Dynamic Stability of a Rectangular Plate Subjected to In-plane Dynamic Shearing Force

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Dynamic instability of a rectangular plate subjected to in-plane sinusoidally time-varying shearing force is analyzed. The small deflection theory of the thin plate is used. The problem is solved by using a Galerkin method and the harmonic balance method.

After presenting the problem in the form of divided matrix equations, numerical results are presented first for natural frequencies of the loaded plate and second for dynamic unstable regions of the rectangular plate with various boundary conditions.

1. Introduction

It is well-known that out-of-plane vibrations of plate structures are observed in a particular frequency range under an in-plane sinusoidally time varying load. This behavior is due to parametric instability. Among investigations of this problem, the dynamic stability of a thin rectangular plate has been studied by many researchers. For examples, the dynamic stability of rectangular plates under a uniformly distributed in-plane periodic load has been considered by Bolotin[1] and Yamaki and Nagai[2]. The dynamic instability of the rectangular plate subjected to an in-plane periodic moment or an in-plane linearly distributed dynamic force has been presented by the authors[3, 4]. However, the dynamic stability of a rectangular plate subjected to in-plane shearing force remains to be considered.

In this paper, theoretical solutions are reported for the dynamic stability of a rectangular plate under an in-plane periodic shearing force applied along the edges. The problem based upon the small deflection theory is solved by using a Galerkin method and the harmonic balance method. After presenting the problem in the form of divided matrix equations, numerical results are presented first for natural frequencies of the loaded plate and second for dynamic unstable regions of the rectangular plate with various boundary conditions.

2. Basic Equations and Boundary Conditions

Assume that a rectangular plate with length a, width b and thickness d is subjected to in-plane loads applied along the boundaries. A cartesian co-ordinate system (x, y) is introduced as shown in Fig. 1. The in-plane force N_{xy} due to static load N_{xyo} and periodic dynamic load $N_{xyt}\cos\Omega t$ is given by

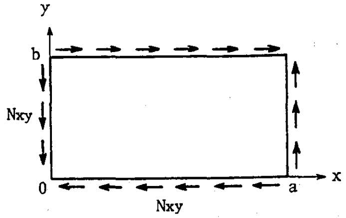

Fig. 1　Geometry and co-ordinates.

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$$N_{xy}=N_{xyo}+N_{xyt}\cos\Omega t, \tag{1}$$

in which N_{xyo} is the magnitude of the static load, N_{xyt} is the amplitudes of the dynamic load, Ω is the radian frequency of excitation, and t is the time.

It is assumed that the effect of longitudinal and rotatory inertia forces and transverse shear can be neglected. The basic equations for linear free vibrations of a plate subjected to these forces then can be written as

$$L(w)=\rho d\frac{\partial^2 w}{\partial t^2}+D\nabla^4 w-2N_{xy}\frac{\partial^2 w}{\partial x\partial y}=0, \tag{2}$$

where w denotes the plate deflection, ρ is the mass density, $D=Ed^3/\{12(1-\nu^2)\}$ is the elastic rigidity, E is Young's modulus, ν is Poisson's ratio, and $\nabla^4=(\partial^2/\partial x^2+\partial^2/\partial y^2)^2$.

The following three boundary conditions are considered in the present analysis:

case I, simply supported along all edges, i. e.,

$$w=\frac{\partial^2 w}{\partial x^2}=0(x=0,a)\text{and } w=\frac{\partial^2 w}{\partial y^2}=0(y=0,b), \tag{3a}$$

case II, simply supported along two edges and clamped along the other edges, i. e.,

$$w=\frac{\partial^2 w}{\partial x^2}=0(x=0,a)\text{and } w=\frac{\partial w}{\partial y}=0(y=0,b), \tag{3b}$$

case III, clamped along all edges, i. e.,

$$w=\frac{\partial w}{\partial x}=0(x=0,a)\text{and } w=\frac{\partial w}{\partial y}=0(y=0,b). \tag{3c}$$

