Heat transfer to modified power law fluids in plane Couette-Poiseuille laminar flow between parallel plates with constant heat flux at the fixed wall

by

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The fully developed laminar heat transfer to modified power-law fluids flowing between parallel plates with one moving plate was analyzed taking into account the viscous dissipation of the flowing fluid. Applying the shear stress described by the modified power-law model, the energy equation together with the fully developed velocity profile is solved numerically for the thermal boundary condition of constant heat flux at the fixed wall with the moving wall insulated. The effects of the flow index, relative velocity of the moving plate, dimensionless shear rate parameter and Brinkman number on Nusselt numbers at the plate walls were discussed.

1. Introduction

In the previous study(1), fully developed laminar heat transfer to modified power-law fluids flowing between parallel plates with the moving wall was analyzed numerically for the thermal boundary condition of constant wall heat flux at the moving wall together with the fixed wall insulated. This case was referred to as Case A.

In this paper, numerical solutions for another thermal boundary condition have been reported. Applying the shear stress described by the modified power-law model and the fully developed velocity profile obtained from the previous study(1), the energy equation including the viscous dissipation term is solved numerically for the thermal boundary condition of constant heat flux at the fixed wall with the moving wall insulated. This case is referred to as Case B. The effects of the relative velocity of the moving plate, flow index, dimensionless shear rate parameter and Brinkman number on Nusselt numbers at the plate walls are discussed.

Nomenclature

A	area normal to the flow direction
Br_B	Brinkman number for Case B
c_p	specific heat at constant pressure
D_h	hydraulic diameter $\equiv 2L$
k	thermal conductivity
L	distance between the parallel plates
m	consistency index
n	flow index
Nu_0	Nusselt number at the fixed wall
q_0	wall heat flux at the fixed wall
T	temperature
T_B	bulk temperature
u	axial velocity of the fluid
u_m	average velocity of the fluid
u^*	dimensionless velocity $\equiv u/u_m$
U	axial velocity of the moving plate
U^*	dimensionless relative velocity of the moving plate $\equiv U/u_m$
V	dimensionless parameter
y	coordinate normal to the fixed plate
y^*	dimensionless coordinate $\equiv y/D_h$
z	axial coordinate

Received October 25, 2001

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Greek Symbols

β	dimensionless shear rate parameter
η_a	apparent viscosity
η_a^*	dimensionless apparent viscosity $\equiv \eta_a / \eta^*$
η_0	viscosity at zero shear rate
η^*	reference viscosity
ρ	density
τ	shear stress
θ	dimensionless temperature
θ_B	dimensionless bulk temperature
θ_0	dimensionless temperature at the fixed wall

2. Analysis

The physical model for the analysis is shown in Fig. 1. The lower plate is axially moving at a constant velocity, U. The assumptions used in the analysis are:

1. The flow is incompressible, steady-laminar, and fully developed, hydrodynamically and thermally.
2. The fluid is non-Newtonian and the shear stress may be described by the modified power-law model[2], and physical properties are constant except viscosity.
3. The body forces and axial heat conduction are neglected.

Heat transfer

The energy equation together with the assumptions above is written as

$$k\frac{d^2T}{dy^2} + \tau\left(\frac{du}{dy}\right) = \rho c_p u \frac{dT_B}{dz}. \tag{1}$$

The velocity, u, and its gradient, $\frac{du}{dy}$, have been obtained in the previous report[1].

The thermal boundary conditions:

Case B (constant heat flux at the fixed plate with the moving plate insulated):

$$\begin{cases} -k\dfrac{\partial T}{\partial y} = q_0 \quad \text{at} \quad y = 0 \\ k\dfrac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = L \end{cases} \tag{2}$$

τ in Eq.(1) is the shear stress defined by

$$\tau = \eta_a \frac{du}{dy} \tag{3}$$

where η_a is the apparent viscosity defined by

$$\eta_a = \frac{\eta_0}{1 + \dfrac{\eta_0}{m}\left|\dfrac{du}{dy}\right|^{1-n}} \qquad \text{for } n<1, \tag{4}$$

