On the Effort Function

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Ⅰ. INTRODUCTION

Fukaura(1990) examined the seniority paying system within the context of labor management firm model, by using the notion of effort function, which relates the input of young workers who embody the new technology to the old(existent)worker's work effort. He concluded that the optimal labor growth rate was determined so as to equal the cost and benefit of labor input at the margin. However, the latter deeply depends on the behavior of effort function. This supplemental note will be spent in clarifying the theoretical foundations of the effort function.

Ⅱ. BASIC FRAMEWORK

Let's us begin to state the steady state employment schedule by using the two period model. In this case, the LMF must obey the following schedule, where L_{ij} = the worker in period j, and i = young($_Y$) or old($_O$).

period 0　　；　$L_{Y,0}$ and $L_{O,0}$ are hired,

period 1　　；　$L_{O,0}$ is retired.

$L_{Y,0}$ is re-employed as $L_{O,1}$.

$L_{Y,1}$ ($=L_{O,0}$) is newly employed, which is perfectly substitute to $L_{O,1}$.

To focus on the growing case, strictly speaking, we must consider next four type workers, that is,

$L_{O,1}$; re-employed $L_{Y,0}$ (seniority rule),

ΔL_O ; job hoppers from the other firm at period 1, called commonly "TORABAYU" in Japanese.

$L_{Y,1}$; newly employed young workers without up-to-date technology,

ΔL_Y ; newly employed young workers with up-to-date technology.

Accordingly, the labor growth rate is defined as

$$n+1 = \frac{\Delta L_Y + L_{Y,1}}{\Delta L_O + L_{O,1}}$$

For analytical convenience, we will set $L_{Y,1} = \Delta L_O = 0$. $L_{Y,1} = 0$ is easily rationalized because the technological innovation is in general accepted by young generation, on the other hand, $\Delta L_O = 0$ is more limited assumption because a job hopping is frequent recently. Then we have

$$n+1 = \frac{\Delta L_Y}{L_{O,1}} \equiv \frac{L_Y}{L_O}.$$

This also defines the rate of generation alternation.

At the start of period the LMF must satisfy

$$W = L_Y + L_O, \qquad\qquad\qquad\qquad [1]$$

for given investment fund, W. If $L_Y = L_O$ then $n = 0$, $L_Y < L_O$ then $n < 0$, $L_Y > L_O$ then $n > 0$. Moreover we define

$$\frac{\theta}{L_O} = P_\theta. \qquad\qquad\qquad\qquad [2]$$

This is the price of the effort, θ, in terms of L_O. It is further assumed that the more L_Y is the more induced effort the LMF can acquire for given L_O.

$$P_\theta = P_\theta(L_Y), \quad P_\theta' > 0, \quad P_\theta'' > 0. \qquad\qquad [3]$$

This specification captures the idea that the more the number of L_Y the more intensively an each L_O will make an effort. It is central to the results. We can re-write eq. [2] like

$$(W-L_Y)P_\theta(L_Y)=\theta. \qquad\qquad\qquad [4]$$

Fig— 1 shows the relationship between θ and L_Y. L_Y is measured by the distance from W to $W-L_Y$. So the constraint is expressd by the linear down-sloped line. The intercept on the vertical axis shows the θ derived from L_Y for given L_O. On the left half panel we can trace the combination of L_Y, θ and L_O. Because of [3], as the budget constraint becomes more steeper, θ increases first, then decreased.