3. Method of Solution

For these boundary conditions, one can assume the solution of equation (2) to be of the forms

$$w=\sum_m\sum_n T_{mn}(t)W_{mn}(x,y), \tag{4}$$

where T_{mn} is an unknown function of the time variables and W_{mn} is an eigenfunction associated with free vibrations satisfying the geometric boundary conditions of the plate subjected to no in-plane force, defined as

$$\text{case I},\ W_{mn}=\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \tag{5a}$$

$$\text{case II},\ W_{mn}=\sin\frac{m\pi x}{a}\sum_j a^n{}_j\{\cos\frac{(j-1)\pi y}{b}-\cos\frac{(j+1)\pi y}{b}\}, \tag{5b}$$

$$\text{case III},\ W_{mn}=\sum_i a^m{}_i\{\cos\frac{(i-1)\pi x}{a}-\cos\frac{(i+1)\pi y}{a}\}\sum_j a^n{}_j\{\cos\frac{(j-1)\pi y}{b}-\cos\frac{(j+1)\pi y}{b}\}, \tag{5c}$$

where $a^m{}_i$ and $a^n{}_j$ are the modal coefficients, and m and n are the half-wave numbers in the x and y directions, respectively.

With these expressions, applying the Galerkin method, one has

$$\int_0^a\int_0^b L(w)W_{rs}dxdy=0, \tag{6}$$

where r and s=1,2, ……. Performing the indicated integrations of equation (6) gives the following set of ordinary differential equations:

$$[A]\{\ddot{T}\}+[B]\{T\}+(N_{xyo}+N_{xyt}\cos\Omega t)[C]\{T\}=\{0\}, \tag{7}$$

where [A], [B] and [C] are square matrices (see the Appendix A), and {T} is a column vector consisting of the dependent time valuables.

The following non-dimensional qualities are now introduced:

$$\bar{N}_{xyo}=\frac{N_{xyo}}{N_{cr}},\ \bar{N}_{xyt}=\frac{N_{xyt}}{N_{cr}},$$

$$\bar{\omega}=\frac{\Omega}{\Omega_{11}},\ \omega_{mn}=\frac{\Omega_{mn}}{\Omega_{11}},\ \tau=\Omega_{11}t. \tag{8}$$

Here $N_{cr}=\lambda_{cr}D\pi^2/b^2$ is the buckling load for each boundary condition, λ_{cr} is the eigenvalue of buckling, $\Omega_{11}=k_{11}{}^2\sqrt{D/\rho hb^4}$ is the lowest natural radian frequency of the plate subjected to no in-plane force, and k_{11} is the lowest eigenvalue of free vibrations.

Inverting the matrix [A] and premultiplying both sides of equation (7) by the inverse matrix lead to

$$[I]\{\ddot{T}\}+[F]\{T\}+(\bar{N}_{xyo}+\bar{N}_{xyt}\cos\bar{\omega}\tau)[G]\{T\}=\{0\}, \tag{9}$$

where $[F]=[A]^{-1}\times[B]=\mathrm{diag}(\omega_{11}{}^2\omega_{12}{}^2\cdots\omega_{21}{}^2\cdots\omega_{NN}{}^2)$, $[G]=[A]^{-1}\times[C]$ and $[I]$ is the unit matrix.

The solution of equation (9) in now sought in the form[1), 6)]:

$$\{T\}=\exp(\lambda t)\{\frac{1}{2}\{b_o\}+\sum_k(\{a_k\}\sin k\bar{\omega}\tau+\{b_k\}\cos k\omega\tau)\} \tag{10}$$

where $\{b_o\}$, $\{a_k\}$, and $\{b_k\}$ are some vectors that are independent of the time variable.