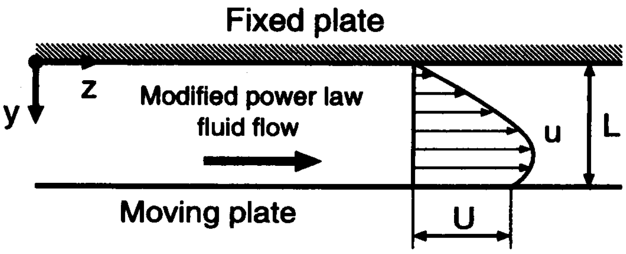

Fig.1 Schematic of parallel plates with one moving plate

$$\eta_a = \eta_0\left(1 + \frac{m}{\eta_0}\left|\frac{du}{dy}\right|^{n-1}\right) \qquad \text{for } n>1. \tag{5}$$

Dimensionless apparent viscosity, η_a^*, is defined as

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{1+\beta}{1+\beta\left|\dfrac{du^*}{dy^*}\right|^{1-n}} \qquad \text{for } n<1, \tag{6}$$

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{\beta + \left|\dfrac{du^*}{dy^*}\right|^{n-1}}{\beta+1} \qquad \text{for } n>1, \tag{7}$$

$$\eta^* = \frac{\eta_0}{1+\beta} \qquad \text{for } n<1, \tag{8}$$

$$\eta^* = \eta_0\left(1+\frac{1}{\beta}\right) \qquad \text{for } n>1, \tag{9}$$

$$\beta = \frac{\eta_0}{m}\left(\frac{u_m}{D_h}\right)^{1-n}. \tag{10}$$

Bulk temperature, T_B, is defined as

$$T_B \equiv \frac{\iint_A uTdA}{\iint_A udA}. \tag{11}$$

dT_B/dz in Eq.(1) is evaluated, from an energy balance, as

$$\frac{dT_B}{dz} = \frac{q_0}{\rho c_p u_m L}\left[1 + \frac{\int_0^L \tau\left(\frac{du}{dy}\right)dy}{q_0}\right] \tag{12}$$

The average fluid velocity, u_m, is defined as

$$u_m \equiv \frac{1}{L}\int_0^L u dy. \tag{13}$$

Introducing dimensionless temperature, θ, defined as

$$\theta \equiv T/[q_0 D_h / k], \tag{14}$$

the energy equation and the boundary conditions may be expressed in the dimensionless forms as

$$\frac{d^2\theta}{dy^{*2}} = 2u^* + Br_B \cdot V. \tag{15}$$

$$\textbf{Case B:} \quad \begin{cases} \dfrac{d\theta}{dy^*} = -1 \quad \text{at} \quad y^* = 0 \\ \dfrac{d\theta}{dy^*} = 0 \quad \text{at} \quad y^* = \dfrac{1}{2} \end{cases} \tag{16}$$

$$Br_B \equiv \frac{\eta^* u_m^2}{D_h q_0} \tag{17}$$

$$V = 2u^* \left\{ \int_0^{1/2} \eta_a^* \left(\frac{du^*}{dy^*} \right)^2 dy^* \right\} - \eta_a^* \left(\frac{du^*}{dy^*} \right)^2 \tag{18}$$

Nusselt number, Nu_0, is defined as

$$Nu_0 \equiv \frac{[q_0/(T_0 - T_B)]D_h}{k} = \frac{1}{\theta_0 - \theta_B} \tag{19}$$

where dimensionless bulk temperature, θ_B, is defined as

$$\theta_B \equiv T_B / [q_0 D_h / k] \tag{20}$$

and $(\theta_0 - \theta_B)$ is calculated as

$$\theta_0 - \theta_B = 2 \int_0^{1/2} u^* (\theta_0 - \theta) dy^*. \tag{21}$$

3. Results and discussion

In order to examine the reliability of the numerical solutions obtained in this study, the solutions computed for the energy equation together with the boundary conditions were compared with the analytical solutions determined at the extremes at $\beta \to 0$ and $\beta \to \infty$. Both solutions were in good agreement.

