

Frequency Response Improvement of DSP Controlled DC-DC Converter with Pole-Zero Cancellation Technique

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Abstract — Reentry, the performance of the DSP and FPGA is developed remarkably. So, fully digital control is enabled in switch mode power supplies. However, in many cases, the control system is built by very complicatedly and very difficult theories such as the adaptive control. Furthermore, in most popular PID control, its design method of the parameters is not clear, so derivation of the optimal parameters is very difficult.

This paper proposes revolutionary control technique which is cancelled the transfer function of the converter by using pole-zero-cancellation method. This technique is very simple and easy to stability design.

I. INTRODUCTION

In recent years many electronic circuits is digitized. However, only switch mode power supplies are consisted of the analog circuit till now. This is because the switch mode power supplied consisting of analog circuit is realized with low cost. However, almost circuits are digitized and the whole system is moved by processors. Therefore, it is difficult for switch mode power supplies to stay in the system as the analog circuit. Hence the digitization of the switch mode power supplies is necessary.

So far, various examinations have been discussed about digital control of switch mode power supplies[1-5]. However, important parameters such as the switching frequency were impractical because the performance of CPU such as DSP or FPGA was not so good.

Recently, due to the development of the semiconductor manufacture technology, the performance of CPU such as DSP and FPGA is developed remarkably. Therefore, expectation of the realization of the fully digital control switching power supply becomes higher.

However, the control system is built by very complicatedly and very difficult theories such as the adaptive control or predictive control in many cases of the discussions of the digital controlled switch mode power supplies so far. On the other hand, also in the most popular and easiest control method such as PID control, the design method is not so clear, and the optimal design is difficult[6, 7].

This paper proposes revolutionary control technique which is cancelled the transfer function of the converter by using pole-zero-cancellation method. This technique is very simple and easy to stability design of converter system. Furthermore, the arbitrary frequency characteristics can be created by introducing a new frequency characteristic. Here, the design

method and system stability of the proposed control technique is examined by using buck converter and boost converter.

II. CONVERTER ANALYSIS

For the design of the control system, it is necessary to grasp correctly the characteristics of the converter in detail. The buck converter and boost converter as the controlled objects are shown in Fig. 1, 2, respectively. The dynamic characteristics of each converter can be derived by applying the state space averaging method[8,9]. The dynamic characteristic of duty to output voltage of each converter is derived following equation;

$$G_{dv}(s) = \frac{\Delta V_o(s)}{\Delta D(s)} = \frac{G_{dvo}(s)}{P(s)} \quad (1)$$

where;

Buck converter:

$$P(s) = \frac{s^2}{\omega_o^2} + s \frac{2\delta}{\omega_o} + 1 \quad (2)$$

$$G_{dvo}(s) = \left(\frac{s}{\omega_{esr}} + 1 \right) \frac{R}{R + r_L} V_i \quad (3)$$

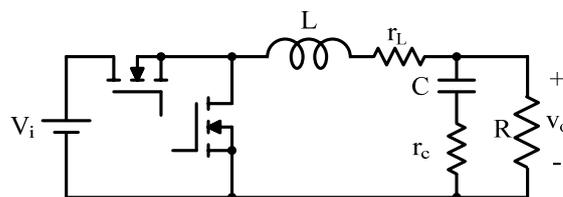


Fig. 1. Buck converter.

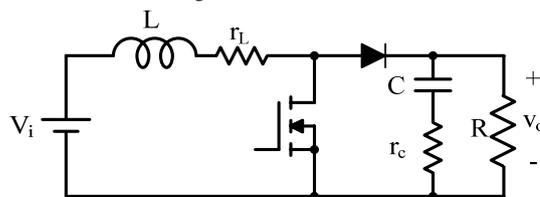


Fig. 2. Boost converter.

