Effects of viscous dissipation and fluid axial heat conduction on laminar heat transfer in ducts with constant wall temperature (Part I: Parallel-plates duct)

by

Ganbat Davaa*, Toru Shigechi**, Odgerel Jambal* and Satoru Momoki**

The problem of heat transfer for non-Newtonian fully developed laminar flow has been solved for parallel-plates duct with constant wall temperature. The effects of viscous dissipation and fluid axial heat conduction were taken into account and a numerical scheme based on the finite difference method was applied to solve the governing elliptic type energy equation for an infinite axial domain of $-\infty < z < \infty$. The effects of Brinkman number and Peclet number on developing temperature distribution and Nusselt number at the walls are discussed.

1. Introduction

This paper is concerned with the heat transfer to hydrodynamically fully developed laminar fluid flow in parallel-plates ducts with constant and equal temperatures at the walls. The fluid behavior is assumed to obey the power law model and the effects of viscous dissipation and fluid axial heat conduction are taken into account. The problem of laminar heat transfer for power-law fluids with an emphasis on viscous dissipation and fluid axial heat conduction is controlled by the magnitudes of Peclet number, Pe, which characterizes the ratio of axial heat convection to axial heat conduction, Brinkman number, Br, which represents the ratio of overall dissipation to heat conduction and the flow index, n.

The energy equation together with a constant temperature at upstream infinity, fully developed temperature profile at downstream infinity and the appropriate thermal boundary conditions at the walls is numerically solved by the finite difference method as an elliptic type problem. In considering the effect of fluid axial heat conduction, it is often necessary to investigate this effect in two semi-infinite regions of the duct. In this study, the walls of the upstream region at $-\infty < z \leq 0$ are kept at the entering fluid temperature and the walls of the region at $0 < z < \infty$ are kept at a specified temperature. The solution method to compute the temperature field and the applied mesh system have already been described in the previous reports[1]. Our results are compared with those reported in tabular forms by the previous researchers[2],[3]. The numerical values of the fully developed Nusselt numbers for different Peclet number were reported by Nguen[3] and in his study viscous dissipation effect was not considered. He considered the laminar heat transfer of Newtonian fluids flowing in an infinitely long duct and applied a transformed axial coordinate to solve the problem. For gas flows in parallel-plate channels, Ou and Cheng[4] obtained analytical solutions by considering the effects of pressure work and viscous dissipation. They stated that for a given Brinkman number the effect of viscous dissipation increases with the distance from the thermal entrance and in particular the asymptotic Nusselt number is 17.5, regardless of the value of Brinkman number, instead of conventionally accepted 7.54 for constant wall temperature case. In their study the effect of fluid axial heat conduction was neglected. Deavours[5] investigated analytically thermally developing heat transfer of Newtonian fluids in parallel plates duct. In his paper, a method of finding certain expansion coefficients in a series of nonorthogonal eigenfunctions was applied to obtain the fluid temperature.

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* Graduate student, Graduate School of Science and Technology
** Department of Mechanical Systems Engineering

Nomenclature

Br	: Brinkman number
c_p	: specific heat at constant pressure
D_h	: hydraulic diameter ($= 2L$)
f	: friction factor
k	: thermal conductivity
L	: distance between the parallel plates
n	: flow index
Nu	: Nusselt number
Pe	: Peclet number
T	: temperature
u	: fully developed velocity profile
u_m	: fluid average velocity ($\equiv \frac{1}{L}\int_0^L u dy$)
u^*	: dimensionless velocity ($=u/u_m$)
y	: coordinate normal to the fixed plate
y^*	: dimensionless coordinate
z	: axial coordinate
z^*	: dimensionless axial coordinate

Greek Symbols

m	: consistency index
ρ	: density
τ	: shear stress
θ	: dimensionless temperature

Subscripts

b	: bulk
e	: entrance or inlet
fd	: fully developed
w	: wall

2. Analysis

The geometry of the problem and the coordinate system for the analysis is shown in Fig.1. The assumptions and conditions used in the analysis are:

- The flow is steady, laminar and fully developed hydrodynamically.
- The fluid is non-Newtonian with constant physical properties. The shear stress may be described by the power-law model.
- The body forces are neglected.
- The entering fluid temperature, T_e, is constant at upstream infinity ($z \to -\infty$).
- There is a step change in the wall temperature at $z = 0$. For $z \leq 0$ the walls are kept at T_e. For $0 < z$, the walls are at a constant temperature T_w.
- A symmetry line exists at the half width of the channel.