Temperature difference $(\theta - \theta_B)$ for Case B is shown in Figs.2, 3 and 4 for $\beta = 10^{-5}$, 1.0, 10^5 respectively. Parameter β defined by Eq. (10) represents dimensionless average shear rate under the specified fluid flow condition. For pseudoplastic fluids $(n < 1)$ the extreme at $\beta \to 0$ corresponds to a Newtonian fluid and that at $\beta \to \infty$ to a power law fluid. For dilatant fluids $(n > 1)$ the extreme at $\beta \to \infty$ corresponds to a Newtonian fluid and that at $\beta \to 0$ to a power law fluid. This correspondence at the extreme can be examined from Figs.2 and 4. As expected for $\beta \to 10^{-5}$ the temperature difference is almost same between pseudoplastic $(n = 0.5)$ and Newtonian $(n = 1.0)$ fluids (see Fig 2). It is seen from Fig.4 that when β approaches infinity (10^5) the temperature difference is almost same between dilatant $(n = 1.5)$ and Newtonian $(n = 1.0)$ fluids.

For $U^* = -1.0$ and $U^* = 0.0$, $(\theta - \theta_B)$ increases near the walls with an increase in Br_B and decreases in the middle region of the channel. This is attributed to the heat generated by viscous dissipation near the walls and large axial heat convection in the middle region. For the case of $U^* = 1.0$, $(\theta - \theta_B)$ increases with an increase in Br_B near the fixed wall. This is also owing to the heat generated by viscous dissipation near the fixed wall.

The numerical values of Nusselt number on Newtonian and non-Newtonian fluids are tabulated in Table 1. The results of this study were compared with the numerical values obtained by the analytical studies on Newtonian[3] and power law[4] Nusselt numbers, respectively. Both solutions were in good agreement.

Nusselt numbers for Case B, Nu_0, are shown in Fig. 5. Nu_0 decreases with an increase in Br_B for $U^* = -1.0$, $U^* = 0.0$ and $U^* = 1.0$. It is due to viscous dissipation near the fixed wall. It can be seen from Eq. (12) that the bulk temperature always increases when viscous dissipation effects increase. With a decrease in relative velocity, U^*, the difference between bulk temperature, θ_B, and the heated wall temperature, θ_0 in Eq. (19) decreases and Nusselt number, Nu_0, increases (see Fig.5). For the case of $U^* = -1.0$, it is seen that the relationship between Nu_0 and β is different for small values of Brinkman number $(Br_B = 0 \sim 0.01)$ for both pseudoplastic $(n < 1)$ and dilatant $(n > 1)$ fluids. It may be explained by how Br_B influences on Nu_0. It is seen from Fig.6 for $Br_B = 0$ we have higher values of Nu_0 for fluids with higher n. In the case of $Br_B = 0.01$ for greater values of n, Nu_0 tends to increase for pseudoplastic fluids but it decreases for dilatant fluids. For $U^* = 0.0$ and 1.0, Nusselt number, Nu_0, decreases with an increase flow index, n (see Fig. 6 and Table 1).

4. Conclusions

In this report the results on heat transfer for Case B only are discussed. The counterpart for Case A have been discussed in the previous report[1].

In this study the energy equation together with the boundary conditions of constant heat flux at the fixed wall with the moving wall insulated was solved numerically taking into account viscous dissipation effect. Nusselt number, Nu_0 behaviour corresponding to small values as well as higher values of Brinkman number, Br_B for different relative velocities, U^* is made clear.

In the analyses of heat transfer of non-Newtonian fluids, the results calculated by adopting the simple power-law fluid model do not predict correctly the values of Nusselt number in the region of lower shear rate. In order to calculate the whole region of shear rate from zero to infinity, the modified power-law model as adopted in this study should be used.