$$\omega_o = \sqrt{\frac{R+r_L}{LC(R+r_c)}} \quad (4)$$

$$\delta = \frac{L+C\{Rr_c+r_L(R+r_c)\}}{2\sqrt{LC(R+r_c)(R+r_L)}} \quad (5)$$

$$\omega_{esr} = \frac{1}{Cr_c} \quad (6)$$

Boost converter:

$$P(s) = \frac{s^2}{\omega_o^2} + s \frac{2\delta}{\omega_o} + 1 \quad (7)$$

$$G_{dvo}(s) = \left(\frac{s}{\omega_{esr}} + 1 \right) \left(-\frac{s}{\omega_{rhp}} + 1 \right) \alpha V_i \quad (8)$$

$$\alpha = \frac{R+r_c}{\{RD'(RD'+r_c)+r_L(R+r_c)\} \{RD'r_c+(RD'2+r_L)(R+r_c)\}} \quad (9)$$

$$\omega_o = \frac{1}{R+r_c} \sqrt{\frac{RD'(RD'+r_c)+r_L(R+r_c)}{LC}} \quad (10)$$

$$\delta = \frac{L+C\{Rr_c+r_L(R+r_c)\}}{2\sqrt{LC\{RD'(RD'+r_c)+r_L(R+r_c)\}}} \quad (11)$$

$$\omega_{esr} = \frac{1}{Cr_c} \quad (12)$$

$$\omega_{rhp} = \frac{D^2 R^3 - r_L R (R+r_c)}{LC(R+r_c)} \quad (13)$$

Figure 3 (a) shows the construction of the analog system, and Fig. 3 (b) shows the block diagram of analog system. From, Fig. 3, the loop gain of analog controlled converter can be derived following equation;

$$T(s) = \frac{\Delta V_o(s)}{\Delta V_o^*(s)} = \frac{G_{dvo}(s)}{P(s)} \cdot G_c(s) \cdot K \cdot K_s \cdot PWM \quad (14)$$

where;

$G_c(s)$: Transfer function of phase compensator

K : DC gain of error amp.

K_s : Sense gain of output voltage

PWM : transfer gain of voltage to duty

Figure 4 (a) shows the construction of the digital system, and Fig. 4 (b) shows the block diagram of digital system. From, Fig. 4, the loop gain of digital controlled converter can be derived following equation;

$$T(s) = \frac{\Delta V_o(s)}{\Delta V_o^*(s)} = \frac{G_{dvo}(s)}{P(s)} \cdot G_c(s) \cdot H_e(s) \cdot K \cdot K_s \cdot DPWM \quad (15)$$

where;

$G_c(s)$: Transfer function of phase compensator

K : DC gain of error amp.

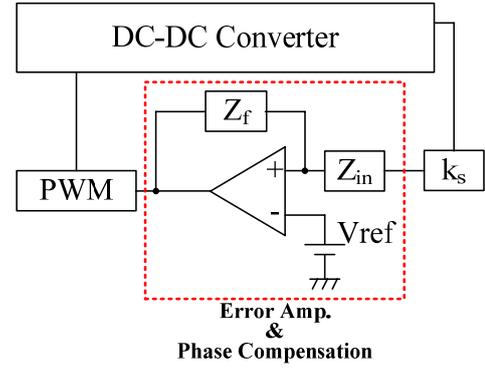
K_s : Sense gain of output voltage

$DPWM$: transfer gain of voltage to duty

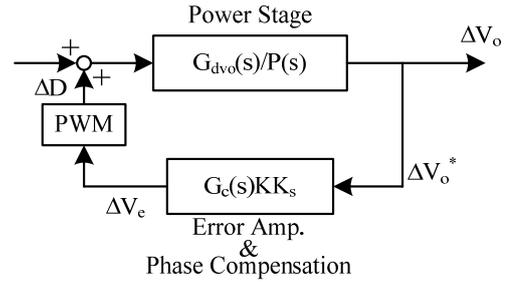
$H_e(s)$: Dead time component of digital controller

The difference of analog and digital control is laid in the facts that digital control is discrete control whereas analog control is continuous control. Therefore, there is a dead time element $H_e(s)$ in transfer function of control system. $H_e(s)$ depends on the sampling period of the AD converter. $H_e(s)$ expressed as following equation;

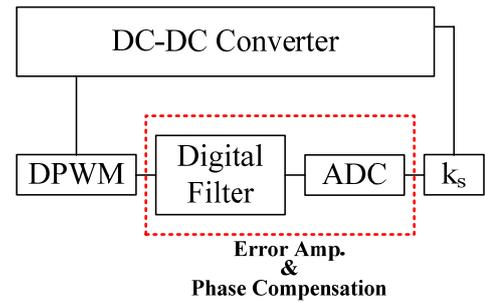
$$H_e(s) = e^{-sT_{sample}} \quad (16)$$



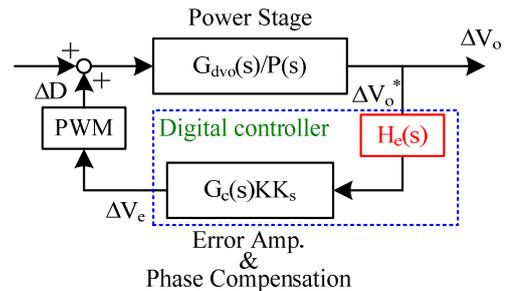
(a) Construction of analog controlled system.



