Fully developed laminar heat transfer to non-Newtonian fluids flowing in a concentric annulus with a moving core (The case of first kind of thermal boundary condition)

by

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The fully developed laminar heat transfer to non-Newtonian fluids flowing in a concentric annulus with a moving core is analyzed by taking into account the viscous dissipation of the flowing fluid. Applying the shear stress described by the modified power-law model, the energy equation together with the fully developed velocity profile is solved numerically for the thermal boundary conditions of the tube walls being kept at constant but different temperatures. The effects of the flow index, the relative velocity of the moving core, the dimensionless shear rate parameter and the Brinkman number on the temperature distribution and Nusselt numbers at the tube walls are discussed.

1. Introduction

The problems of the fully developed heat transfer to non-Newtonian fluids in a concentric annulus with an axially moving core were studied previously[1]-[2] for the thermal boundary condition of constant heat flux at either tube wall.

In this paper, the fully developed heat transfer is studied for the thermal boundary condition of the tube walls being kept at constant but different temperatures or for the first kind of boundary condition[3]. The case of the inner tube wall temperature being kept higher than that of the outer tube wall is referred to CASE A and the counter part is referred to CASE B. Because of the space restraint only CASE A has been discussed in the present paper.

Applying the velocity profile of the modified power law fluids obtained in the previous report[4], the energy equation including the viscous dissipation term is solved numerically. The effects of the relative velocity of the moving core, the flow index and the dimensionless shear rate parameter and the Brinkman number on the temperature distribution and Nusselt number are discussed.

Nomenclature

Br : Brinkman number
c_p : specific heat at constant pressure
D_h : hydraulic diameter $= 2(R_o - R_i)$
k : thermal conductivity
n : flow index
Nu : Nusselt number
r : radial coordinate
r^* : dimensionless radial coordinate
R : tube radius
T : temperature
u : fully developed velocity profile
u_m : average velocity of the fluid
u^* : dimensionless velocity $(=u/u_m)$

Greek Symbols

α : radius ratio
β : dimensionless shear rate parameter
η_a : apparent viscosity
η_a^* : dimensionless apparent viscosity
η_0 : viscosity at zero shear rate
ρ : density
τ : shear stress
θ : dimensionless temperature
ξ : transformed dimensionless radial coordinate $= [2(1-\alpha)r^* - \alpha]/(1-\alpha)$

Subscripts

b : bulk
i : inner wall or inner tube
o : outer wall or outer tube

2. Analysis

The geometrical configuration and the coordinate system for the analysis are shown in Fig.1.

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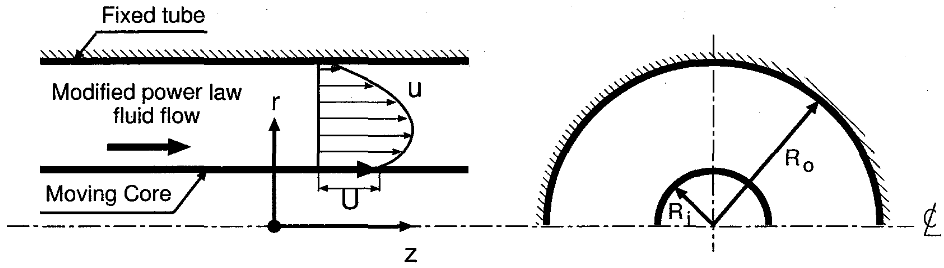

Fig.1 Geometrical configuration

The assumptions and conditions used in the analysis are:

- The flow is steady, laminar and fully developed hydrodynamically.
- The fluid is non-Newtonian with constant physical properties. The shear stress may be described by the modified power-law model.
- The body forces and the fluid axial heat conduction are neglected.

The energy equation together with the assumptions above is written as

$$k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\tau\frac{du}{dr}=0 \tag{1}$$

The thermal boundary conditions are:
CASE A

$$\begin{cases} T=T_i & \text{at} \quad r=R_i \\ T=T_o & \text{at} \quad r=R_o \end{cases} \tag{2}$$

The velocity , u, of the modified power-law fluids in concentric annuli with moving cores was evaluated and reported in the previous paper(1).