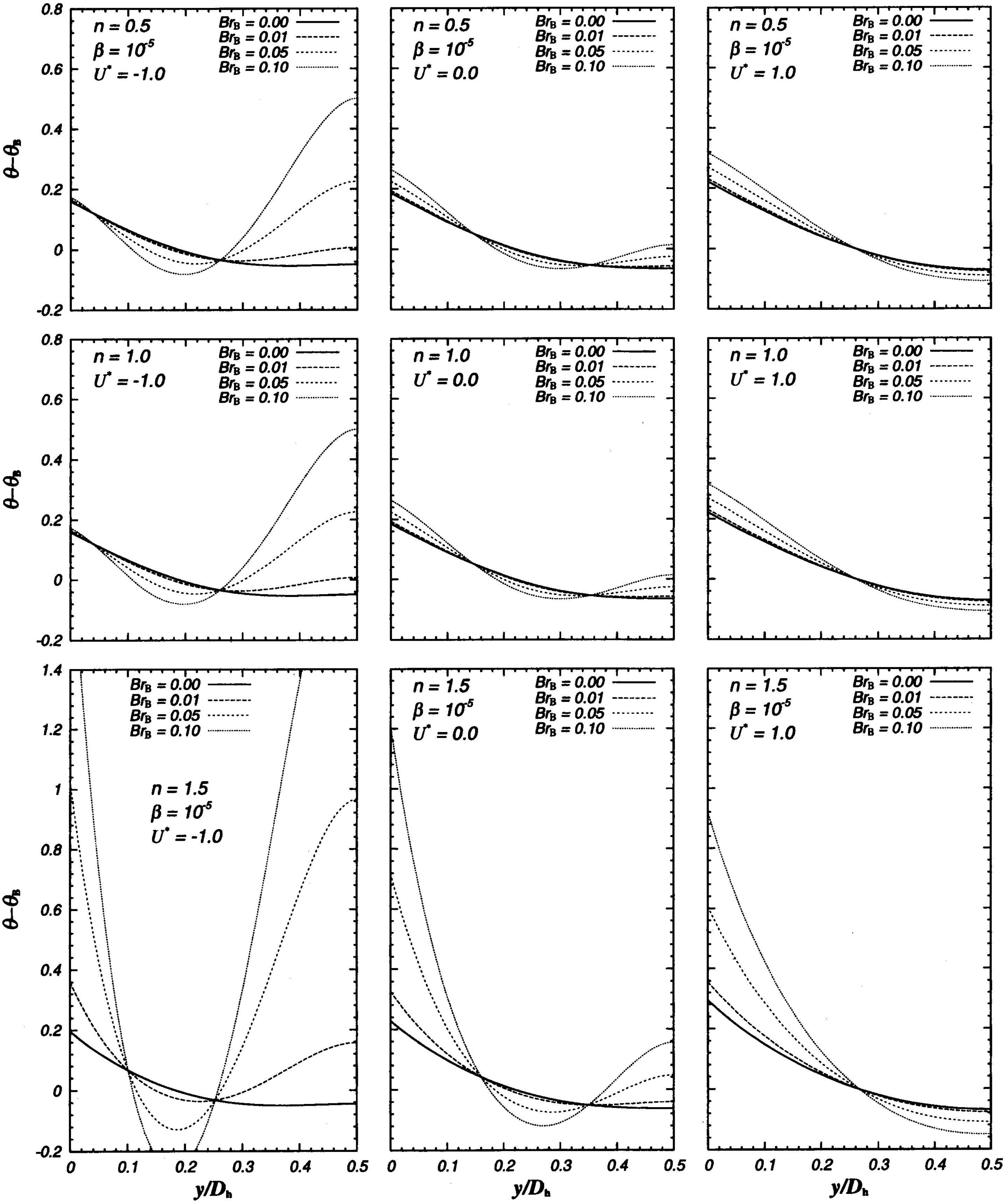

Fig.2 Dimensionless temperature difference for Case B ($\beta = 10^{-5}$)