(b) Block diagram of analog controlled system
Fig. 3. Analog controlled system.



(a) Construction of digital controlled system.



(b) Block diagram of digital controlled system
Fig. 4. Digital controlled system.

Because $H_e(s)$ depends on the sampling period, the gain is not changed on frequency response as shown in Fig. 5. However, the phase is drastically rotating around Nyquist frequency ($=f_s/2$).

In order to evaluate the performance of control system, the experiment circuits are implemented using the specifications and parameters in Table 1.

A. Buck Converter

Figure 6 shows the analytical and experimental results of proportional control in analog control case. As shown in Fig. 6, the phase rotation of frequency response is improved at higher frequency side by influence of ESR-Zero, and the system has stable operation.

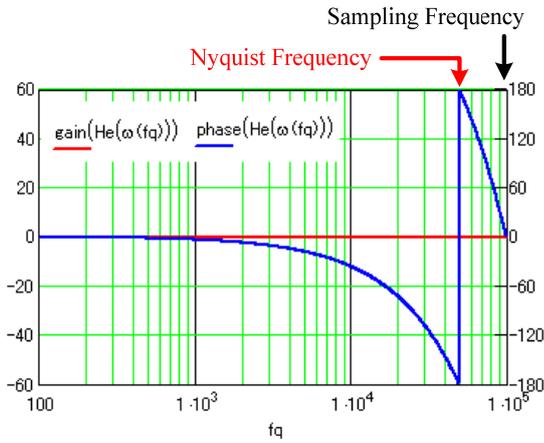


Fig. 5. Frequency response of $H_e(s)$.

TABLE I
CIRCUIT PARAMETERS AND SPECIFICATIONS

	Symbol	Description	Value
Buck Converter	V_i	Input Voltage	12V
	V_o/I_o	Load Condition	2.5V/5A
	L	Filter Inductor	22 μ H
	C	Filter Capacitor	470 μ F
	r_L	DC Resistance of L	100m Ω
	r_c	ESR of C	25m Ω
	R	Load Resistance	1 Ω
	K_s	Sense Gain	0.25
	K	Feedback DC Gain	5
	PWM	PWM Gain	0.5
	f_s	Switching Frequency	100kHz
T_{sample}	Sampling Period	10 μ s	
Boost Converter	V_i	Input Voltage	5V
	V_o/I_o	Load Condition	10V/2A
	L	Filter Inductor	100 μ H
	C	Filter Capacitor	220 μ F
	r_L	DC Resistance of L	125m Ω
	r_c	ESR of C	45m Ω
	R	Load Resistance	6 Ω
	K_s	Sense Gain	0.14
	K	Feedback DC Gain	7
	PWM	PWM Gain	0.5
	f_s	Switching Frequency	100kHz
T_{sample}	Sampling Period	10 μ s	

In this case, the phase margin is around 23 degrees. On the other hand, the digital control case, the phase of frequency response is drastically rotated by influence of the dead time component $H_e(s)$. As a result, the phase margin disappears, and the system becomes unstable.

In digital control case, the phase rotation is larger than analog control case by influence of dead time component $H_e(s)$, so the phase compensation is necessary to keep the system stability.

B. Boost Converter

Figure 8 shows the analytical and experimental results of proportional control in analog control case. As shown in Fig. 8, the phase of frequency response is rotated at middle frequency range by influence of Right-Half-Plane Zero (RHP-Zero), and the phase rotation is improved at higher frequency side by influence of ESR-Zero. However, in this case, this system is unstable. On the other hand, the digital control case, the phase is drastically rotated from middle frequency range by influence of the dead time component $H_e(s)$ in addition to RHP-zero.

In the case of digital control, the stabilization is possible by phase compensation, but extension of bandwidth becomes difficult because the phase rotation from middle frequency range is too large.

