AN ESSAY IN THE ECONOMICS OF POST-LOSS MINIMIZATION: AN ANALYSIS OF THE EFFECTIVENESS OF THE INSURANCE LAW AND CLAUSES

Mahito Okura

Faculty of Economics, Nagasaki University
Address: 4-2-1 Katafuchi, Nagasaki, 850-8506, JAPAN
Tel & Fax: +81-95-820-6328
Email: okura@nagasaki-u.ac.jp

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Abstract

This article proposes a simple model and an analysis of the incentive problem in terms of post-loss minimization in the insurance market. This essay discusses a model of the insurance market in which the insurer decides which of the two types of clause which are inconsistent with the commercial law, or no clause, best influences the insured's choice as to whether or not to make an effort to reduce the loss. The essay seeks to investigate the exogenous situations where the insurer presents these types of clause, or no clause, in terms of the incentive problem between insurer and insured.

Introduction

All entities, such as firms and individuals, face the potential for loss. Firms can be plagued by numerous types of accident that decrease profit. Individuals who drive a car may fear injuring themselves, other drivers, or pedestrians. They should have some incentive to prevent loss. (See for example, Dorfman, 2002)

Insurance is one of the risk management systems available to transfer exposure to risk from the firm or individual to the insurer. Arrow (1970) argues that insurance permits a reduction in the social amount of risk-bearing in order to divorce risk-bearing from productive activity.

However, the shifting of risk towards the insurer might also encourage carelessness. Arrow (1963) named this problem "moral hazard". Many articles and books have discussed moral hazard over several decades. For example, Zeckhauser (1970) indicates the trade-off between risk-bearing and incentives that reduce the losses. Many previous articles have analyzed risk-sharing rules to induce loss minimization. For example, Holmstrom (1979) and Shavell (1979) have formalized the value of deductibles when moral hazard is present. Pauly (1974), Arnott and Stiglitz (1986) and Lee (1992) have proved the role of government intervention in coping with moral hazard. Holmstrom (1982) has considered the case of a team, in which there is a joint production function for the members. Radner (1981) and Rubinstein and Yaari (1983) have shown that multi-period insurance contracts may prevent the generation of moral hazard. Holmstrom and Milgrom (1991) have illustrated this problem through multitask agents. Milgrom and Roberts (1992) and Winter (1992) have provided a good survey of this field.

Some insurance literature has distinguished between two types of loss minimization. Bennett (1992), for example, defined pre-loss minimization as "the steps taken by

insurers before a loss-producing event to reduce the likelihood of that event and/or its severity if it does occur", and post-loss minimization as "the steps taken after a loss to ensure that the loss is contained to the minimum amount". Dorfman (2002) called the former "loss prevention" and the latter "loss reduction".

Moral hazard is one of the serious problems in the insurance market. Early studies concentrated on considering incentive to lower the probability of loss before an accident takes place. That is, the studies were in terms of pre-loss minimization. They have shown that unobservability by the insurer of the insured directly attracts moral hazard. Even if the effort can be observed by the insurer, the incentive for the insured to lower the probability of large loss remains after an accident takes place. This essay proposes a simple model and an analysis of this incentive problem. That is, this essay will investigate moral hazard in terms of post-loss minimization. In the economic models developed in previous research, little attention has been paid to post-loss minimization because the two types of loss minimization look the same. Both have the effect of reducing the expected loss; the only difference is the timing of the loss minimization. However, the incentive systems for each type of loss minimization are not similar and their effects are not necessarily the same in the real world. Therefore, it is useful to analyze them independently.

There is an interesting incentive method designed to assist post-loss minimization in Japanese (and several foreign countries') insurance law. It can be called "duty to sue and labor". In Japanese insurance law, the insured must be wished to reduce the loss when an accident takes place, and whenever an accident takes place, the insurer must incur all of the costs that are paid by the insured in order to reduce the losses.