τ in Eq.(1) is the shear stress defined by

$$\tau \equiv \eta_a\frac{du}{dr} \tag{3}$$

where η_a is the apparent viscosity defined as

$$\eta_a \equiv \frac{\eta_0}{1+\frac{\eta_0}{m}\left|\frac{du}{dr}\right|^{1-n}} \qquad \text{for} \quad n<1, \tag{4}$$

$$\eta_a \equiv \eta_0\left(1+\frac{m}{\eta_0}\left|\frac{du}{dr}\right|^{n-1}\right) \qquad \text{for} \quad n>1. \tag{5}$$

The bulk temperature is defined as

$$T_b \equiv \frac{\int_{R_i}^{R_o} uTrdr}{\int_{R_i}^{R_o} urdr}=\frac{2}{u_m(R_o^2-R_i^2)}\int_{R_i}^{R_o} uTrdr \tag{6}$$

The Nusselt numbers at the walls are

$$Nu_j \equiv \frac{h_jD_h}{k} \tag{7}$$

where $j=i$ for the inner tube wall and $j=o$ for the outer tube wall and

$$q_i=-k\left.\frac{\partial T}{\partial r}\right|_{r=R_i} \qquad q_o=k\left.\frac{\partial T}{\partial r}\right|_{r=R_o},$$

$$h_j=\frac{q_j}{|T_j-T_b|} \tag{8}$$

The following dimensionless quantities are introduced

$$r^*=\frac{r}{D_h} \tag{9}$$

$$\theta=\frac{T-T_o}{T_i-T_o} \tag{10}$$

$$Br=\frac{\eta^* u_m^2}{D_h k(T_i-T_o)} \tag{11}$$

where

$$\eta^*=\frac{\eta_0}{1+\beta} \qquad \text{for} \quad n<1, \tag{12}$$

$$\eta^*=\eta_0\left(1+\frac{1}{\beta}\right) \qquad \text{for} \quad n>1, \tag{13}$$

Dimensionless apparent viscosity, η_a^*, is defined as

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*}=\frac{1+\beta}{1+\beta\left|\frac{du^*}{dr^*}\right|^{1-n}} \qquad \text{for} \quad n<1, \tag{14}$$

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*}=\frac{\beta+\left|\frac{du^*}{dr^*}\right|^{n-1}}{\beta+1} \qquad \text{for} \quad n>1 \tag{15}$$

The substitution of the above quantities into the dimensional formulation gives

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial\theta}{\partial r^*}\right) + Br\eta^*\left(\frac{du^*}{dr^*}\right)^2 = 0 \qquad (16)$$

$$\begin{cases} \theta = 1 \quad \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)} \\ \theta = 0 \quad \text{at} \quad r^* = \frac{1}{2(1-\alpha)} \end{cases} \qquad (17)$$

The bulk temperature in the dimensionless form is calculated as

$$\theta_b \equiv \frac{8(1-\alpha)}{1+\alpha}\int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} u^* \ \theta \ r^* \ dr^* \qquad (18)$$

Nusselt number at the inner and outer tube walls are

$$Nu_i = -\frac{1}{1-\theta_b}\frac{\partial\theta}{\partial r^*}\bigg|_{r^*=\frac{\alpha}{2(1-\alpha)}} \qquad (19)$$

$$Nu_o = -\frac{1}{\theta_b}\frac{\partial\theta}{\partial r^*}\bigg|_{r^*=\frac{1}{2(1-\alpha)}}$$