Differentiating eq. [4] with respect to L_Y gives.

$$\frac{\partial P_\theta}{\partial L_Y}=\frac{\partial P_\theta}{\partial L_Y}(W-L_Y)-P_\theta(L_Y)$$

$$=P_\theta(L_Y)\left[\frac{W}{P_\theta(L_Y)}\frac{\partial P_\theta}{\partial L_Y}-\frac{L_Y}{P_\theta(L_Y)}\frac{\partial P_\theta}{\partial L_Y}-1\right]$$
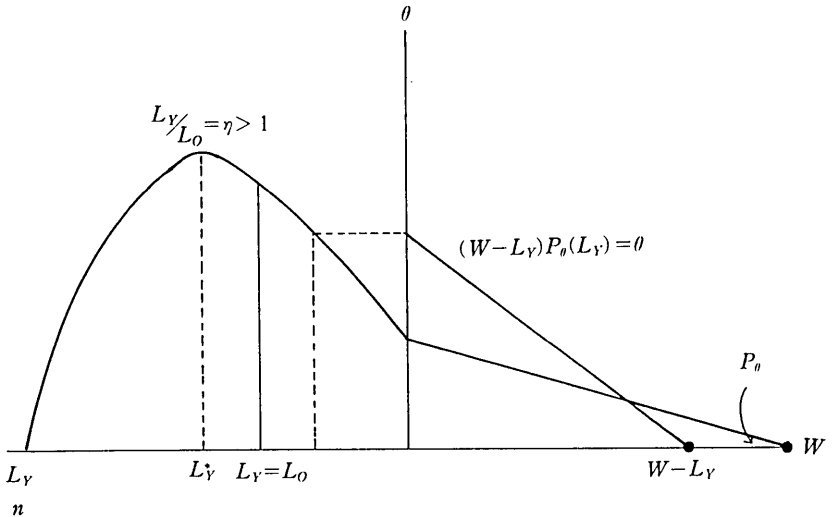

Fig— 1

$$= P_\theta(L_Y) [(\frac{W-L_Y}{L_Y})\eta - 1], \qquad\qquad [5]$$

where η is $\dfrac{L_Y}{P_\theta} \dfrac{\partial P_\theta}{\partial L_Y}$, the price elasticity of the young labor input. Eq.[5]

is equal to zero if $\dfrac{W}{L_Y} = \dfrac{1+\eta}{\eta}$, hence L_Y which maximizes θ, L_Y^*, is given

by

$$L_Y^* = \frac{\eta}{1+\eta} W \quad \text{so} \quad \frac{L_Y^*}{L_O} = \eta \qquad\qquad [6]$$

Because $P_\theta" > 0$ then $\eta > 1$. Accordingly, the left half area to the point of

$n = 0$ corresponds to Fig-1 in Fukaura(1990) because L_Y^* lies left of the

point where $L_Y = L_O$, that is $n = 0$.

III. CONSIDERATIONS

Because the effort function is essentially the theoretical device to
comprehend a phychological feature of the work effort, the implications de-
pend crucially on the model applied, just like the rational expectation. In
what follows, we derive some interesting facts by applying this new tool in-
to the LMF model.

III - 1 Time preference and the SPS

In the LMF model our system is characterized by the next equa-
tions(see Fukaura(1990), pp. 221).

$$\frac{L_Y}{L_O} = \eta, \qquad\qquad [7]$$

$$r = f'(k_1), \qquad\qquad [8]$$

$$\frac{u_1}{u_2} = 1 + n, \qquad\qquad [9]$$

$$w_1 = f(k_1) - (\frac{k}{1+n})f'(k_1) + \theta'f(k_2).$$ [10]

Then we have

$$\frac{u_1}{u_2} - 1 = \eta - 1 = n > 0.$$ [11]

This says the time preference of the LMF $(\equiv \rho)$ about w_1, w_2 is equal to the labor growth rate and depends on the elasticity of P_θ. If the wage is all devoted to consumption, a younger must receive a premium of at least ρ for each YEN before he(she)will postpone a YEN's worth of expenditure from period 1 to 2 (then $\rho > 0$ implies w_1 is worth w_2 ($> w_1$) to keep the utility constant at L_Y^*).

Above discussion can be summarized like following. See Fig.$-$2.

