress $0.1\sigma_y$ and $0.4\sigma_y$. As mentioned in 3.2 and 3.3, the initial deflection influence coefficient (IDIC) linearly decreases as the amplitude of local/overall initial deflection increases. The decreasing slope is nearly the same for different amplitudes of overall/local initial deflection. Therefore, the IDICs are plotted with normalized local initial deflection under the overall initial deflection of $L/1000$ as shown in Fig.19. Slight influence of residual stress on the relationship between IDICs and local initial deflection can be observed. In general, the slope of the plot for smaller residual stress is steeper, indicating larger influence of local initial deflection. For the columns with high slenderness ratio, the difference on IDICs becomes up to approximately 5% with the increase of residual stress from $0.1\sigma_y$ to $0.4\sigma_y$. The relationship between IDICs and normalized overall initial deflection is plotted under the local initial deflection of $b/150$ as shown in Fig.20. No clear influence of residual stress can be found on the relationship between IDICs and normalized overall initial deflection.

7. Conclusions

In this study, 486 models were analyzed to investigate the quantitative influence of the initial deflections on the load-bearing capacity of unstiffened welded square box section columns with a variety

357 of normalized slenderness and width-thickness ratios. The main conclusions can be summarized as follows.

358 (1) The FE model established to predict the load-bearing capacity was verified through the comparison
359 with experimental results. The FEA results show good agreement with the experimental results with the
360 errors less than 6%.

361 (2) The load-bearing capacity linearly decreases as the amplitude of local/overall initial deflection
362 increases. The decreasing slope is nearly the same for different amplitudes of overall/local initial
363 deflection.

364 (3) With the normalized width-thickness and slenderness ratios equal to 1.2 and 1.8, respectively, the
365 influence of the local initial deflection reaches the maximum with the decreasing slope of 0.1375
366 corresponding to the 20.7% reduction on load-bearing capacity.

367 (4) With the normalized width-thickness and slenderness ratio equal to 0.3 and 1.4, respectively, the
368 influence of the overall initial deflection reaches the maximum with decreasing slope of 0.1166
369 corresponding to the 17.9% reduction on load-bearing capacity.

370 (5) Based on numerous FEA results, the formulae for the initial deflection influence coefficient (IDIC)
371 are developed, which describe the quantitative influence caused by the initial deflections. Compared with
372 the FEA results, the accuracy of the formulae is verified.

373 (6) Compared with the experimental results, it is verified that the product between the IDIC and the
374 simplified formula can predict the load-bearing capacity with good accuracy in the case of columns with
375 various amplitudes of the initial deflections.

376 (7) The residual stress has slight influence on the relationship between IDICs and local initial
377 deflection, while it has negligible influence on the relationship between IDICs and overall initial deflection.

The validity of the proposed formulae has been evaluated only for the conditions examined in this research. Their applicability to different cross-sections, residual stress pattern and high strength steel columns as well as wider range of initial deflection value should be studied in the future work.

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Annex: An example to apply show the application procedure of the proposed formulae

Test specimen W-S in Table 1 is set as an example to improve the accuracy of prediction on the load-bearing capacity. The basic parameter of W-S is tabulated below.

Basic buckling parameter of W-S	
Parameter	Value
Normalized slenderness ratio λ	0.82
Normalized width-thickness ratio R	0.95
Overall initial deflection	$L/1825$
Local initial deflection	$b/520$
σ_T/σ_y	0.671

Calculation according to Kishi's formula (Eq.(11)) and design code (Eq.(12)):

$$\sigma_{cr}/\sigma_u = 1 - (0.0502\lambda^2 - 0.2485\lambda + 0.6077)(R - 0.5) = 0.803$$

$$\sigma_u/\sigma_y = 1.059 - 0.258\lambda - 0.190\lambda^2 = 0.720$$

$$\therefore \sigma_{cr}/\sigma_y = 0.803 * 0.720 = 0.578$$

Calculation of IDIC: Eqs. (6) - (10)