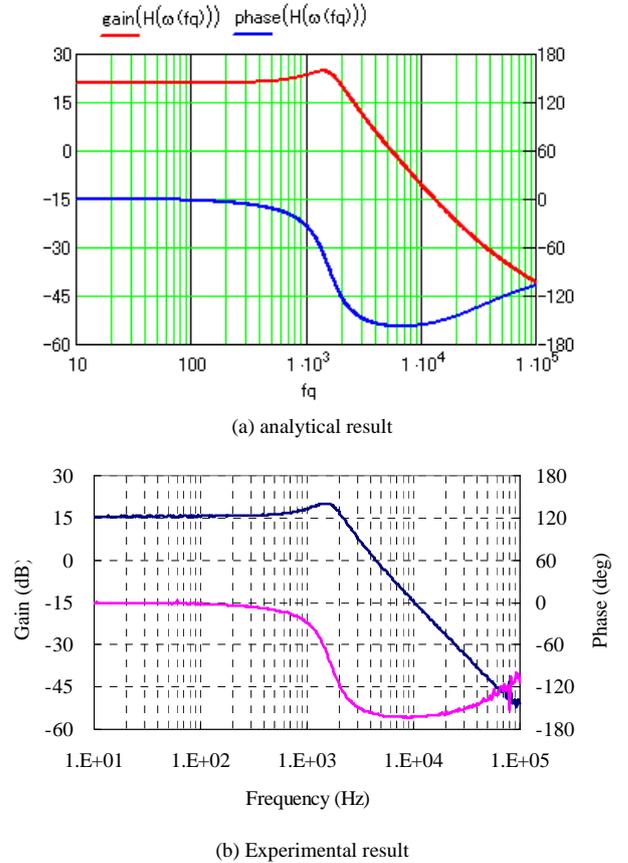
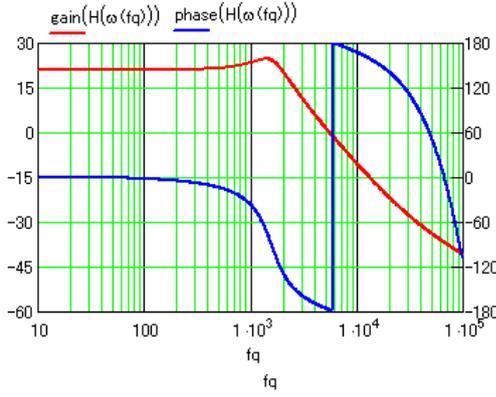
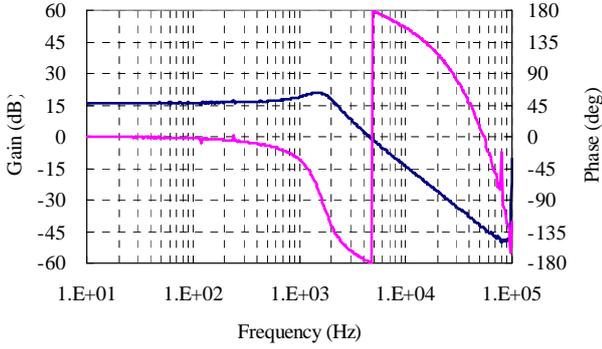


Fig. 6. Frequency response of buck converter (analog control)

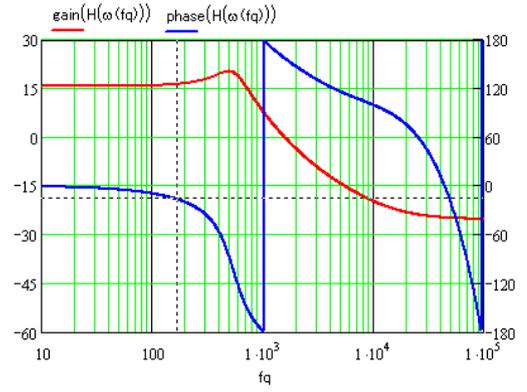


(a) analytical result

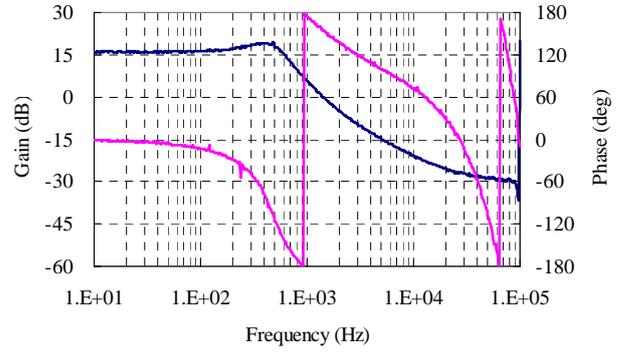


(b) Experimental result

Fig. 7. Frequency response of buck converter (digital control)