Substituting equation (10) into equation (9) and applying the harmonic balance method leads to a set of homogeneous algebraic equations as

$$([M_o]-\lambda[M_1]-\lambda^2[M_2])\{x\}=\{0\}, \tag{11}$$

in which $[M_o]$, $[M_1]$ and $[M_2]$ are the coefficient matrices of the zeroth (constant), first and second powers of λ, respectively, and $\{x\}$ is the column vector consisting of $\{b_o\}$, $\{b_k\}$, and $\{a_k\}$.

The eigenvalue λ can be obtained by solving a double sized matrix as an eigenvalue problem in the form

$$\begin{bmatrix} [0] & [I] \\ [M_2]^{-1}[M_o] & -[M_2]^{-1}[M_1] \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \lambda \begin{Bmatrix} x \\ y \end{Bmatrix}, \qquad (12)$$

where $\{Y\} = \lambda \{X\}$.

As the matrix of equation (12) is a non-symmetric matrix with real elements, the eigenvalues consist of pairs of complex numbers. If the eigenvalues of equation (12) are distinct, then the necessary and sufficient condition for stability is that real parts of the complex roots should be negative or zero. In the present problem, however, the unstable motions occur in the same frequency range[7]. To distinguish the repeated unstable regions, the division of matrix equations is introduced in the following section.

4. Division of Matrix Equations

If we take a 16 degrees of freedom system in which one considers m, n=1, 2, 3 and 4, the time variables of equation (9) are given as follows:

$$\{T\} = \{T_{11}\ T_{12}\ T_{13}\ T_{14}\ T_{21}\ T_{22}\ T_{23}\ T_{24}\ T_{31}\ T_{32}\ T_{33}\ T_{34}\ T_{41}\ T_{42}\ T_{43}\ T_{44}\}^T. \qquad (13)$$

The coefficient matrix $[G]$ with respect to parametric instability has the following form

$$[G] = \begin{bmatrix} [0] & [G]_{2j}^{1n} & [0] & [G]_{4j}^{1n} \\ [G]_{1j}^{2n} & [0] & [G]_{3j}^{2n} & [0] \\ [0] & [G]_{2j}^{3n} & [0] & [G]_{4j}^{3n} \\ [G]_{1j}^{4n} & [0] & [G]_{3j}^{4n} & [0] \end{bmatrix} \qquad (14)$$

in which

$$[G]_{ij}^{mn} = \begin{bmatrix} 0 & f_{m112} & 0 & f_{m114} \\ f_{m211} & 0 & f_{m213} & 0 \\ 0 & f_{m312} & 0 & f_{m314} \\ f_{m411} & 0 & f_{m413} & 0 \end{bmatrix}$$

and $[O]$ is the 4×4 zero matrix.

The matrix $[G]$ is a sparse matrix as shown in equation (14). Three quarters of the elements are zero. The matrix $[G]$ is transformed into the following four non-zero submatrices by rearranging the order of the rows and columns as

$$[G] = \begin{bmatrix} [0] & [0] & [0] & [G_{14}] \\ [0] & [0] & [G_{23}] & [0] \\ [0] & [G_{32}] & [0] & [0] \\ [G_{41}] & [0] & [0] & [0] \end{bmatrix} \qquad (15)$$

in which $[G_{14}]$, $[G_{23}]$, $[G_{32}]$ and $[G_{41}]$ are 4×4 submatrices with non-zero elements.

According to this transformation, the time variables $\{T\}$ and the coefficient matrix $[F]$ are rearranged as follows:

$$\{T\} = \{T_{11}\ T_{13}\ T_{31}\ T_{33}\ T_{12}\ T_{14}\ T_{32}\ T_{34}\ T_{21}\ T_{23}\ T_{41}\ T_{43}\ T_{22}\ T_{24}\ T_{42}\ T_{44}\}^T \qquad (16a)$$

$$[F] = \mathrm{diag}\ (\omega_{11}^2\ \omega_{13}^2\ \omega_{31}^2\ \omega_{33}^2\ \omega_{12}^2\ \omega_{14}^2\ \omega_{32}^2\ \omega_{34}^2\ \omega_{21}^2 \cdots\cdots \omega_{22}^2 \cdots\cdots \omega_{44}^2). \qquad (16b)$$