$$\begin{cases} p_1 = -0.0384\lambda^4 + 0.2037\lambda^3 - 0.3362\lambda^2 + 0.1511\lambda - 0.0958 = -0.1030 \\ p_2 = -5.8278\lambda^5 + 35.7294\lambda^4 - 80.1948\lambda^3 + 78.3656\lambda^2 - 30.3199\lambda + 7.0455 = 4.6527 \\ p_3 = -6.1618\lambda^4 + 24.2455\lambda^3 - 30.0485\lambda^2 + 14.1142\lambda - 7.6727 = -5.7214 \end{cases}$$

$$f(R, \lambda) = p_1 * R^{(p_2 + p_3 * R)} = -0.1072$$

$$\begin{cases} q_1 = 0.037R^3 - 0.0716R^2 - 0.0126R + 0.0635 = 0.0186 \\ q_2 = 0.0626R^3 - 0.16R^2 + 0.0584R + 0.1264 = 0.0912 \\ q_3 = -0.0252R^3 + 0.0659R^2 - 0.0228R - 0.1108 = -0.0946 \end{cases}$$

$$g(\lambda, R) = q_1(\lambda - 1.4)^3 + q_2(\lambda - 1.4)^2 + q_3 = -0.0676$$

$$\text{IDIC} = f(R, \lambda) * (I_p - 1) + g(\lambda, R) * (I_c - 1) + 1 = 1.1068$$

$$\therefore \sigma_{u,formula}/\sigma_y = \text{IDIC} * \sigma_{cr}/\sigma_y = 1.1068 * 0.578 = 0.640$$

Comparison with test results:

$$\begin{aligned} \frac{\sigma_{cr} - \sigma_T}{\sigma_T} &= \frac{0.578 - 0.671}{0.671} * 100\% = -13.9\% \\ \frac{\sigma_{u,formula} - \sigma_T}{\sigma_T} &= \frac{0.640 - 0.671}{0.671} * 100\% = -4.6\% \end{aligned}$$

It can be seen that the prediction by the simplified formula tends to underestimate the load-bearing capacity of the columns due to large initial deflections considered. When the actual amplitude of initial deflections is considered into the calculation by the proposed formulae, more accurate prediction on the load-bearing capacity can be obtained.

Fig. 1 Test specimens

(a) Meshing profile

(b) Geometry and boundary condition of specimen

Fig. 2 FE model

Fig. 3 Distribution of residual stress in the cross-section

Fig. 4 Distribution of residual stress in every plate

(a) Overall initial deflection shape (b) Local initial deflection shape

Fig. 5 Initial deflection shape

(a) Local initial deflection on one plate ($m=3$ as an example)

(b) Local initial deflection for half sin-wave

Fig. 6 Local initial deflection shape

Fig. 7 Comparison of load-bearing capacity between experimental and FEA results

Fig. 8 Influence of local initial deflection on load-bearing capacity

Fig. 9 Initial deflection influence coefficient (IDICs) versus normalized local initial deflection

Fig. 10 Decreasing slope α versus normalized width-thickness ratio R

Fig. 11 Examples of deformed shape. (Amplifying factor:5)

Fig. 12 Influence of overall initial deflection on load-bearing capacity

Fig. 13 Initial deflection influence coefficient (IDICs) versus overall initial deflection

Fig. 14 Decreasing slope β versus normalized slenderness ratio λ

(a) Equation of p_1

(b) Equation of p_2

(c) Equation of p_3

Fig. 15 Equation of coefficient terms p_1, p_2, p_3

(a) Equation of q_1

(b) Equation of q_2

(c) Equation of q_3

Fig. 16 Equation of coefficient terms q_1 , q_2 , q_3

Fig. 17 Comparison of IDICs between formulae and FEA results

(a) $\lambda = 0.9$

(b) $\lambda = 1.2$

Fig. 18 Comparison of IDICs between formulae and FEA results

Fig. 19 Initial deflection influence coefficient (IDICs) versus normalized local initial deflection

(a) R07λ07

(b) R09λ10

(c) R12λ10

(d) R15λ18

Local initial deflection $b/150$
 —■— RES0.1 —●— RES0.25 —▲— RES0.4

Fig. 20 Initial deflection influence coefficient (IDICs) versus normalized overall initial deflection