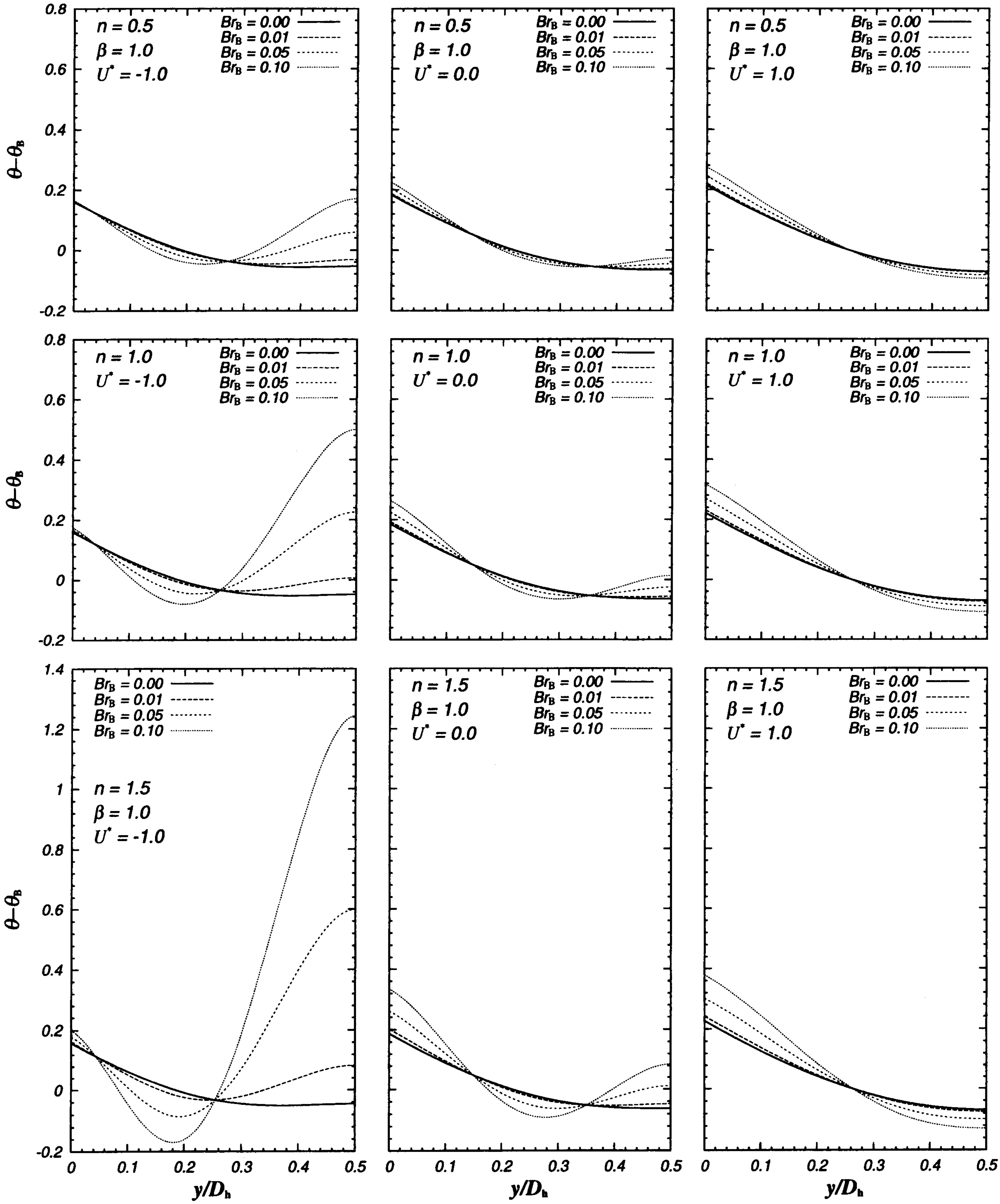

Fig.3 Dimensionless temperature difference for Case B ($\beta = 1$)