Considering the properties of these matrices, the matrix equation of motion (9) is now divided into the following two equations:

Type 1, $$[I_1]\{\ddot{T}_1\}+[F_1]\{T_1\}+(\bar{N}_{xyo}+\bar{N}_{xyt}\cos\bar{\omega}\tau)[G_1]\{T_1\}=\{0\}, \qquad (17a)$$

Type 2, $$[I_2]\{\ddot{T}_2\}+[F_2]\{T_2\}+(\bar{N}_{xyo}+\bar{N}_{xyt}\cos\bar{\omega}\tau)[G_2]\{T_2\}=\{0\}. \qquad (17b)$$

Here $\{T_1\} = \{T_{11}\ T_{13}\ T_{31}\ T_{33}\ T_{22}\ T_{24}\ T_{42}\ T_{44}\}^T$, $\{T_2\} = \{T_{12}\ T_{14}\ T_{32}\ T_{34}\ T_{21}\ T_{23}\ T_{41}\ T_{43}\}^T$,

$$[G] = \begin{bmatrix} [0] & [G_{14}] \\ [G_{41}] & [0] \end{bmatrix} \text{and } [G_2] = \begin{bmatrix} [0] & [G_{23}] \\ [G_{32}] & [0] \end{bmatrix}$$

Unstable regious are independently obtained from these two equations.

There are two different types of unstable motions obtained from equation (9) or equations (17a) and (17b): that is, the simple parametric resonance in the vicinity of $\bar{\omega}=2\omega_{mn}/s$ and the combination resonance in the vicinity of $\bar{\omega}=(\omega_{mn}\pm\omega_{k\ell})/s$, in which s=1, 2 corresponds to the principal and secondary unstable regions, respectively. Here ω_{mn} is the normalized natural frequency having the half-wave numbers m and n in the x and y directions, respectively. The kind and width of the unstable regions depend on the elements of the matrices $[G_1]$ and $[G_2]$. The coupling elements of the matrices $[G_1]$ and $[G_2]$ have the same sign. According to Hsu's formulation[7], the sum type combination resonances in the vicinity of $\bar{\omega}=(\omega_{mn}+\omega_{k\ell})/s$ will be obtained and the difference type one of $\bar{\omega}=(\omega_{mn}-$

$\omega_{k\ell})/s$ will be not obtained for the present problem. Since the diagonal elements $g_{pp}{}^{i}$ of $[G_i]$ (i=1, 2) are zero as shown in equations (17a) and (17b), and $[F_i]$ and $[I_i]$ are diagonal matrices, parametric resonances occur only through the coupling term $g_{pq}{}^{i}$ ($p \neq q$). Therefore, the simple parametric response which occurs through the direct term $g_{pp}{}^{i}$ would not be important for the present case.

The combination resonance which occurs through the non-zero coupling term is predominant. As to the combination resonance $\omega_{mn}+\omega_{k\ell}$ of the present problem, the following properties are found from the coupling term of equations (17a) and (17b):

Type 1, $m+n=$ even and $m \neq k$, $n \neq \ell$, (18a)

Type 2, $m+n=$ odd and $m \neq k$, $n \neq \ell$. (18b)

Using this matrix partition, it is easy to distinguish kinds of unstable regions and to save computational time for the eigenvalue problem.

Based upon the above theoretical analysis, numerical solutions have been obtained for the rectangular plate. First, the frequencies of the plate under static loading are presented. Then, the unstable regions due to dynamic loading are determined.