However, there are clauses of the insurance contract that are inconsistent with this law. In the real world, two types of clause mainly prevail. One gives that the insurer has the right not to compensate effort-costs paid by the insured. The other provides compensation for an insured's effort-cost within the range of the insurance amount. Hereafter, I call the former "F-clause" and the latter "P-clause". Because both clauses are valid in Japan, the insurer who presents either type of clause may not necessarily incur the effort-cost paid by the insured.

The purpose of this essay is to analyze the effectiveness of these clauses. This essay seeks to investigate the exogenous situations where the insurer utilizes the above clauses, or none, in terms of the incentive problem between insurer and insured.

The Model

There are many insured and one insurer in the economy. Except for their disutility for effort, the insured are assumed to be identical. Each insured purchases one unit of an insurance contract from this monopolistic insurer. The insured's utility is assumed to be linear for wealth W^A. It consists of insurance money the insured might get and disutility

for effort. Thus, this essay will take the insured's utility function to be

$$u(W^A) - \phi(e) \tag{1}$$

where $u(\bullet)$ is continuous, strictly increasing and (weakly) concave, and $\phi(e) = \lambda_i e$ for effort and $\phi(e) = 0$ for no effort. It is assumed that e is given exogenously and λ_i is a subjective variable for the insured and is distributed according to a cumulative distribution function $F(\bullet)$. The insurer cannot know each insured's λ_i but can know the form of the distribution $F(\bullet)$.

Furthermore, it is assumed that the insurer is risk-neutral. The insurer's utility is its own wealth W^P minus compensations to the insured if this is incurred. Thus, the insurer's utility function is given by

$$W^P - \varphi(e) \tag{2}$$

where $\varphi \in (0, e]$ for incurring and $\varphi = 0$ for not incurring.

The insured and insurer want to maximize their respective utility and this paper sets out a four stage game as follows.

In the first stage, the insurer decides which of the two clauses, or none, will be presented.

In the second stage, a nature choice is made as to whether or not an accident will take place, according to probability $\pi \in (0,1)$, which is given exogenously. If an accident does not take place then the game ends. If the choice is that an accident takes place, then the game is continued.

In the third stage, the insured decides whether or not to make an effort to reduce the loss. This model assumes, for simplicity, that in this stage an insured who does not make an effort is faced with two states of the world, $\frac{1}{2}$ loss and total loss, with probabilities π_1 and π_2, respectively. In addition, it is assumed that $\pi_1, \pi_2 \in (0, \pi)$, $\pi = \pi_1 + \pi_2$ and $\pi_1 > \pi_2$. In contrast, if an insured makes an effort, both probabilities can be reversed. That is, to make an effort can lower the probability of large loss.

In the fourth stage, a nature choice is made as to which two states take place according to probabilities that are decided in the previous stage.

Each insured's utility consists of its own initial wealth w, coverage rate $\alpha \in (0,1]$, insurable asset D, and premium of the full insurance p. This essay assumes for simplicity that these are exogenous variables. Furthermore, they can be rewritten in forms that will be convenient for the next section as follows:

$$A \equiv u(w - \alpha p), \tag{3}$$

$$B \equiv u\left(w - \alpha p - \frac{1}{2}(1 - \alpha)D \right), \tag{4}$$

$$C \equiv u(w - \alpha p - (1 - \alpha)D), \tag{5}$$

$$\Delta \equiv B - C. \tag{6}$$

Since $\alpha \in (0,1]$ and $u(\bullet)$ is strictly increasing, the sign of Δ cannot be negative.

Finally, to classify the difference between two clauses, it takes the further specification $0 < e < \frac{\alpha D}{2}$. This inequality means that when the insured's efforts and the nature choice leads to $\frac{1}{2}$ loss, the full amount of the effort-cost is incurred by the insurer only if the insurer presents the P-clause. In the model, the difference between the two clauses only occurs with the above situation.

The insurance market can be represented by the game situation. Figure 1 depicts the above situation in the form of the game tree.