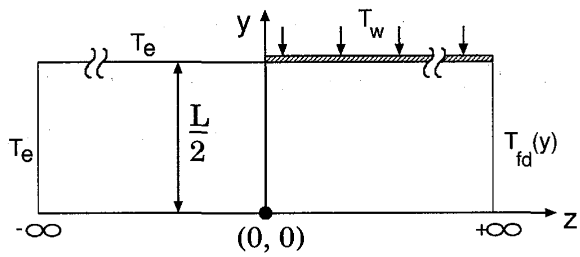

Fig.1 Geometrical configuration

Fluid Flow

With the assumptions described above, the governing momentum equation with the non-slip condition is

$$\frac{d\tau}{dy} = -\frac{dP}{dz}. \tag{1}$$

$$\text{B.C.}: \begin{cases} \dfrac{du}{dy} = 0 \quad \text{at} \quad y = 0 \\ u = 0 \quad \text{at} \quad y = \dfrac{L}{2}. \end{cases} \tag{2}$$

The shear stress, τ, in Eq.(1) is

$$\tau = -m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} \tag{3}$$

The solution of the momentum equation for the fully developed laminar flow of power-law fluids is

$$u = \frac{n}{n+1}\left[\frac{1}{m}\left(-\frac{dP}{dz}\right)\right]^{\frac{1}{n}}\left[\left(\frac{L}{2}\right)^{\frac{n+1}{n}} - y^{\frac{n+1}{n}}\right] \tag{4}$$

Then the average velocity of the flow is

$$u_m \equiv \frac{2}{L}\int_0^{L/2} u\, dy$$

$$= \frac{n}{2n+1}\left[\frac{1}{m}\left(-\frac{dP}{dz}\right)\right]^{\frac{1}{n}}\left(\frac{L}{2}\right)^{\frac{n+1}{n}}. \tag{5}$$

Introducing the following dimensionless parameters

$$u^* = \frac{u}{u_m}, \qquad y^* = \frac{y}{D_h} \tag{6}$$

yields the exact solution for the dimensionless velocity as

$$u^* = \frac{2n+1}{n+1}\left[1 - (4y^*)^{\frac{n+1}{n}}\right] \tag{7}$$

Heat Transfer

The energy equation together with the assumptions above is written as

$$\rho c_p u \frac{\partial T}{\partial z} = k\left[\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] - \tau \frac{du}{dy} \tag{8}$$

$$\text{in} \quad 0 < y < \frac{L}{2} \quad \text{and} \quad -\infty < z < \infty$$

The boundary conditions are:

$$\begin{cases} \dfrac{\partial T}{\partial y} = 0 & \text{at} \quad y = 0 \quad \text{for} \quad -\infty < z < \infty \\ T = T_w & \text{at} \quad y = \dfrac{L}{2} \quad \text{for} \quad 0 < z \\ T = T_e & \text{at} \quad y = \dfrac{L}{2} \quad \text{for} \quad z \le 0 \\ \lim\limits_{z \to -\infty} \quad T = T_e & \text{for} \quad 0 < y < \dfrac{L}{2} \\ \lim\limits_{z \to +\infty} \quad T = T_{fd}(y) & \text{for} \quad 0 < y < \dfrac{L}{2}. \end{cases} \tag{9}$$

The bulk temperature and Nusselt number are defined as

$$T_b \equiv \frac{\int_0^{L/2} uT dy}{\int_0^{L/2} u dy} \tag{10}$$

$$Nu \equiv \frac{hD_h}{k} \tag{11}$$

where

$$h = \frac{q_w}{T_w - T_b}, \qquad q_w = k\left.\frac{\partial T}{\partial y}\right|_{y=L/2} \tag{12}$$