(a) analytical result



(b) Experimental result

Fig. 9. Frequency response of boost converter (digital control)

III. CONTROLLER DESIGN

The phase compensation is usually used for improvement of the system stability. There are various phase compensation. Above all, the PID control is the most popular and easiest technique in digital control. However, in PID control, the design method is not so clear, and the optimal design is difficult.

Reduction of the phase rotation is very important for system stability. Especially in the second order system, the phase is drastically rotated around 180 degrees at resonance peak. The stability of the system is improved remarkably if the phase rotation can be reduced.

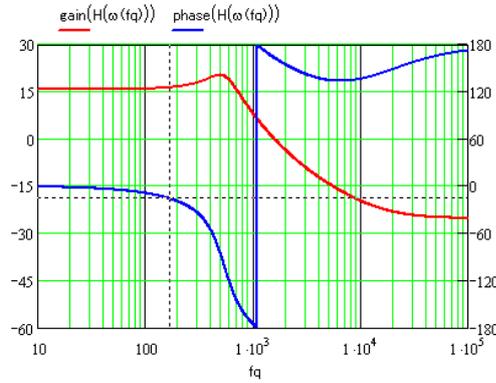
This paper proposes revolutionary control technique which is cancelled the transfer function of the converter by using pole-zero-cancellation method. The phase rotation and gain change can be suppressed by canceling the converter characteristics. Furthermore, new characteristic is set in the system by designing the arbitrary transfer function.

Figure 10 shows the block diagram of pole-zero-cancellation control. From Fig. 10, the transfer function of compensator is given following equation;

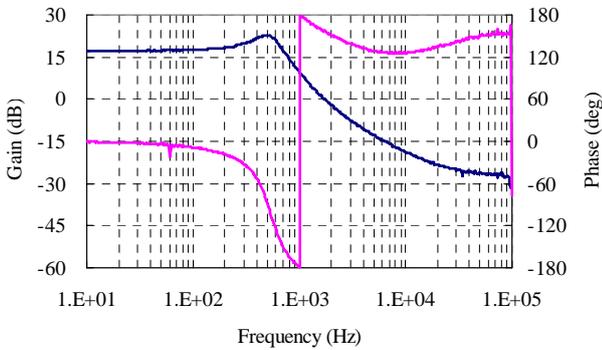
$$G_c(s) = G_{new}(s) \cdot G_{pzc}(s) \quad (16)$$

Here, the $G_{new}(s)$ is defined as simple low pass filter.

$$G_{new}(s) = \frac{1}{\frac{s}{\omega_c} + 1} \quad (17)$$



(a) analytical result



(b) Experimental result

Fig. 8. Frequency response of boost converter (analog control)

A. Buck converter

In buck converter case, the resonance peak and ESR-Zero are cancelled. The phase rotation of 180 degree is reduced by cancelling resonance peak. The transfer function of the pole-zero-cancellation $G_{pzc}(s)$ is given following equation;

$$G_{pzc}(s) = \frac{\frac{s^2}{\omega_o^2} + s \frac{2\delta}{\omega_o} + 1}{\frac{s}{\omega_{esr}} + 1} \quad (18)$$

Moreover, the transfer function of the compensator is given following equation;

$$G_c(s) = \frac{\frac{s^2}{\omega_o^2} + s \frac{2\delta}{\omega_o} + 1}{\left(\frac{s}{\omega_{esr}} + 1\right) \left(\frac{s}{\omega_c} + 1\right)} \quad (19)$$

Figure 11 shows the frequency response of the compensator. As shown in Fig. 11, the ant resonance peak is appeared at the same frequency of power stage frequency response. Figure 12 show the loop gain of the total system. Figure 12 (a) shows the analytical result, and Fig. 12 (b) shows the experimental result. Both result agreed well. From Fig. 12, the characteristic of the buck converter is canceled and shows only the characteristic of the low pass filter. Moreover, the slope of gain curve is -6dB/oct, so this system is first order system. Moreover, the slope of gain curve is -6dB/oct, so this system is first order system. In this case, the bandwidth is around 10kHz, and the phase margin is around 60 degrees.