Table 1 Dimensions of experimental specimens

Specimen	B (mm)	D (mm)	t (mm)	L (mm)	L_B (mm)	σ_y (MPa)	E (GPa)	λ	R
S1a/b	282	230	6	2200	2500	390	187	0.35	0.9
S2a/b	282	230	6	3700	4000			0.55	0.9
S3a/b	282	230	6	4900	5200			0.72	0.9
W-S	200	152	4	3650	4000	373.4	205	0.82	0.95
W-L	200	152	4	4850	5200			1.07	0.95
W3-120-6-1800A/B	120	120	6.2	1800	1940	411	210	0.60	0.43
W3-120-6-2500A/B	120	120	6.2	2500	2640	411		0.80	0.43
			/6.1						/0.44
W3-120-6-2800A/B	120	120	6.3	2800	2940	411		0.90	0.42
W3-150-6-2800A/B	150	150	6.1	2800	2940	393		0.70	0.55
W3-80-5-2000A/B	80	80	5.1	2000	2140	415		0.99	0.34
			/5.3						/0.33
R-50-22	151	94.2	6	2090	-	741	215	0.91	0.69
R-50-27	181	116	6	2570				0.93	0.84
R-50-33	217	144	6	3140				0.92	1.03
R-50-38	247	166	6	3610				0.92	1.18
R-50-44	283	193	6	4180				0.93	1.36
R-65-22	151	94.4	6	2720				1.19	0.68
R-65-27	181	116	6	3340				1.20	0.84
R-65-33	217	143	6	4080				1.20	1.03

Table 2 Measured initial deflections

Specimen	w_{l0}	w_{g0}	Specimen	w_{l0}	w_{g0}	Specimen	w_{l0}	w_{g0}
S1a	$b/250$	$L/2800$	W3-120-6-1800A	-	$L/534$	R-50-22	-	$L/2538$
S1b		$L/2500$	W3-120-6-1800B		$L/416$	R-50-27		$L/1938$
S2a		$L/4000$	W3-120-6-2500A		$L/1068$	R-50-33		$L/20833$
S2b		$L/4000$	W3-120-6-2500B		$L/1740$	R-50-38		$L/4081$
S3a		$L/4350$	W3-120-6-2800A		$L/491$	R-50-44		$L/7246$
S3b		$L/3700$	W3-120-6-2800B		$L/387$	R-65-22		$L/4132$
W-S	$b/520$	$L/1825$	W3-150-6-2800A		$L/733$	R-65-27		$L/2604$
W-L		$L/1426$	W3-150-6-2800B		$L/475$	R-65-33		$L/8928$
			W3-80-5-2000A		$L/1159$			
			W3-80-5-2000B		$L/721$			

Where w_{g0} and w_{l0} are the amplitudes of overall and local initial deflections, respectively.

Table 3 Boundary conditions

DOF	T_x	T_y	T_z	R_x	R_y	R_z
Mov	Fix	Fix	Free	Fix	Free	Fix
Fix	Fix	Fix	Fix	Fix	Free	Fix

Table 4 Residual stress used in FE model validation

Specimen	σ_{rt}/σ_y	σ_{rc}/σ_y	Specimen	σ_{rt}/σ_y	σ_{rc}/σ_y	Specimen	σ_{rt}/σ_y	σ_{rc}/σ_y
S1a	0.6	0.04	W3-120-6-1800A	1.0	0.27	R-50-22	0.6	0.1
S1b			W3-120-6-1800B			R-50-27		
S2a			W3-120-6-2500A			R-50-33		
S2b			W3-120-6-2500B			R-50-38		
S3a			W3-120-6-2800A			R-50-44		
S3b			W3-120-6-2800B			R-65-22		
W-S	1.0	0.35	W3-150-6-2800A		0.38	R-65-27		
W-L		(0.43)	W3-150-6-2800B			R-65-33		
			W3-80-5-2000A		0.45			
			W3-80-5-2000B					