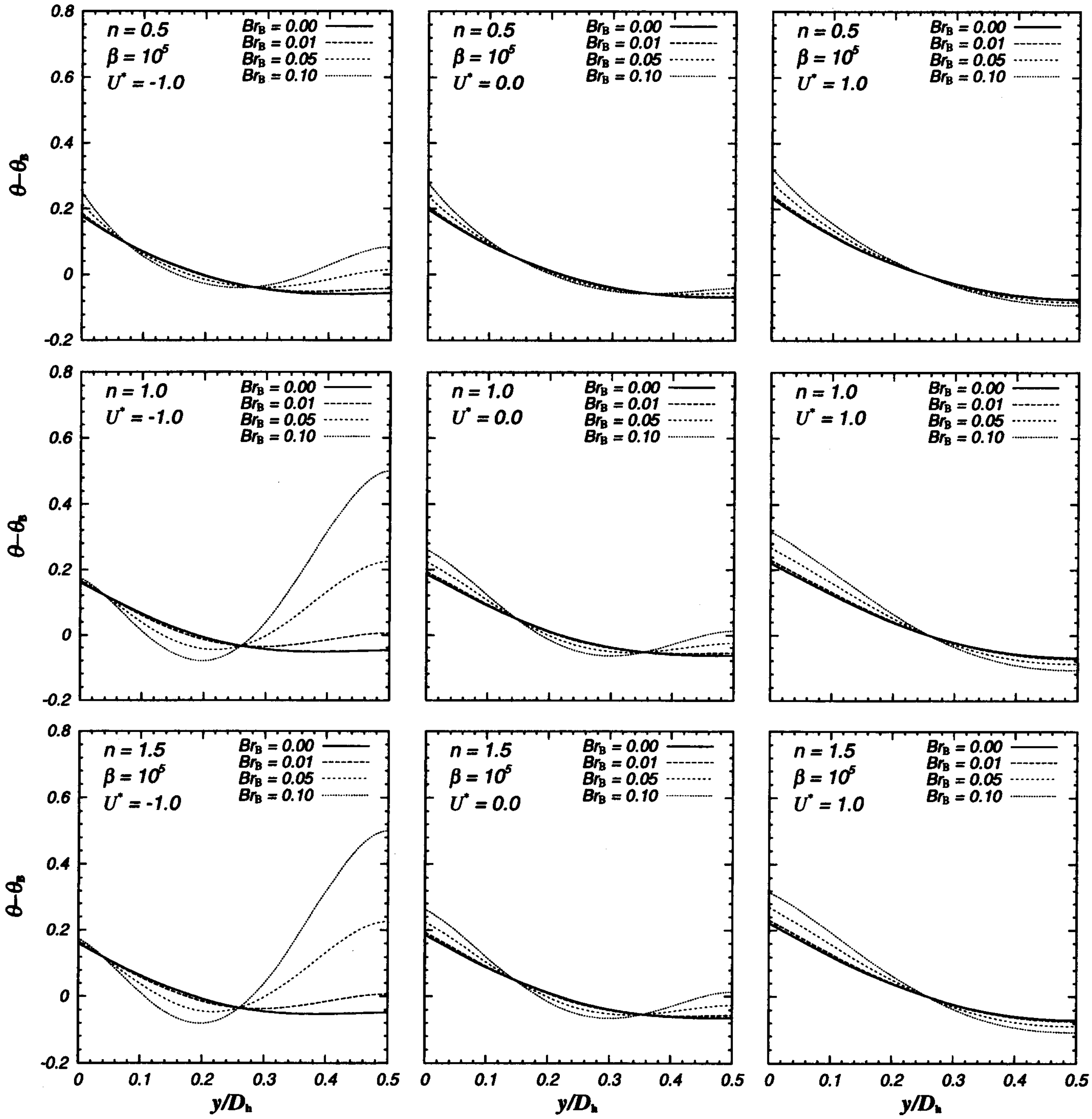

Fig.4 Dimensionless temperature difference for Case B ($\beta = 10^5$)