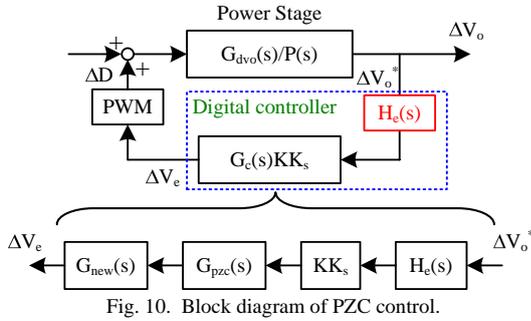


Fig. 10. Block diagram of PZC control.

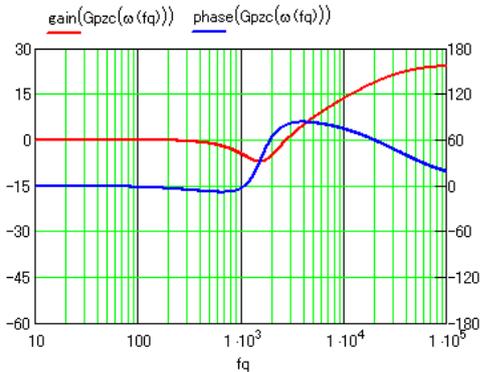
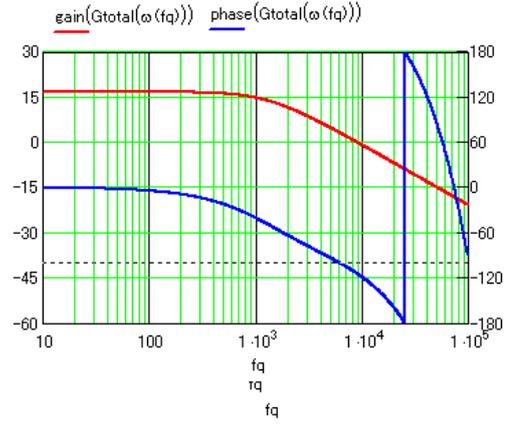
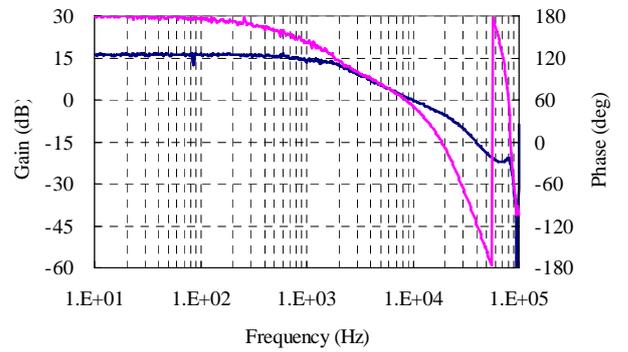


Fig. 11. Frequency response of $G_c(s)$. (Buck converter)



(a) analytical result



(b) Experimental result

Fig. 12. Frequency response of buck converter (digital control)

B. Boost converter

On the other hand, the resonance peak, ESR-Zero and RHP-Zero are cancelled in boost converter case. In boost converter, RHP-Zero is very important factor. The phase is drastically rotated at frequency of RHP-Zero in addition to resonance peak. So, the system becomes unstable easily. Therefore, in boost converter, importance of the characteristic cancellation is higher than buck converter.

The transfer function of the pole-zero-cancellation $G_{pzc}(s)$ is given following equation;

$$G_{pzc}(s) = \frac{\frac{s^2}{\omega_o^2} + s \frac{2\delta}{\omega_o} + 1}{\left(\frac{s}{\omega_{esr}} + 1\right) \left(-\frac{s}{\omega_{rhp}} + 1\right)} \quad (20)$$

Moreover, the transfer function of the compensator is given following equation;

$$G_c(s) = \frac{\frac{s^2}{\omega_o^2} + s \frac{2\delta}{\omega_o} + 1}{\left(\frac{s}{\omega_{esr}} + 1\right) \left(-\frac{s}{\omega_{rhp}} + 1\right) \left(\frac{s}{\omega_c} + 1\right)} \quad (21)$$