3. Results and Discussion

In the following figures the solutions are illustrated for the annulus of radius ratio $\alpha = 0.5$ and for the three values of the core velocity namely, U^* = -1, 0 and 1. The effect of the viscous dissipation is demonstrated by the Brinkman number. The effect of the Brinkman number on the temperature distributions across the channel is demonstrated in Figs.2-4. Figure 2 is for the pseudoplastic fluid of $n = 0.5$ and $\beta = 1$ while Figs. 3-4 demonstrate the temperature distributions for the Newtonian fluid $n = 1$ and for the dilatant fluid of $n = 1.5$ and $\beta = 1$. $\xi = 0$ corresponds to the inner tube wall and $\xi = 1$ is the outer tube wall. The parameter in these figures are the Brinkman number. It is seen that the results for the Newtonian ($n = 1$) fluid and for the non-Newtonian fluids are similar qualitatively. The dimensionless temperature increase due to the viscous dissipation is more pronounced for U^* = - 1 and $U^* = 0$ and this is attributed to the velocity gradient reported in the previous report[4]. For U^* = - 1.0, θ greatly increases with an increase in Br near the moving core while for $U^* = 1.0$, the increas in θ is small near the moving core or $\xi = 0$. Also it is seen, the temperature of the Newtonian fluid is more sensitive to the Brinkman number than that of the pseudoplastic fluids, but less than for the dilatant fluid.

Nusselt numbers at the inner core, Nu_i, and at the outer tube wall, Nu_o, are shown in Figs.5-6. In these figures the Nusselt numbers are shown as a function of the dimensionless shear rate, β, and the Brinkman number is a parameter. With an increase in Br, the Nusselt number at the inner tube decreases while Nu_o increases.

4. Conclusions

The fully developed laminar heat transfer of the modified power-law fluids in a concentric annulus with an axially moving core was analyzed taking into account the viscous dissipation. In this work, the numerical solutions for the thermal boundary condition of constant but different temperatures at the tube walls or for the thermal boundary condition of first kind were obtained. The effects of the flow index, the relative core velocity, the dimensionless shear rate parameter and of the Brinkman number on the temperature distribution and on the Nusselt numbers at the tube walls have been discussed.

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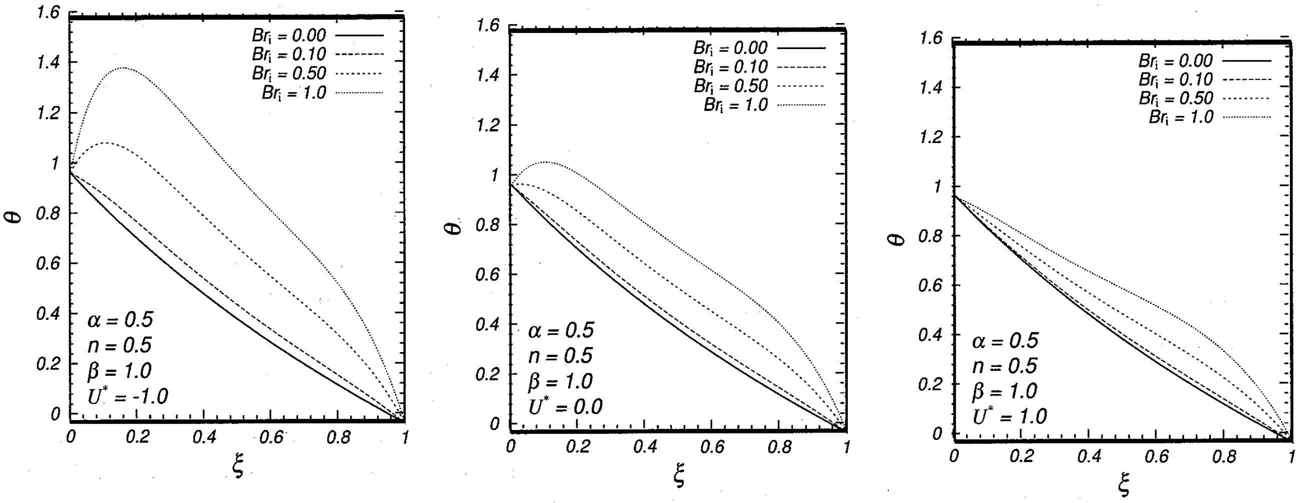


Fig. 2 Temperature profiles for the pseudoplastic fluid (U^* = -1; 0; 1)