5. Natural Frequency of the Square Plate under Static Loading

Based on equation (B-3) shown in Appendix B, the natural frequency for the square plate subjected to the static shearing force $\bar{N}_{xyo}$ for each boundary condition has been determined. Numerical results for the lowest eigenvalue of free vibrations k_{11} and buckling eigenvalue λ_{cr} of square plates (the aspect ratio $\mu=1.0$) in each case are given in Table 1. The shearing force $\bar{N}_{xyo}$ vs. the natural frequency $\bar{n}$ for each case are shown in Figs. 2, 3 and 4. In these figures, the ordinate $\bar{N}_{xyo}$ shows the static shearing force normalized to the buckling load, while the abscissa $\bar{n}=\Omega_{mn}/\Omega_{11}$ denotes the natural frequency normalized to the relevant lowest natural radian frequency.

Table 1 Constants k_{11} and λ_{cr}: $\mu=1.0$.

	case I	case II	case III
k_{11}	2.0	2.935	3.650
λ_{cr}	9.32	12.59	14.69

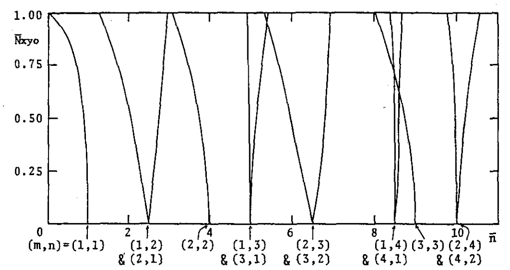


Fig. 2 Shearing force $\bar{N}_{xyo}$ vs. natural frequency $\bar{n}$: case I and $\mu=1.0$.

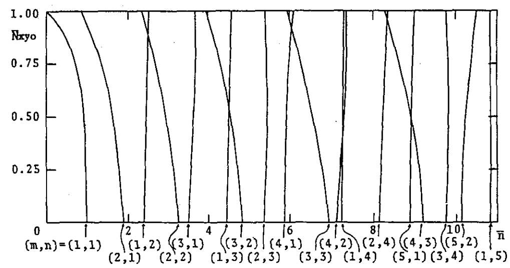


Fig. 3 Shearing force $\bar{N}_{xyo}$ vs. natural frequency $\bar{n}$: case II and $\mu=1.0$.

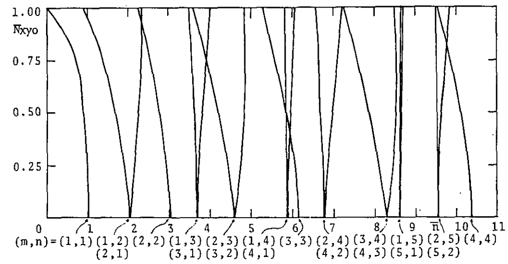
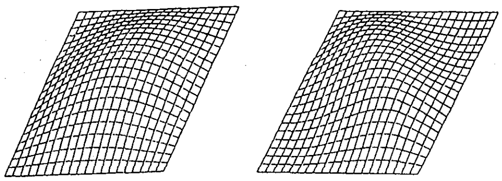


Fig. 4 Shearing force $\bar{N}_{xyo}$ vs. natural frequency $\bar{n}$: case III and $\mu=1.0$.

quency. The notation (m, n) in these figures represents the number of half-waves in the x and y directions when $\bar{N}_{xyo}=0$, respectively. As the mode of vibration is neither symmetric nor antisymmetric about $x=a/2$ or $y=b/2$ when $\bar{N}_{xyo} \neq 0$, the notation (m, n) is not valid. The following will be observed from the figures. Natural frequencies change with an increase in the static shearing force $\bar{N}_{xyo}$. They decrease or slightly increase. The effect of the static shearing force is the most pronounced when the mode of vibration nearly coincides with the corresponding buckling wave form. In this particular case, mode (1, 1), the frequency is zero when the shearing force $\bar{N}_{xyo}$ is equal to unity.

Considering the effect of the static shearing



(a) $\bar{N}_{xyo}=0$ (b) $\bar{N}_{xyo}=1$

Fig. 5 Effect of shearing force $\bar{N}_{xyo}$ on the modes of vibration: case I and $\mu=1.0$.