Analysis of the Equilibrium

In this section, I seek a subgame perfect Nash equilibrium. This equilibrium can be derived by backward induction. At the beginning of the equilibrium analysis, for a given insurer's decision, the insured decides whether or not to make the effort to reduce the loss.

If insured i at each information set makes an effort, the insured obtains expected utility,

EU_j^S for $j = 1,2,3$.

$$EU_1^S = \pi_1 B + \pi_2 C - \pi \lambda_i e, \tag{7}$$

$$EU_2^S = \pi_1 B + \pi_2 C - \pi_2 \lambda_i e, \tag{8}$$

$$EU_3^S = \pi_1 B + \pi_2 C. \tag{9}$$

In contrast, if the insured does not make an effort, the insured obtains expected utility, EU^N.

$$EU^N = \pi_2 B + \pi_1 C. \tag{10}$$

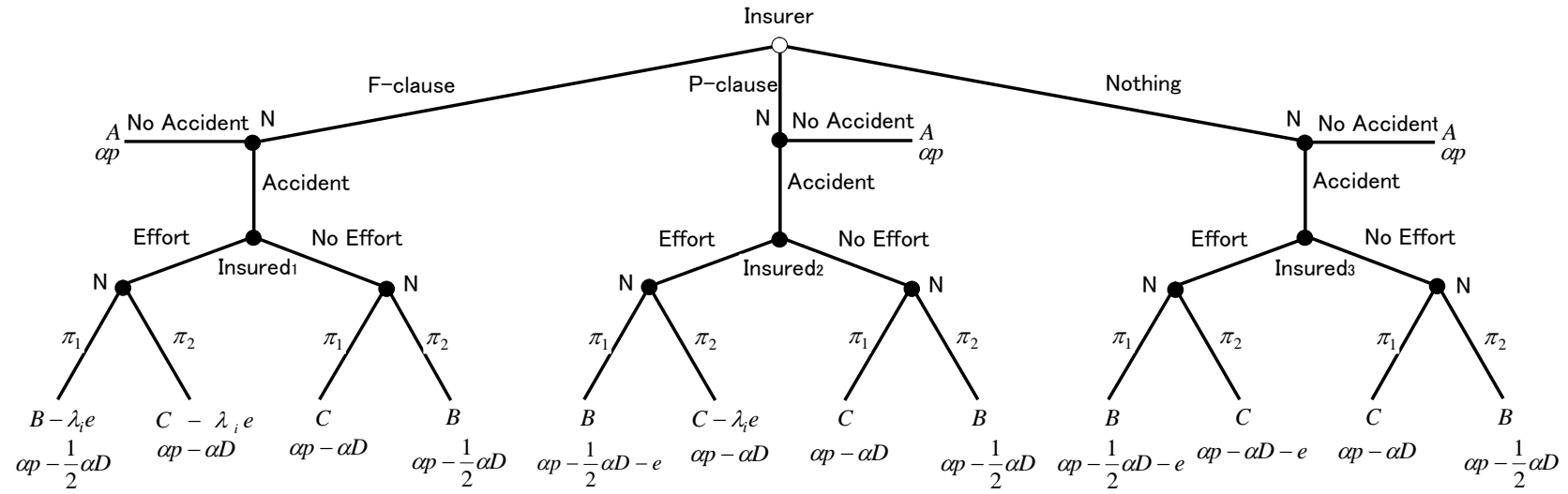Therefore, the insured at each information set is better off by making an effort if and only if

$$EU_1^S - EU^N = (\pi_1 - \pi_2)\Delta - \pi\lambda_i \geq 0 \Leftrightarrow \lambda_i \leq \frac{(\pi_1 - \pi_2)\Delta}{\pi e} \equiv \lambda^*, \tag{11}$$

$$EU_2^S - EU^N = (\pi_1 - \pi_2)\Delta - \pi_2\lambda_i e \geq 0 \Leftrightarrow \lambda_i \leq \frac{(\pi_1 - \pi_2)\Delta}{\pi_2 e} \equiv \lambda^{**}, \tag{12}$$

$$EU_3^S - EU^N = (\pi_1 - \pi_2)\Delta \geq 0 \text{ for any } \lambda_i. \tag{13}$$

The Equation (11) means that a small number of insured whose λ_i is lower than λ^* only make an effort when the insurer presents the F-clause. The interpretation of Equation (12) is that some insured whose λ_i is lower than λ^{**} only make an effort when the insurer presents the P-clause. The Equation (13) states that all insured are sure to make an effort when none of the clauses are used.