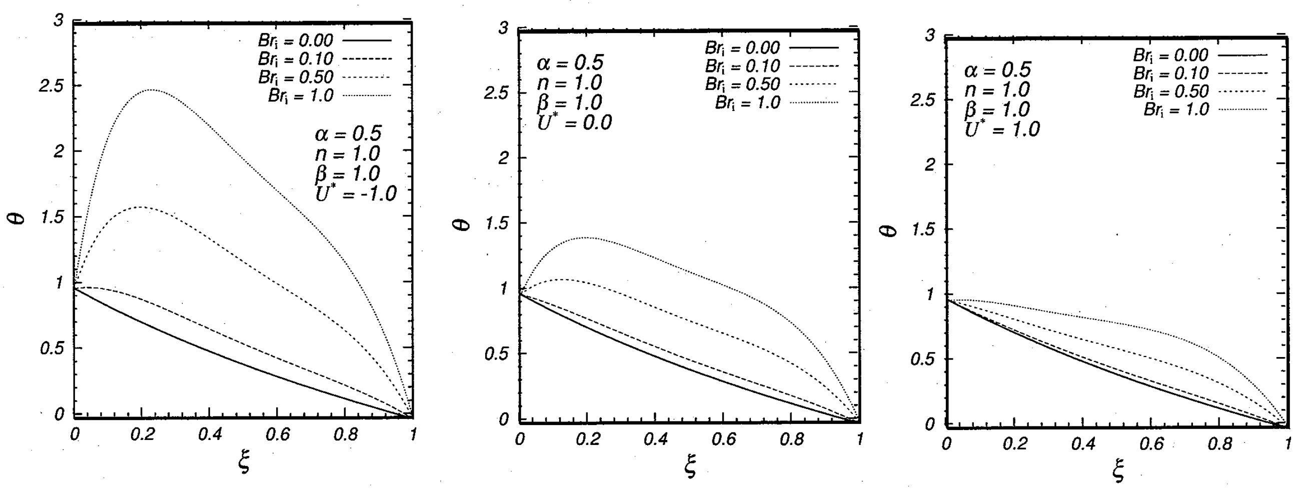


Fig. 3 Temperature profiles for the Newtonian fluid (U^* = -1; 0; 1)