The following dimensionless quantities are introduced

$$z^* = z/(Pe \cdot D_h) \tag{13}$$

$$Pe = \rho c_p u_m D_h / k \tag{14}$$

$$\theta = \frac{T - T_e}{T_w - T_e} \tag{15}$$

$$Br = \frac{m u_m^{n+1} D_h^{1-n}}{k(T_w - T_e)} \tag{16}$$

The substitution of the above quantities into the dimensional formulation gives

$$u^* \frac{\partial \theta}{\partial z^*} = \frac{\partial^2 \theta}{\partial y^{*2}} + \frac{1}{Pe^2}\frac{\partial^2 \theta}{\partial z^{*2}} + Br\left(-\frac{du^*}{dy^*}\right)^{n+1} \tag{17}$$

$$\text{in} \quad 0 < y^* < \frac{1}{4} \quad \text{and} \quad -\infty < z^* < \infty$$

$$\begin{cases} \dfrac{\partial \theta}{\partial y^*} = 0 & \text{at} \quad y^* = 0 \quad \text{for} \quad -\infty < z^* < \infty \\ \theta = 1 & \text{at} \quad y^* = \dfrac{1}{4} \quad \text{for} \quad 0 < z^* \\ \theta = 0 & \text{at} \quad y^* = \dfrac{1}{4} \quad \text{for} \quad z^* \le 0 \\ \lim\limits_{z^* \to -\infty} \quad \theta = 0 & \text{for} \quad 0 < y^* < \dfrac{1}{4} \\ \lim\limits_{z^* \to +\infty} \quad \theta = \theta_{fd}(y^*) & \text{for} \quad 0 < y^* < \dfrac{1}{4}. \end{cases} \tag{18}$$

The bulk temperature in the dimensionless form is calculated as

$$\theta_b \equiv 4\int_0^{1/4} u^* \, \theta \, dy^* \tag{19}$$

Nusselt number at the wall

$$Nu_w = \frac{1}{(1-\theta_b)}\left.\frac{\partial \theta}{\partial y^*}\right|_{y^*=1/4} \tag{20}$$

In the fully developed region the dimensionless temperature is a function of y^* alone, i.e., $\frac{\partial \theta}{\partial z^*} = \frac{\partial^2 \theta}{\partial z^{*2}} = 0$. Then the dimensionless temperature θ_{fd} corresponding to the boundary condition of constant wall temperature is the particular solution of the following equation.

$$\frac{d^2 \theta_{fd}}{dy^{*2}} = -Br\left(-\frac{du^*}{dy^*}\right)^{n+1} \tag{21}$$

$$\begin{cases} \dfrac{d\theta_{fd}}{dy^*} = 0 & \text{at} \quad y^* = 0 \\ \theta_{fd} = 1 & \text{at} \quad y^* = \dfrac{1}{4} \end{cases} \tag{22}$$

The temperature gradient is obtained as

$$\frac{d\theta_{fd}}{dy^*} = -Br\, 4^{\frac{(n+1)^2}{n}}\left[\frac{(2n+1)}{n}\right]^n y^{*\frac{2n+1}{n}} \tag{23}$$

Eq.(21) has been solved analytically and the solution for θ_{fd} is

$$\theta_{fd} = 1 + Br\left(\frac{2n+1}{n}\right)^n \times \frac{n}{3n+1}\left[-4^{\frac{(n+1)^2}{n}} y^{*\frac{3n+1}{n}} + 4^{n-1}\right] \tag{24}$$

The bulk temperature in the fully developed region is

$$\theta_{b_{fd}} = 1 + 2Br\left(\frac{4}{n}\right)^{n-1}\frac{(3n+1)(2n+1)^n}{(4n+1)(5n+2)} \tag{25}$$