(1) $n = 0$

where $\frac{L_Y}{L_O} = 1$ so less than η, therefore,

$$\frac{u_1}{u_2} = 1,$$

(2) $n = n^*$

$$\frac{u_1}{u_2} - 1 = \rho^* = \eta - 1,$$

then $1 + n = \eta$,

(3) $0 < n < n^*$

where $1 < \frac{L_Y}{L_O} < \eta$, therefore,

$$\frac{u_1}{u_2} - 1 = \rho^{**} < = \eta - 1,$$

then $1 + n < \eta$,

(4) $n > n^*$

$$\rho^{***} > \rho^* = \eta - 1.$$
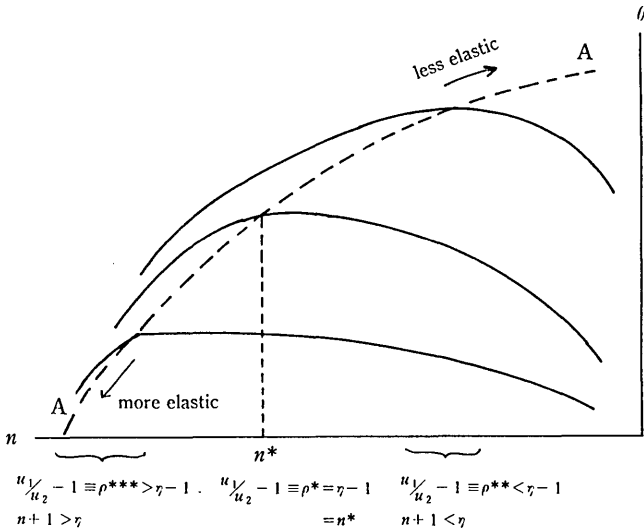
then $1 + n > \eta$.

Fig — 2

Accordingly, as n is increased, ρ is also increased. As the results the SPS is intensified (This corresponds to eq.[8] in Fukaura(1990), pp. 222). Needless to say, in the area under n^* the SPS works as the incentive scheme, in this sense we call this area the efficient SPS area. Suppose an economy with all available goods be perfectly perishable and no money(workers can not save). If so, the degree of the SPS, $\frac{w_1}{w_2}$, just corresponds to $\frac{c_1}{c_2}$ and n to an interest rate. Put differently, the young workers will save their ability in the from of "old worker's induced effort" at the rate n and later will withdraw them in the from of the seniority paying. The SPS can be characterized by the intergenerational saving system. The LMF will, so to speak, become bankrupt with $1 + n > \eta$ and be under an excess liquidity when $1 + n < \eta$. Accordingly, it is natural to assume the LMF will be at n^*, an optimal rate of generation alternation.

Using the definition of η, we obtain

$$\frac{\partial \theta}{\partial \eta} = \frac{-(W - L_Y)L_Y P_\theta{}'}{\eta^2} < 0 \quad \text{and} \quad \frac{\partial L_Y^*}{\partial \eta} = W\eta(1+\eta)^{-2} > 0 . \qquad [12]$$

Resulted set of n^* at various η is shown as the down sloped line, AA in Fig.— 2 . In other words, in order to accelerate generation alternation, the old worker must make an effort more intensively. Because the LMF can attain the sufficient effort level with a few old worker when η is high, the alternation can be accelerated, vice versa. A generation alternation works as a quantitative adjustment, however, in our model, a qualitative adjustment through an incentive elasticity also plays an important role.

Ⅲ— 2   The seniority paying system and the labor market problems

     Bearing the above in mind, following conditions are to satisfy in order to establish the efficient SPS.

(1)The young generation embodies the new technology.

(2)The old generation makes an effort to master the new technology.

(3)The generation alternation in the LMF advances constantly.

A drastically developing society like Japan in 1960' considerably installed these conditions. A rush of technological improvement and supply of young, high-quality labor power have accelerated the "intergenerational saving". Accordingly, the SPS has been the efficient scheme and characterized our economy(the recent circumstances, however, seem not to be favorable to the SPS). It would be an attractive study to consider whether the SPS would last in future or not both in the economic and sociological sense.

     The opennes of domestic labor market and the prolongation of the limit age are the conspicuous facts which deeply relate not only to the SPS but also to the labor market policy in Japan of today.