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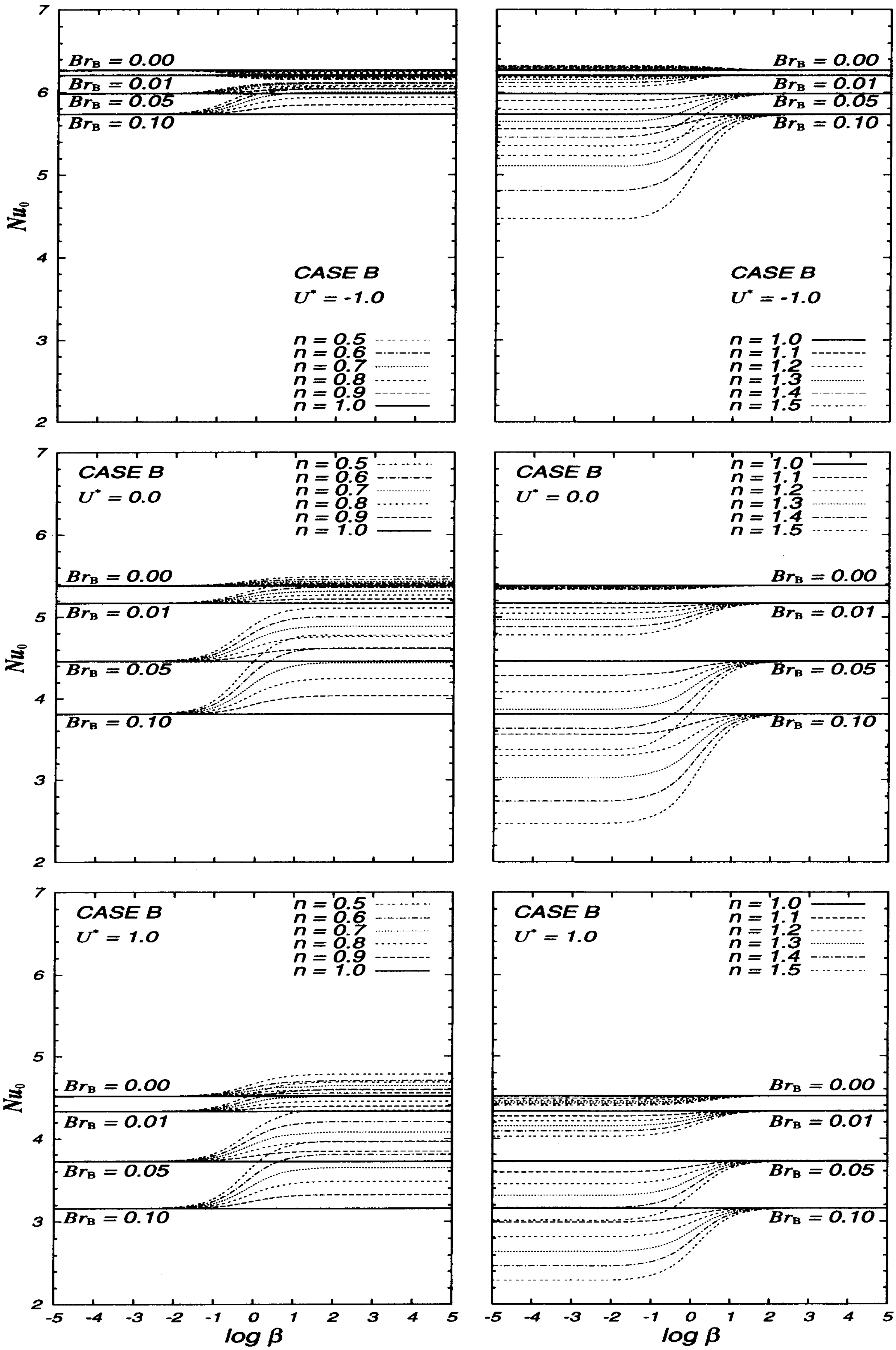

Fig.5 Nusselt numbers for Case B

Table 1 Nusselt number values for Case B

U^*	Br_B	Values	Nu_0 Flow index, n 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
-1	0.00	Newtonian						6.269					
		Power law	6.169	6.195	6.217	6.237	6.254	6.269	6.282	6.294	6.304	6.314	6.323
	0.01	Newtonian						6.209					
		Power law	6.159	6.180	6.195	6.206	6.211	6.209	6.201	6.186	6.160	6.123	6.071
	0.05	Newtonian						5.983					
		Power law	6.119	6.120	6.109	6.084	6.043	5.983	5.899	5.788	5.643	5.461	5.237
	0.10	Newtonian						5.722					
		Power law	6.070	6.047	6.004	5.939	5.847	5.722	5.560	5.357	5.107	4.810	4.469
0	0.00	Newtonian						5.385					
		Power law	5.492	5.460	5.436	5.416	5.399	5.385	5.373	5.362	5.353	5.345	5.338
	0.01	Newtonian						5.170					
		Power law	5.411	5.362	5.316	5.270	5.222	5.170	5.112	5.046	4.970	4.882	4.781
	0.05	Newtonian						4.459					
		Power law	5.110	5.003	4.887	4.760	4.618	4.459	4.280	4.082	3.864	3.627	3.373
	0.10	Newtonian						3.804					
		Power law	4.779	4.616	4.440	4.246	4.034	3.804	3.557	3.295	3.023	2.744	2.465
1	0.00	Newtonian						4.516					
		Power law	4.787	4.708	4.645	4.594	4.552	4.516	4.486	4.460	4.437	4.417	4.399
	0.01	Newtonian						4.330					
		Power law	4.690	4.598	4.520	4.452	4.389	4.330	4.272	4.214	4.155	4.093	4.028
	0.05	Newtonian						3.717					
		Power law	4.339	4.207	4.083	3.962	3.841	3.717	3.588	3.454	3.314	3.166	3.012
	0.10	Newtonian						3.158					
		Power law	3.967	3.802	3.642	3.483	3.322	3.158	2.990	2.819	2.644	2.468	2.290