Figure 1: The Game Tree

Insurer

F-clause | P-clause | Nothing

No Accident A / αp

Accident

Effort | No Effort

Insured$_1$ | Insured$_2$ | Insured$_3$

π_1 | π_2

$B - \lambda_i e$; $\alpha p - \frac{1}{2}\alpha D$

$C - \lambda_i e$; $\alpha p - \alpha D$

C ; $\alpha p - \alpha D$

B ; $\alpha p - \frac{1}{2}\alpha D$

B ; $\alpha p - \frac{1}{2}\alpha D - e$

$C - \lambda_i e$; $\alpha p - \alpha D$

C ; $\alpha p - \alpha D$

B ; $\alpha p - \frac{1}{2}\alpha D$

B ; $\alpha p - \frac{1}{2}\alpha D - e$

C ; $\alpha p - \alpha D - e$

C ; $\alpha p - \alpha D$

B ; $\alpha p - \frac{1}{2}\alpha D$

Note:
1. A node with label "N" is a nature node.
2. The above value in each pair is the insured's
   utility and the below value is the insurer's utility.

To consider the insurer's strategy in the second stage of the game, the insurers' utilities are defined in terms of insured decisions. If the insurer presents the F-clause, the P-clause, or no clause, it obtains the expected utility, EU_0^F, EU_0^P, EU_0^S, respectively.

$$EU_0^F = \alpha p - F(\lambda^*)\left(\frac{1}{2}\pi_1\alpha D + \pi_2\alpha D\right) - (1 - F(\lambda^*))\left(\frac{1}{2}\pi_2\alpha D + \pi_1\alpha D\right), \qquad (14)$$

$$EU_0^P = \alpha p - F(\lambda^{**})\left(\frac{1}{2}\pi_1\alpha D + \pi_2\alpha D + \pi_1 e\right) - (1 - F(\lambda^{**}))\left(\frac{1}{2}\pi_2\alpha D + \pi_1\alpha D\right), \qquad (15)$$

$$EU_0^S = \alpha p - \frac{1}{2}\pi_1\alpha D - \pi_2\alpha D - \pi e. \qquad (16)$$

Using Equations (14) to (16), the following equations can be derived.

$$EU_0^F - EU_0^P = -\frac{1}{2}\alpha D(F(\lambda^{**}) - F(\lambda^*))(\pi_1 - \pi_2) + F(\lambda^{**})\pi_1 e, \qquad (17)$$

$$EU_0^F - EU_0^S = -\frac{1}{2}\alpha D(1 - F(\lambda^*))(\pi_1 - \pi_2) + \pi e, \qquad (18)$$

$$EU_0^P - EU_0^S = -(1 - F(\lambda^{**}))\left(\frac{1}{2}\alpha D(\pi_1 - \pi_2) - \pi_1 e\right) + \pi_2 e. \qquad (19)$$

The insurer presents the F-clause if both Equations (17) and (18) are not negative, or equivalently,

$$\frac{F(\lambda^*)}{F(\lambda^{**})} \geq 1 - \frac{2\pi_1 e}{\alpha D(\pi_1 - \pi_2)}, \qquad (20)$$

$$F(\lambda^*) \geq 1 - \frac{2\pi e}{\alpha D(\pi_1 - \pi_2)}. \qquad (21)$$

Because $\lambda^* < \lambda^{**}$ and $F(\bullet)$ is non-decreasing, then $\dfrac{F(\lambda^*)}{F(\lambda^{**})}$ is less than unity.