Table 5 load-bearing capacity obtained by the experimental and FEA results

Specimen	σ_T/σ_y	σ_{FEA}/σ_y	Errors(%)	Specimen	σ_T/σ_y	σ_{FEA}/σ_y	Errors(%)
S1a	0.857	0.850	-0.82	W3-120-6-2800B	0.6	0.638	6.33
S1b	0.893	0.849	-4.93	W3-150-6-2800A	0.81	0.789	-2.59
S2a	0.862	0.842	-2.32	W3-150-6-2800B	0.78	0.763	-2.18
S2b	0.861	0.842	-2.21	W3-80-5-2000A	0.74	0.652	-11.89
S3a	0.882	0.829	-6.01	W3-80-5-2000B	0.64	0.614	-4.06
S3b	0.853	0.827	-3.05	R-50-22	0.743	0.711	-4.31
W-S	0.671	0.689	2.68	R-50-27	0.731	0.707	-3.28
W-L	0.536	0.530	-1.12	R-50-33	0.709	0.675	-4.80
W3-120-6-1800A	0.86	0.839	-2.44	R-50-38	0.639	0.609	-4.69
W3-120-6-1800B	0.87	0.824	-5.29	R-50-44	0.579	0.552	-4.66
W3-120-6-2500A	0.73	0.762	4.38	R-65-22	0.593	0.622	4.89
W3-120-6-2500B	0.84	0.785	-6.55	R-65-27	0.637	0.605	-5.02
W3-120-6-2800A	0.63	0.659	4.60	R-65-33	0.585	0.552	-5.64
Average Errors(%)	-2.50						
Standard DEV(%)	4.07						

Table 6 Variable main properties of the numerical model

Parameter	Range of variable
Dimension of cross-section	500 × 500 mm
Normalized slenderness ratio λ	0.3, 0.5, 0.7, 1.0, 1.4, 1.6, 1.8
Normalized width-thickness ratio R	0.3, 0.5, 0.7, 0.9, 1.2, 1.5
Length of the column L	4572-27432 mm
Thickness of plate t	7.39-36.93 mm

Table 7 Combination of normalized slenderness and width-thickness ratios

Width-thickness ratio R	Slenderness ratio λ						
	0.3	0.5	0.7	1.0	1.4	1.6	1.8
0.3	R03 λ 03	R03 λ 05	R03 λ 07	R03 λ 10	R03 λ 14	R03 λ 16	R03 λ 18
0.5	R05 λ 03	R05 λ 05	R05 λ 07	R05 λ 10	R05 λ 14	R05 λ 16	R05 λ 18
0.7	R07 λ 03	R07 λ 05	R07 λ 07	R07 λ 10	R07 λ 14	R07 λ 16	R07 λ 18
0.9	R09 λ 03	R09 λ 05	R09 λ 07	R09 λ 10	R09 λ 14	R09 λ 16	R09 λ 18
1.2	R12 λ 03	R12 λ 05	R12 λ 07	R12 λ 10	R12 λ 14	R12 λ 16	R12 λ 18
1.5	R15 λ 03	R15 λ 05	R15 λ 07	R15 λ 10	R15 λ 14	R15 λ 16	R15 λ 18

Table 8 Combinations of amplitude of initial deflections

	Combi1	Combi2	Combi3	Combi4	Combi5	Combi6	Combi7	Combi8	Combi9
Local	$b/75$	$b/75$	$b/75$	$b/150$	$b/150$	$b/150$	$b/450$	$b/450$	$b/450$
Overall	$L/500$	$L/1000$	$L/3000$	$L/500$	$L/1000$	$L/3000$	$L/500$	$L/1000$	$L/3000$

Table 9 Failure modes in the FE models

Width-thickness ratio R	Slenderness ratio λ						
	0.3	0.5	0.7	1.0	1.4	1.6	1.8
0.3	O	O	O	O	O	O	O
0.5	C	C/O	C/O	C/O	C/O	C/O	C/O
0.7	C	C	C	C	C/O	C/O	C/O
0.9	C	C	C	C	C	C	C
1.2	C	C	C	C	C	C	C
1.5	C	C	C	C	C	C	C

Table 10 Failure mode change by initial deflections

(a) R05 λ 05-07, R07 λ 14-18

Overall initial deflection	Local initial deflection		
	$b/75$	$b/150$	$b/450$
$L/500$	C	C	O
$L/1000$	C	C	O
$L/3000$	C	C	O