     Consider the exogenous increase of n from n^+ to n^{++} in Fig.— 3 , by the workers' immigration. Three possible consequences are expected, P,

Q, R. In order to maximize θ, P is desired, but not occur because the im-
migrators are less-skilled workers in general so lead no incentive effect.
Rather, the economy will be at R, where the welfare loss will result. And
the existing worker's wage, w_1, is increased and newer's, w_2, is decreas-
ed. Accordingly, we expect the openness of domestic labor market may br-
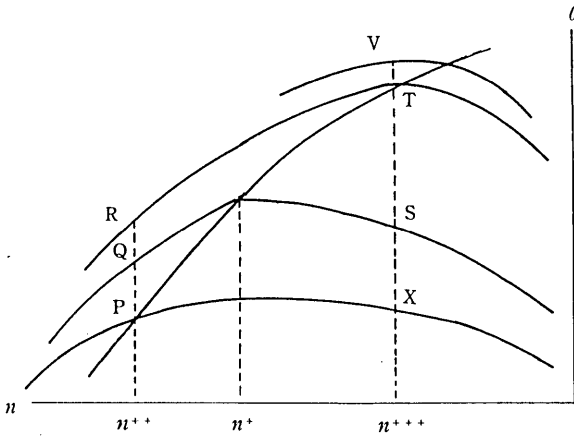ing no desirable results.

Fig $-$ 3

We then consider the problem of prolongation of limit age, which is
represented by the exogenous decrease of n. Various response of the
olders is expected. When the olders lose their incentives to catch up the
youngers because of the longer tenure, $\theta(n)$ will shift up, ex. T or V. On
the contrary case, although improbably, S or X will occur. In any events,
the olders must accept the lower wages, which is an only definitive result.

Accordingly, either policy would tend to shift up the $\theta(n)$ together
with the contraction of the SPS efficient area.

III − 3   Saving and the SPS

It is a little confusing that the SPS can be characterized as the "intergenerational saving system" because the word "saving" here differs from a common usage (remember in above section no saving i. e., $W_i = c_i$, was assumed). To see this we refer the relationship between the SPS and saving in usual sense.

It is broad known fact that the co-cyclical movement of saving is mainly observed. Three features pointed in III − 2 are also the features which characterize the boom in broad sense. We can easily show that our model is consistent with this obsevation if we consider the following simple economy where the comsumption function and the utility function are of the form

$$C_i = kY_i^P, \quad 0 < k < 1 , \qquad\qquad [13]$$

$$Y_i^P = \sum_{n=0}^{r} \alpha_{i-n} w_{i-n} \text{(permanent income in period i)}, \quad r < \infty,$$

$$u = q_1 \log w_1 + q_2 \log w_2 . \qquad\qquad [14]$$

For simplicity we will examine only two periods, i. e., i=1, 2. and r= 1 . Then we have

$$S_2 = w_2 - C_2 = w_2 - k(\alpha_2 w_2 + \alpha_1 w_1) = (1 - k\alpha_2) w_2 - k\alpha_1 w_1 . \quad [15]$$

Then we get from eq.[9]

$$w_1 = \frac{q_1 w_2}{q_2 (1+n)} . \qquad\qquad [16]$$

Substituting this into [16] and differentiating with respect to n yields

$$\frac{\partial S_2}{\partial n} = w_2 q_2^{-1} (1+n)^{-2} > 0 . \qquad\qquad [17]$$

IV. CONCLUDING REMARKS

This study described the effort function on the basis of a microeconomic framework and rationalized some conjectures given by Fukaura

(1990).

　There are many related issues that could be pursued theoretically. Notably, a mechanism for explaining the process that an initial technological shock affects the effort function must be inquired. In our model, the effort and the shocks are not separated. Furthermore, One of our points is that the SPS can be regarded as the saving system in the economy without money and inventory. Both money and inventory are the main focal issues of macroeconomics today. In order to consider these problem, the dynamic version of this model must be required.

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