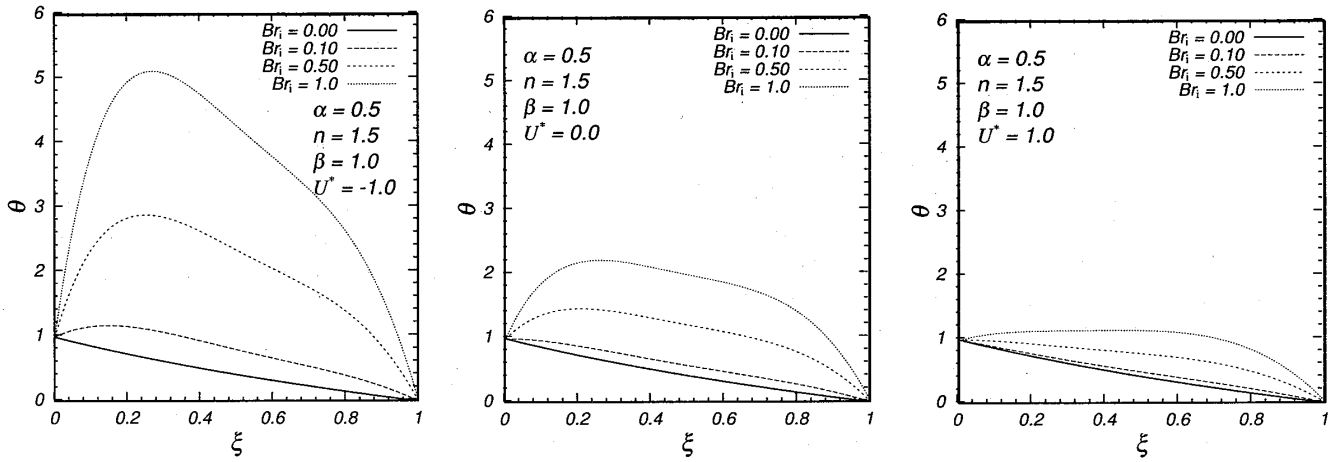
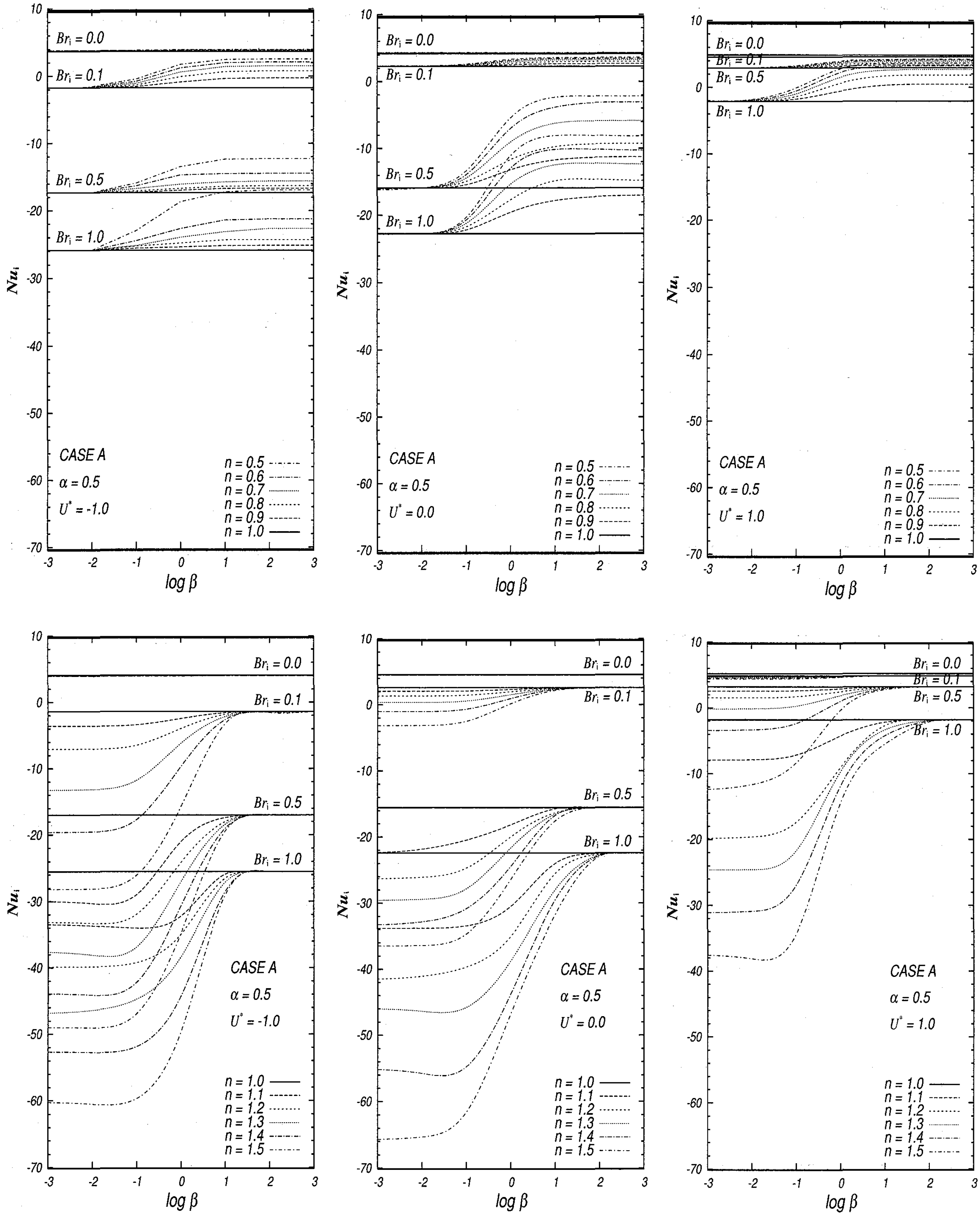
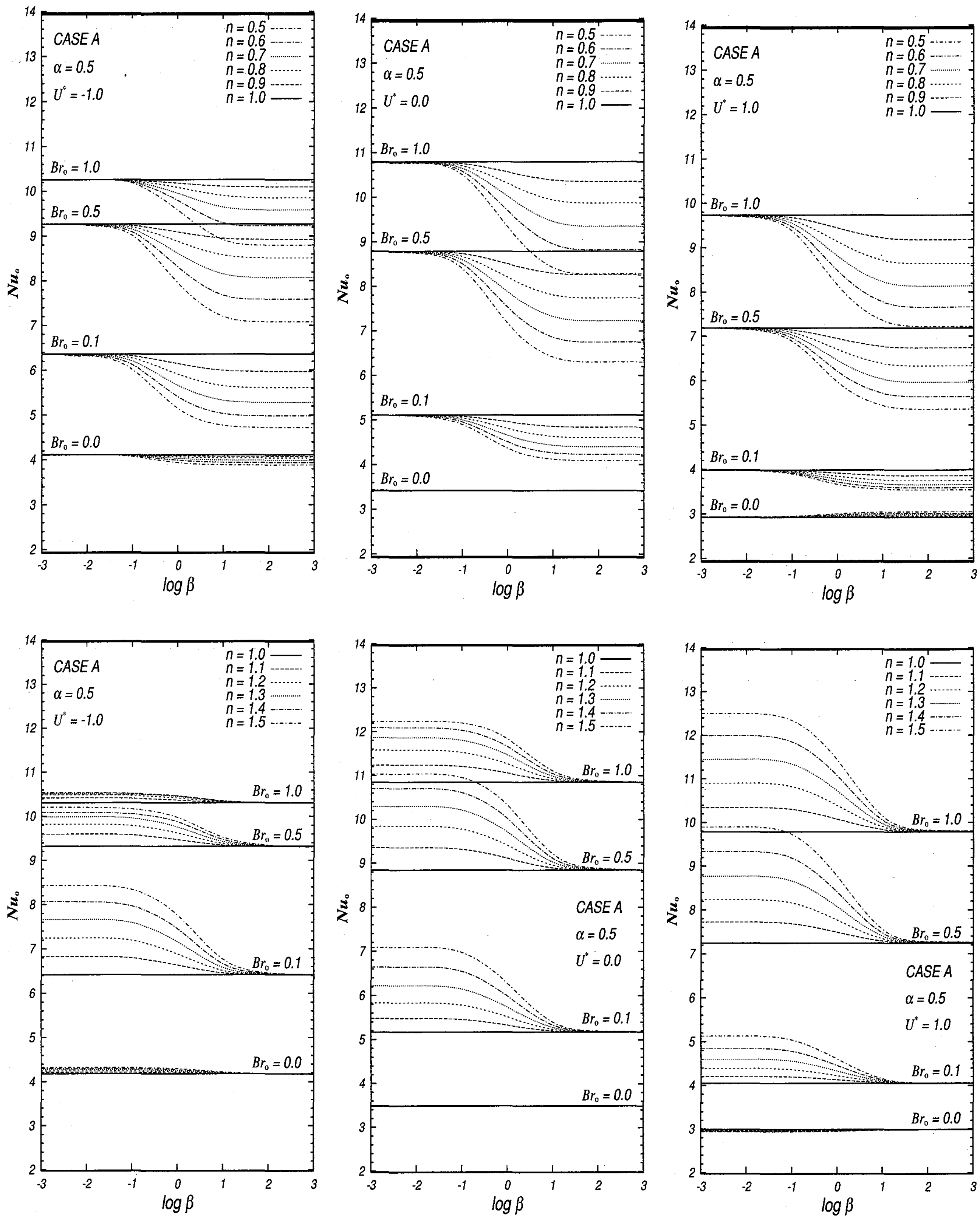


Fig. 4 Temperature profiles for the dilatant fluid (U^* = -1; 0; 1)



U^* = -1 U^* = 0 U^* = 1

Fig. 5 Nusselt number at the inner tube vs β (U^* = -1; 0; 1)



$U^* =$ -1 $U^* = 0$ $U^* = 1$

Fig. 6 Nusselt number at the outer tube vs β ($U^* =$ -1; 0; 1)