The asymptotic value of the Nusselt number is

$$Nu_{fd} = \frac{2(4n+1)(5n+2)}{n(3n+1)} \tag{26}$$

for non-zero Brinkman numbers. For $Br = 0$, the

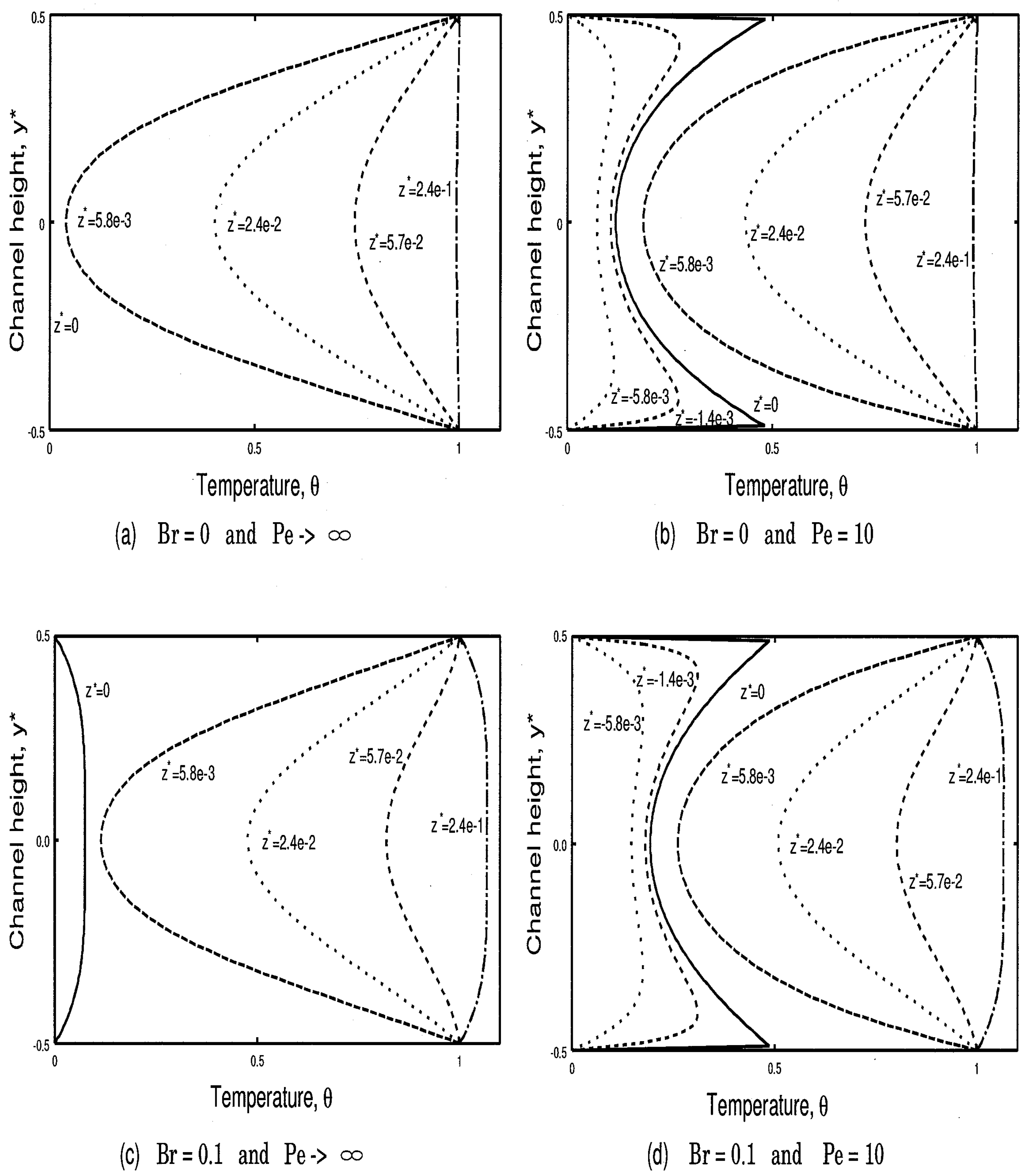

Fig. 2 Developing temperature profiles ($n = 1$)

(a) Negligible viscous dissipation and fluid axial heat conduction: $Br = 0$ and $Pe \to \infty$. (b) Negligible viscous dissipation and considerable fluid axial heat conduction: $Br = 0$ and $Pe = 10$. (c) Considerable viscous dissipation and negligible fluid axial heat conduction: $Br = 0.1$ and $Pe \to \infty$. (d) Both viscous dissipation and fluid axial heat conduction are considerable: $Br = 0.1$ and $Pe = 10$.

value of Nu_{fd} cannot be determined as

$$\frac{d\theta_{fd}}{dy^*} = 0, \quad \theta_{fd} = 1 \quad \text{and} \quad \theta_{b_{fd}} = 1$$

from Eqs. (23) - (25).

3. Results and Discussion

The temperature distributions of the non-Newtonian power-law fluids flowing in parallel-plates duct were calculated for an axial domain of $-\infty < z < \infty$, where at the origin ($z = 0$) there is a step jump in the wall temperature.