The insurer presents the P-clause if Equation (17) is not positive and Equation (19) is not negative, or equivalently,

$$\frac{F(\lambda^*)}{F(\lambda^{**})} \leq 1 - \frac{2\pi_1 e}{\alpha D(\pi_1 - \pi_2)}, \tag{22}$$

$$F(\lambda^{**}) \geq 1 - \frac{2\pi_2 e}{\alpha D(\pi_1 - \pi_2) - 2\pi_1 e}. \tag{23}$$

The insurer presents no clause if both Equations (18) and (19) are not positive, or equivalently,

$$F(\lambda^*) \leq 1 - \frac{2\pi e}{\alpha D(\pi_1 - \pi_2)}, \tag{24}$$

$$F(\lambda^{**}) \leq 1 - \frac{2\pi_2 e}{\alpha D(\pi_1 - \pi_2) - 2\pi_1 e}. \tag{25}$$

A Special Case

In this section, a special case is constructed to focus on interactions between equilibrium strategies and the exogenous variables.

Consider the special case in which all of the insured are risk-neutral and whose λ_i are distributed in uniform shares along $[0,1]$. This special case can be used because it is straightforward to calculate $\Delta = \dfrac{1}{2}(1-\alpha)D$, $F(\lambda^*)$, and $F(\lambda^{**})$ which are given by

$$F(\lambda^*) = \frac{(\pi_1 - \pi_2)(1-\alpha)D}{2\pi e}, \tag{26}$$

$$F(\lambda^{**}) = \frac{(\pi_1 - \pi_2)(1-\alpha)D}{2\pi_2 e}. \tag{27}$$

Furthermore, the Inequalities (20), (21), and (23) can be rewritten as the equalities that represent a typical boundary between two strategies. Substituting Equations (26) and (27) in these expressions gives

$$\frac{\pi_2}{\pi} = 1 - \frac{2\pi_1 e}{\alpha D(\pi_1 - \pi_2)}, \tag{28}$$

$$\frac{(\pi_1 - \pi_2)(1-\alpha)D}{2\pi e} = 1 - \frac{2\pi e}{\alpha D(\pi_1 - \pi_2)}, \tag{29}$$

$$\frac{(\pi_1 - \pi_2)(1-\alpha)D}{2\pi_2 e} = 1 - \frac{2\pi_2 e}{\alpha D(\pi_1 - \pi_2) - 2\pi_1 e}. \tag{30}$$

It is easy to rewrite (28) in terms of e:

$$e_{FP} \equiv e = \frac{(\pi_1 - \pi_2)D}{2\pi}\alpha. \tag{31}$$

Hence, Equation (31) is a linear and strictly increasing function of α.
Equation (29) can be written as

$$e_{FS} \equiv e = \frac{(\pi_1 - \pi_2)D(\alpha \pm \sqrt{\alpha(5\alpha - 4)})}{4\pi}. \tag{33}$$

Calculation shows that solutions are real when $\alpha \geq \dfrac{4}{5}$. Equation (33) can be depicted as

a hyperbola. Furthermore, on this hyperbola, the points with $\alpha = 1$ have $e_{FS} = 0$ and

$$e_{FS} = \frac{(\pi_1 - \pi_2)D}{2\pi}.$$

Equation (30) is equivalent to

$$4\pi_2\pi e^2 - 2(\pi_1 - \pi_2)D((1-\alpha)\pi_1 + \alpha\pi_2)e + \alpha(1-\alpha)D^2(\pi_1 - \pi_2)^2 = 0. \tag{34}$$

Like Equation (32), Equation (34) is also a quadratic equation with respect to e. Thus, the solutions for this equation can be written as

$$e_{PS} \equiv e = \frac{(\pi_1 - \pi_2)D((1-\alpha)\pi_1 + \alpha\pi_2)(1 \pm \sqrt{1-\Xi})}{4\pi_2\pi} \tag{35}$$

where $\Xi \equiv \dfrac{4\alpha(1-\alpha)\pi_2\pi}{((1-\alpha)\pi_1 + \alpha\pi_2)^2}$.