The transfer function of boost converter can be cancelled completely by means of Eq. 21. However, the denominator of Eq. 21 has a negative mark. Consequently, the control logic is replaced and become the positive feedback. As a result, this system is uncontrollable. Therefore, the cancellation of RHP-zero is not possible. Hence, the transfer function of compensator is given following equation;

$$G_c(s) = \frac{\frac{s^2}{\omega_o^2} + s \frac{2\delta}{\omega_o} + 1}{\left(\frac{s}{\omega_{esr}} + 1\right) \left(\frac{s}{\omega_c} + 1\right)} \quad (22)$$

However, the resonance peak of power stage can be cancelled, so the phase rotation at resonance peak is reduced and the system stability is improved. Figure 13 shows the frequency response of the compensator. As shown in Fig. 13, the ant resonance peak is appeared at the same frequency of power stage frequency response. Figure 14 show the loop gain of the total system. Figure 14 (a) shows the analytical result, and Fig. 12 (b) shows the experimental result. Both result agreed well. From Fig. 14, the characteristic of the boost converter is canceled and shows the characteristic of the low pass filter at middle frequency range. Moreover, the slope of gain curve is -6dB/oct, so this system is first order system. Moreover, the slope of gain curve is -6dB/oct, so this system is first order system. However, at high frequency range, the gain is not changed and this system is 0 orders by influence of RHP-Zero.

IV. CONCLUSIONS

This paper proposes revolutionary control technique which is cancelled the transfer function of the converter by using pole-zero-cancellation method. This technique is very simple and easy to stability design of converter system. Furthermore, the arbitrary frequency characteristics can be created by introducing a new frequency characteristic. Here, the design method and system stability of the proposed control technique is examined analytically and experimentally by using buck converter and boost converter. As a result, the effectiveness of proposed control technique is confirmed. Moreover, it is confirmed that the characteristic cancellation of the converter can be realized very easy and can be set the arbitrary characteristic.

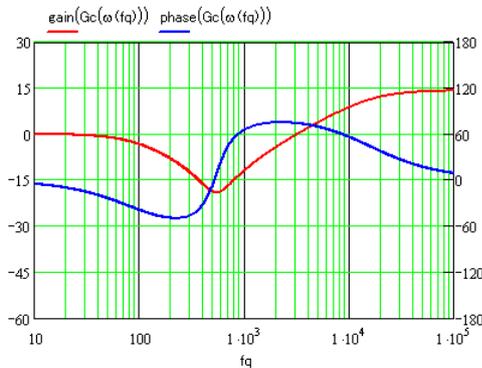
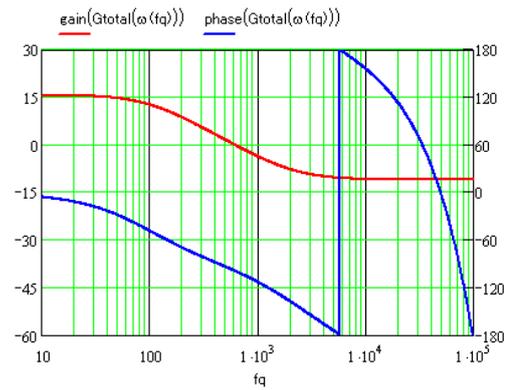
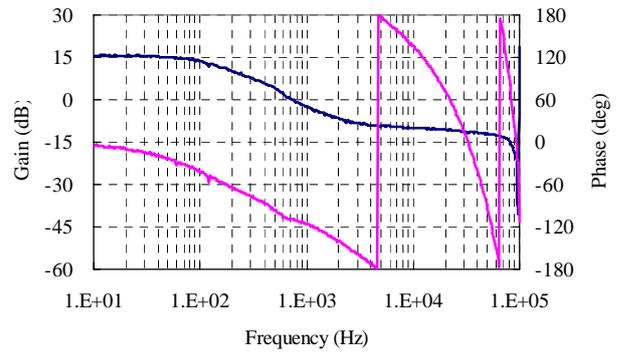


Fig. 13. Frequency response of $G_c(s)$. (Boost converter)



(a) analytical result



(b) Experimental result

Fig. 14. Frequency response of buck converter (digital control)

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