The calculation has been carried out by using the finite difference method. The range of parameters considered are:

Brinkman number: -1, -0.5, -0.1, 0.0, 0.1, 0.5, 1

Peclet number: ∞, 100, 50, 20, 10, 5, 2.

Flow index: 1, 1/3, 0.5, 1.5 and 3

The variations of local temperature profiles with Br and Pe are shown in Fig. 2. At $z^* = 0$, there is a step change in the wall temperature. Figures 2(a) and 2(b) show the case of $Br = 0$, that is the fluid experiences no gain of heat due to viscous dissipation. In Fig. 2(a), for $Pe \to \infty$ and $Br = 0$, at $z^* = 0$ ($-0.5 < y^* < 0.5$) the dimensionless temperature of the fluid is zero. But for $Pe = 10$ and $Br = 0$ in Fig. 2(b) the fluid temperature increase is occurred for negative values of z^*. This indicates that the temperature increase is due to the fluid axial heat conduction and that the influence of axial heat conduction in the fluid for $z^* \leq 0$ vanishes with an increasing Peclet number. From Figs. 2(c) and 2(d) for $Br = 0.1$ it can be seen that the dimensionless temperature of the fluid for $z^* \leq 0$ deviates definitely from zero. This increase is due to the contribution of viscous dissipation in the flowing fluid. As the viscous dissipation effect builds up, heat is transferred to the main body of the fluid flow and heat generation due to viscous dissipation behaves like a heat source, increasing the fluid temperature as seen in Figs. 2(c) and 2(d). In fact, it can be observed from the temperature developments in Fig. 2 that the fluid temperature increases before the fluid reaches the heated wall region because of the heat generated by viscous dissipation and the heat conducted from downstream into the region of $z \leq 0$.

In the following figures the heat transfer results are illustrated in terms of the conventional Nusselt number at the wall. Figure 3 presents the results in the thermally developing range for the three different fluids. It is worthwhile to compare the present results with those reported by Cotta and Ozisik[2] who presented the results for the limiting case of neglected viscous dissipation and fluid axial heat conduction for the power-law fluids. Even at small values of z^*, the agreement is excellent.

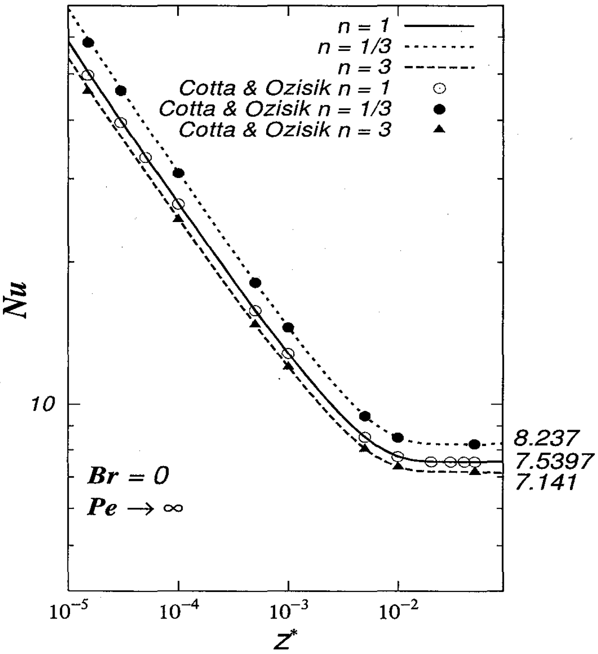

Fig. 3 Nusselt number for different fluids (n = 1, 1/3 and 3)

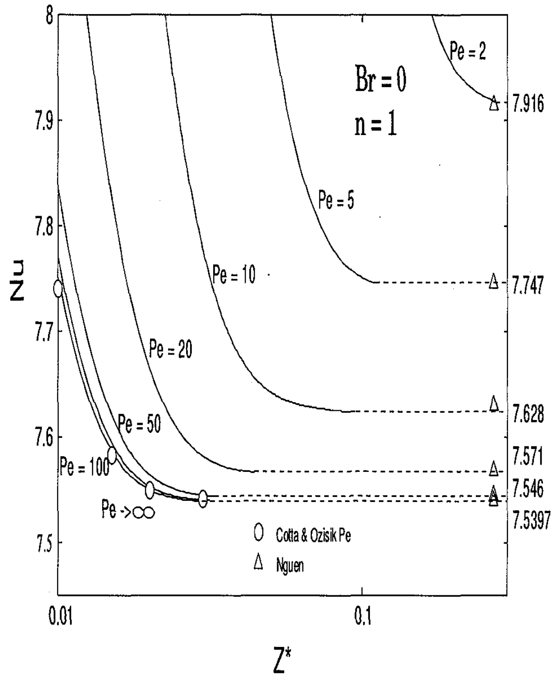

Fig. 4 Nusselt number for various Peclet number for the negligible viscous dissipation case (n = 1)