When $\Xi = 1$, it is easy to compute

$$\alpha = \frac{\pi_1^2 + \pi_1\pi_2 + 2\pi_2^2 \pm 2\pi_2\sqrt{\pi_2\pi}}{\pi_1^2 + 2\pi_1\pi_2 + 5\pi_2^2}. \tag{36}$$

Let α^+ and α^- denote the larger and smaller root of Equation (36), respectively. Unlike Equation (33), Equation (35) can be depicted as the two hyperbolas because $0 < \alpha^- < \alpha^+ < 1$. In other words, two curves are shown on the coordinate plane that may be created by the α-axis and the e-axis. Furthermore, it can also be seen that $\alpha^+ > \dfrac{4}{5}$. Proofs of these are presented in the Appendix.

Moreover, on these hyperbolas, the points with $\alpha = 0$ have $e_{PS} = 0$ and

$e_{PS} = \dfrac{\pi_1(\pi_1 - \pi_2)D}{2\pi_2\pi}$, and the points with $\alpha = 1$ have $e_{PS} = 0$ and $e_{PS} = \dfrac{(\pi_1 - \pi_2)D}{2\pi}$.

These two inequalities will be helpful later in depicting the figure that illustrates some results.

$$\left.\frac{\partial e_{FS}^+}{\partial \alpha}\right|_{\alpha=1} = \left.\frac{\partial e_{PS}^+}{\partial \alpha}\right|_{\alpha=1} = \frac{(\pi_1 - \pi_2)D}{\pi} > \frac{(\pi_1 - \pi_2)D}{2\pi} = \left.\frac{\partial e_{FP}}{\partial \alpha}\right|_{\alpha=1}, \tag{37}$$

$$\left.\frac{\partial e_{PS}^-}{\partial \alpha}\right|_{\alpha=0} = \frac{(\pi_1 - \pi_2)D}{2\pi_1} > \frac{(\pi_1 - \pi_2)D}{2\pi} = \left.\frac{\partial e_{FP}}{\partial \alpha}\right|_{\alpha=0} \tag{38}$$

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where superscript $+$ and $-$ denote the larger and smaller root of Equations (33) and (35). Finally, recall that this model assumed that $0 < e < \dfrac{D}{2}\alpha$ and $\dfrac{D}{2}$ is larger than $\dfrac{(\pi_1 - \pi_2)D}{2\pi}$.

The result is shown in Figure 2. The result indicates that in the special case in which all of the insured are risk neutral and whose λ_i are distributed in uniform shares along $[0,1]$, the combination of the coverage rate ($\equiv \alpha$) and the level of effort-cost ($\equiv e$) has led to the equilibrium outcome.