(b) R05 λ 10-18

Overall initial deflection	Local initial deflection		
	$b/75$	$b/150$	$b/450$
$L/500$	C	O	O
$L/1000$	C	O	O
$L/3000$	C	O	O

Table 11 Coefficient term and determination coefficient of equation ($L/1000$)

Slenderness ratio λ	Coefficient term			Determination coefficient
	p_1	p_2	p_3	
0.3	-0.0749	3.1002	-5.5554	0.9143
0.5	-0.0834	3.5604	-5.4439	0.9254
0.7	-0.0915	4.2250	-5.6733	0.9478
1.0	-0.1168	4.8706	-5.6100	0.9767
1.4	-0.1320	3.9886	-3.7722	0.9924
1.6	-0.1318	3.7669	-3.2494	0.9921
1.8	-0.1289	3.6211	-2.8643	0.9906

Table 12 Coefficient term and determination coefficient of equation ($b/150$)

Width-thickness ratio R	Coefficient term			Determination coefficient
	q_1	q_2	q_3	
0.3	0.0548	0.1333	-0.1124	0.9853
0.5	0.0431	0.1179	-0.1087	0.9874
0.7	0.0303	0.1126	-0.1030	0.9896
0.9	0.0248	0.0989	-0.0963	0.9834
1.2	0.0068	0.0706	-0.0866	0.9873
1.5	0.0086	0.0664	-0.0817	0.9922

Table 13 Maximum errors between formula and FEA results (%)

Width-thickness ratio R	Slenderness ratio λ								
	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8
0.3	0.44	0.89	2.04	2.03	2.82	4.89	3.12	2.17	1.09
0.5	1.81	1.48	1.62	1.67	2.17	4.27	2.55	2.07	1.01
0.7	3.68	3.57	1.81	2.25	2.47	4.17	3.81	3.08	1.36
0.9	3.30	3.22	3.69	4.35	5.40	5.49	5.74	4.97	3.32
1.2	0.46	1.33	2.19	3.68	3.91	4.77	4.99	4.74	6.68
1.5	2.23	1.98	1.29	1.26	1.78	2.48	2.62	4.70	3.67

Table 14 Average errors between formula and FEA results (%)

Width-thickness ratio R	Slenderness ratio λ								
	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8
0.3	0.32	0.51	1.02	0.85	1.37	1.99	1.68	1.17	0.63
0.5	0.62	0.66	0.86	0.75	1.14	1.61	1.45	1.11	0.58
0.7	1.87	1.54	0.85	0.78	1.18	1.69	1.56	1.04	0.79
0.9	1.80	2.18	2.27	2.73	2.94	2.96	2.89	2.15	1.34
1.2	0.28	0.61	1.11	1.78	2.19	2.94	3.31	3.18	3.42
1.5	1.05	1.17	0.84	0.74	0.93	1.33	1.60	2.14	2.07

Table 15 Comparison between formula and experimental results

Specimen	F_T (kN)	$\bar{\sigma}_T$	$\bar{\sigma}_{cr}$	IDIC	IDIC* $\bar{\sigma}_{cr}$	$(\bar{\sigma}_{cr} - \bar{\sigma}_T) / \bar{\sigma}_T$ (%)	$(IDIC * \bar{\sigma}_{cr} - \bar{\sigma}_T) / \bar{\sigma}_T$ (%)
S1a	2053	0.857	0.730	1.0471	0.764	-14.8	-10.9
S1b	2139	0.893	0.730	1.0464	0.764	-18.3	-14.4
S2a	2065	0.862	0.711	1.0689	0.760	-17.5	-11.8
S2b	2062	0.861	0.711	1.0689	0.760	-17.4	-11.7
S3a	2114	0.882	0.684	1.0868	0.743	-22.4	-15.8
S3b	2044	0.853	0.684	1.0844	0.742	-19.8	-13.0
W-S	706	0.671	0.578	1.1068	0.640	-13.9	-4.6
W-L	564	0.536	0.468	1.1123	0.521	-12.7	-2.8

Where F_T is the experimental results, $\bar{\sigma}$ is the non-dimensional load-bearing capacity, $\bar{\sigma}_{cr}$ is calculated based on the Kishi's formula and specification formula, IDIC is an initial deflection influence coefficient.