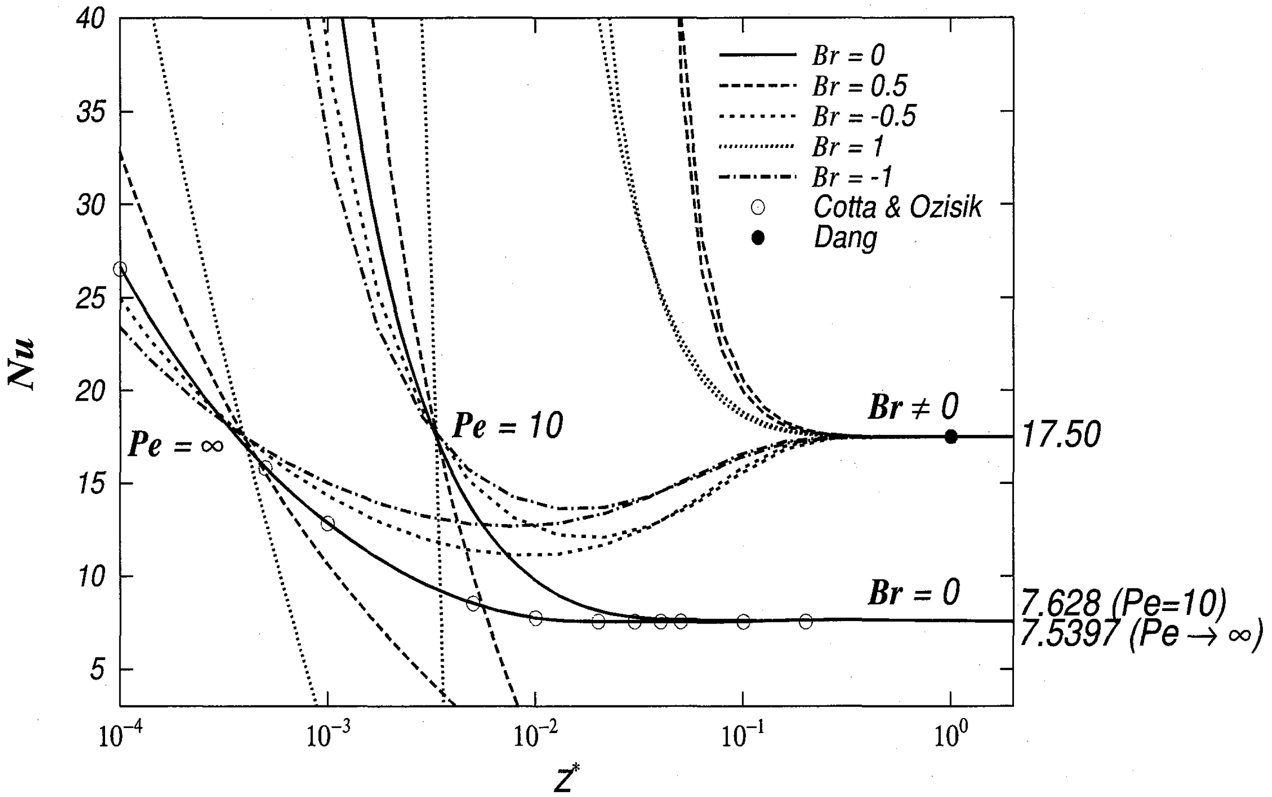

Fig. 5 Effects of Br and Pe on Nusselt number for $n = 1$

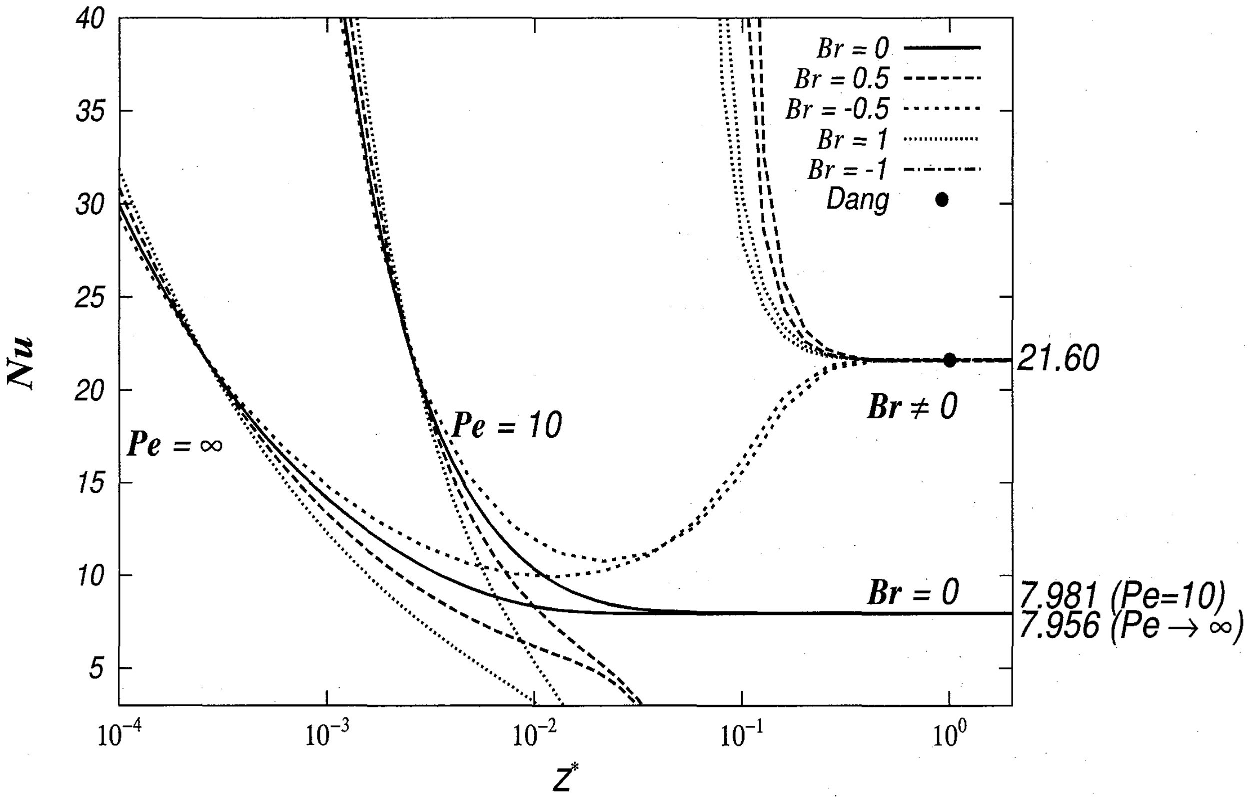

Fig. 6 Effects of Br and Pe on Nusselt number for $n = 0.5$

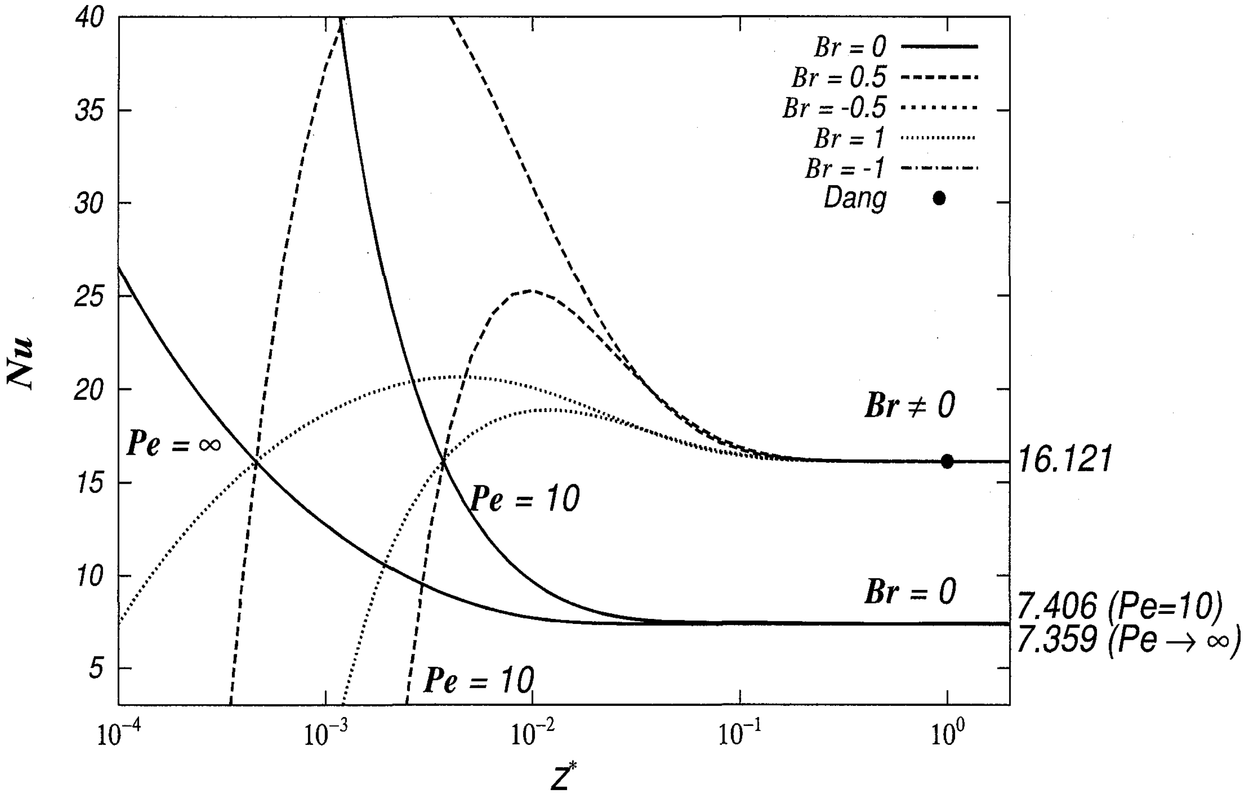

Fig. 7 Effects of Br and Pe on Nusselt number for $n = 1.5$

In Fig. 4, the Nusselt number is shown as a function of the axial coordinate with Peclet number as a parameter. These Nusselt curves are for the case of negligible viscous dissipation and for Newtonian fluids. The circles show the results by Cotta and Ozisik[2] and the triangles are for the results by Nguen[3].

The effects of both Peclet number and Brinkman number on Nusselt number are demonstrated in Figs. 5-7 for Newtonian ($n = 1$), pseudoplastic ($n = 0.5$) and dilatant ($n = 1.5$) fluids, respectively. The solid lines stand for the case of negligible viscous dissipation. The dashed lines are the Nusselt