Figure 2 shows the equilibrium outcome where the insurer presents no clause and all of the insured's effort is horizontally shaded. The vertically shaded area is such that in equilibrium, the insurer presents the P-clause and only those insured whose λ_i is lower than λ^{**} make an effort. The equilibrium where the insurer presents the F-clause and only a small number of insured whose λ_i is lower than λ^* make an effort lies in the diagonally shaded region of Figure 2.
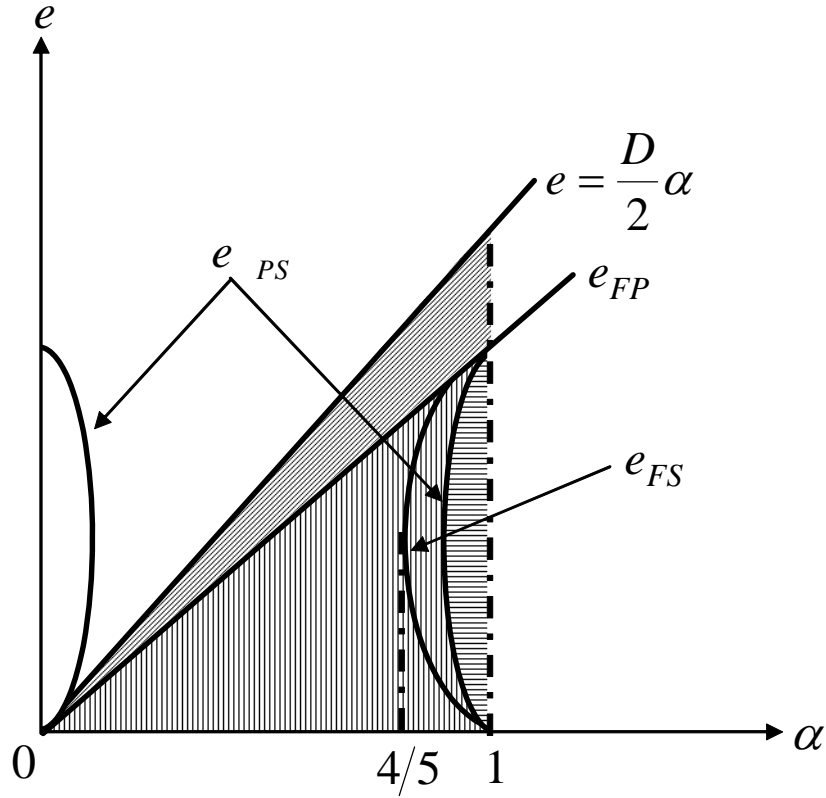
Conclusion and Remarks

This paper has used a model of the insurance market, in which the insurer decides which of two clauses or no clause is used, and the insured chooses whether or not to make an effort to reduce the loss after an accident takes place.

The result is given in Figure 2. Roughly speaking, when the coverage rate is low and the level of effort-cost is high, in equilibrium the F-clause is appropriate for the insurer. When the coverage rate is moderate and the level of effort-cost is moderate or high, the P-clause is appropriate. When the coverage rate is high and the level of effort-cost is low, in equilibrium the use of no clause is appropriate.

This conclusion is based on the following explanations. Firstly, consider the case of the low coverage rate. The insured want to make an effort because the insured must pay much of the loss. In contrast, the insurer has little incentive to reduce the loss because the insurance money is relatively low. Thus, in this case the insurer has a tendency to present either clause and only some insured make an effort. Secondly, consider the case of low effort-cost. Both the insured and the insurer have considerable interest in post-loss minimization. However, the insurer may present the P-clause because the effect of that choice is dependent on the response of the insured's strategy to the insurer's strategy. Thus, even if the level of effort-cost is low, all insured will not necessarily make an effort.

Figure 2: The Region of Each Equilibrium

Note:

region is such that the insurer represents nothing and all of the insured effort.

region is such that the insurer represents P-clause and some insured only effort.

region is such that the insurer represents F-clause and a small number of insured only effort.

The results shed some light on several questions. For example, the insurance system is designed to promote not only post-loss minimization but also pre-loss minimization. It is well known that deductibles and coinsurance, which lower the coverage rate, might be an inducement for pre-loss minimization. However, these devices also effect post-loss

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minimization. The result of this research indicates that the insurer can present either clause if the coverage rate is not high. Thus, the number of insured who make an effort with post-loss minimization will be decreased by these devices. A striking consideration is that deductibles and coinsurance are undesirable for post-loss minimization although these devices induce to active pre-loss minimization. In other words, this model illustrates the tradeoff between pre- and post-loss minimization. Moreover, these devices may not be effective in the equilibrium region where the insurer adopts no clause and all insured make an effort. Let $\overline{\alpha}$ be a maximum coverage rate defined by these devices. It is readily verified that if at least $\overline{\alpha} \leq \dfrac{4}{5}$, the insurer is sure to use either clause. Thus, this explanation can provide the reason why there are a few insurance contracts that do not include either clause in the (Japanese) insurance market.