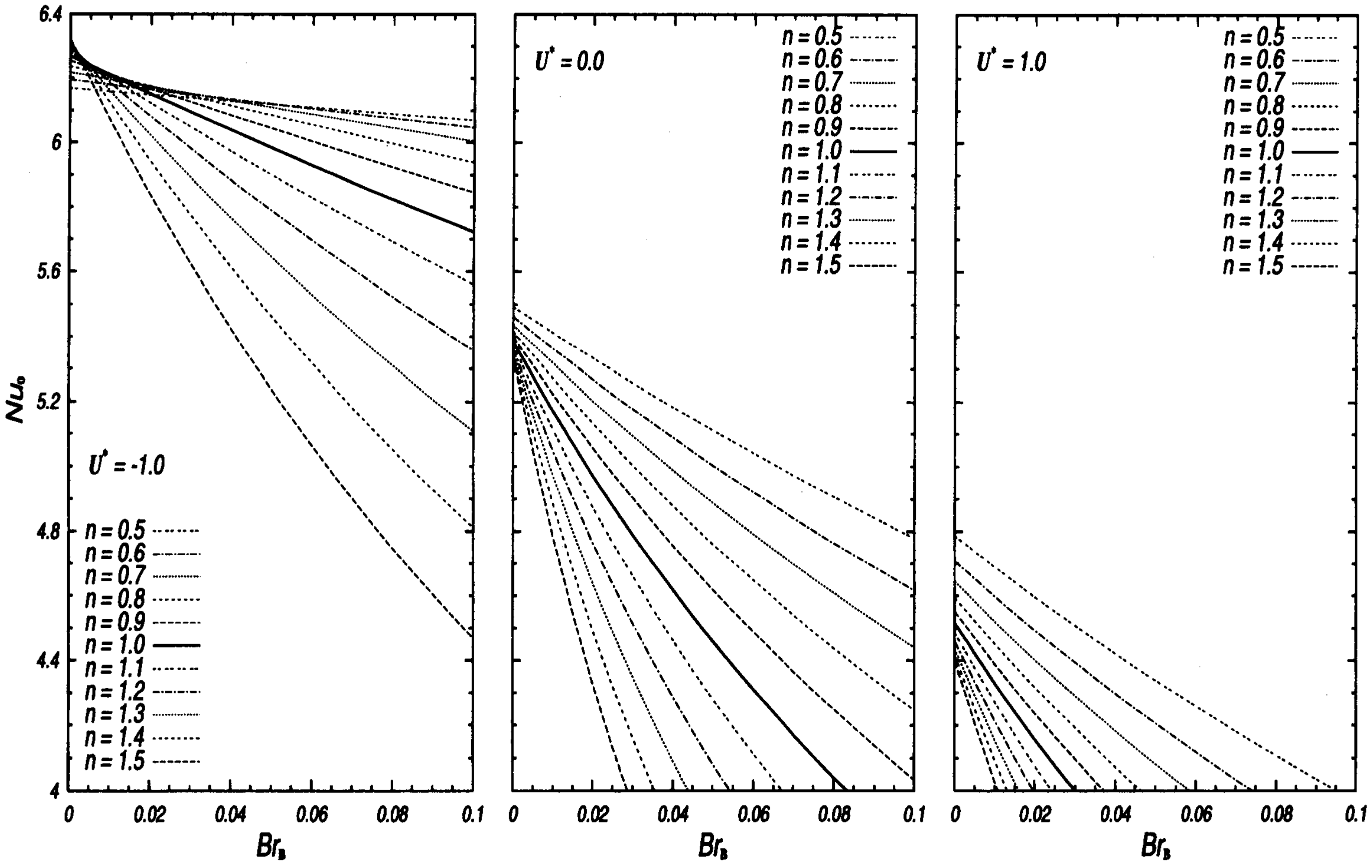

Fig.6 Nusselt numbers vs. Br_B