curves for non-zero Brinkman numbers. In this study, according to the Brinkman number definition, for minus Brinkman numbers the fluid is considered as being cooled and positive Brinkman numbers show that the fluid is being heated from the wall. The results for Nusselt numbers in the fully developed range was in excellent agreement with those of by Dang[6]. Equation (26) ensures that the asymptotic Nusselt number has a single value for any non-zero Br for a given fluid.

4. Conclusions

Thermally developing heat transfer of non-Newtonian power-law fluids flowing in parallel-plates duct under the boundary conditions of constant wall temperature has been analyzed taking into account of the effects of viscous dissipation and fluid axial heat conduction. In view of the mathematical formulation, the energy equation was an elliptic type problem and it was solved by considering two semi-infinite axial domains.

The results are presented graphically in dimensionless form. In order to verify the numerical scheme applied in this study, our results for special case studies are compared with the available data sets.

An inspection of the temperature profile reveals that the fluid temperature increases at $z \leq 0$ due to fluid axial heat conduction and viscous dissipation before the fluid enters the heated wall region.

The results indicate that for a given fluid the asymptotic value of Nusselt number at the wall was a single value for different non-zero values of Brinkman number. For non-zero Brinkman numbers, the asymptotic Nusselt number does not depend on the Peclet number values. However, for zero Brinkman numbers, the asymptotic Nusselt numbers depend on Peclet number values and with a decrease in Peclet number the asymptotic Nusselt numbers increase slightly.

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