Another example is that, since 1984, Japanese fire insurance contracts "for an individual" express that an insurer incurs all of the effort-cost. However, the same contract "for a firm" includes the P-clause. The reason why the difference exists can be explained in terms of two exogenous variables. First, in general the individual's coverage rate is higher than the firm's because many firms utilize many kinds of special clauses (for instance, blanket policy) to lower the coverage rate. Second, the level of the effort-cost for a firm is generally larger than that for an individual. In the most plausible case, when a fire occurs in a large-scale firm, several helicopters are used in order to sprinkle a flood of extinguishing agent. Thus, there is a difference, not only in the coverage rate but also in the level of the effort-cost between an individual and a firm. According to the model developed in this essay, it is rational that an insurer voluntarily presents no clause "for an individual" and the P-clause "for a firm".

Clearly, this model involves a number of simplifying assumptions. A major limitation of this paper is to restrict attention to the special case. This is realistic for the case where the insured are risk-averse and their disutility factors are distributed non-uniformly. Much additional work is required to refine this model to provide stronger results.

Appendix

First, show that $0 < \alpha^- < \alpha^+ < 1$. From Equation (36), it is easy to show $\alpha^- > 0$ because

$$\pi_1^2 + \pi_1\pi_2 + 2\pi_2^2 > 2\pi_2\sqrt{\pi\pi_2} \Leftrightarrow \pi_1^2 + 2\pi_1\pi_2 + 5\pi_2^2 > 0 . \tag{A1}$$

Now suppose that $\alpha^+ \geq 1$. From Equation (36), it can be expressed as

$$-\frac{\pi_1}{\pi_2} + 2\sqrt{1 + \frac{\pi_1}{\pi_2}} - 3 \geq 0. \tag{A2}$$

Let g be the left-hand side of Equation (A2). Differentiating g with respect to $\frac{\pi_1}{\pi_2}$ can be written as

$$\frac{\partial g}{\partial \left(\frac{\pi_1}{\pi_2}\right)} = -1 + \frac{1}{\sqrt{1 + \frac{\pi_1}{\pi_2}}}. \tag{A3}$$

Because Equation (A3) is always strictly negative and $g \to 2\sqrt{2} - 4 < 0$ as $\frac{\pi_1}{\pi_2} \to 1$, g is always strictly negative, and Equation (A2) is impossible, hence, $\alpha^+ < 1$.

Next, prove that $\alpha^+ > \frac{4}{5}$. If $\alpha^+ \leq \frac{4}{5}$, then from Equation (36) the following equation must be satisfied.

$$\frac{1}{5}\left(\frac{\pi_1}{\pi_2}\right)^2 - \frac{3\pi_1}{5\pi_2} - 2 + 2\sqrt{1 + \frac{\pi_1}{\pi_2}} \leq 0. \tag{A4}$$

Let h be the left-hand side of Equation (A4). Differentiating h with respect to $\frac{\pi_1}{\pi_2}$ can be written as

$$\frac{\partial h}{\partial \left(\frac{\pi_1}{\pi_2}\right)} = \frac{2\pi_1}{5\pi_2} - \frac{3}{5} + \frac{1}{\sqrt{1 + \frac{\pi_1}{\pi_2}}}. \tag{A5}$$

Because $\frac{3}{5} < \frac{1}{\sqrt{1 + (\pi_1/\pi_2)}}$ for $\frac{\pi_1}{\pi_2} < \frac{16}{9}$ and $\frac{3}{5} < \frac{2\pi_1}{5\pi_2}$ for $\frac{\pi_1}{\pi_2} > \frac{3}{2}$, then h is a

monotone increasing function of $\dfrac{\pi_1}{\pi_2}$. In addition, it can be seen that $h \to -\dfrac{12}{5} + 2\sqrt{2} > 0$

as $\dfrac{\pi_1}{\pi_2} \to 1$. Hence, h is always strictly positive, and Equation (A4) is impossible.

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