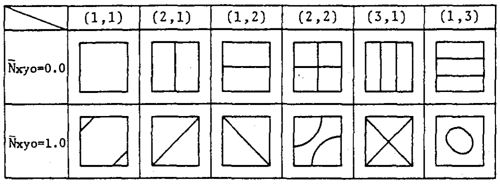


Fig. 6 Nodal lines of the first six modes: case I and $\mu=1.0$.

force on the modes of vibration for the mode (1, 1) of case I, the change in the modal shape is illustrated in Fig. 5. Compressive and tensile forces due to shearing force $\bar{N}_{xyo}$ act along the edges of the plate. The modal shape is affected by these forces such that the amplitude of the side subjected to the compressive force becomes large, while that of the side of the tensile force becomes small. The mode of vibration at $\bar{N}_{xyo}=1$ corresponds to that of buckling. For the first six modes of case I, the changes in modal shape with shearing force are illustrated by nodal lines in Figure 6. The modes at $\bar{N}_{xyo}=0$ are symmetric or antisymmetric about $x=a/2$ or $y=b/2$, while those at $\bar{N}_{xyo}\neq0$ are symmetric or antisymmetric about diagonal lines of the plate.

6. Dynamic Unstable Regions Subjected to Shearing Force

6. 1. Property of Unstable Regions

The results for the rectangular plates with three different boundary conditions subjected to only the periodic shearing force $\bar{N}_{xyt}$ ($\bar{N}_{xyo}=0$) are shown in Fig. 7 through Fig. 12. In these figures, the ordinate $\bar{N}_{xyt}=N_{xyt}/N_{cr}$ denotes the amplitude of the periodic

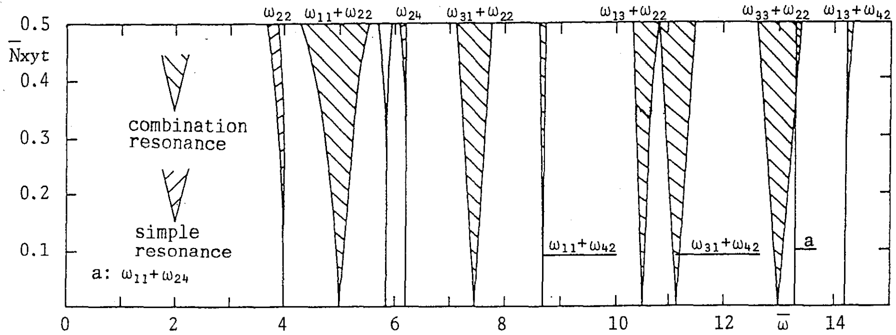


Fig. 7 Unstable regions of the rectangular plate: case I, type 1, $\bar{N}_{xyo}=0.0$ and $\mu=1.5$

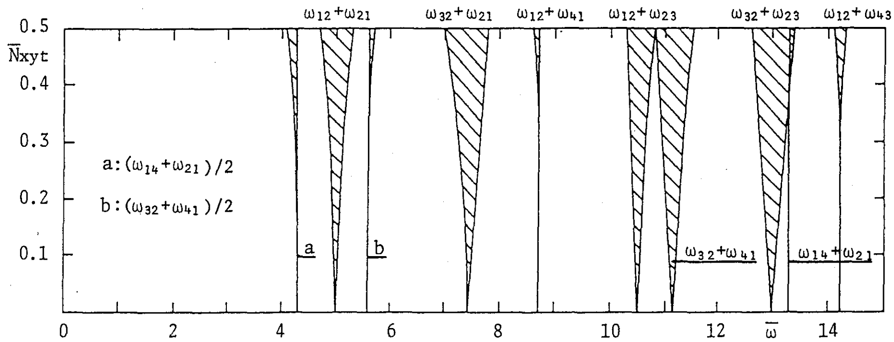


Fig. 8 Unstable regions of the rectangular plate: case I, type 2, $\bar{N}_{xyo}=0.0$ and $\mu=1.5$.

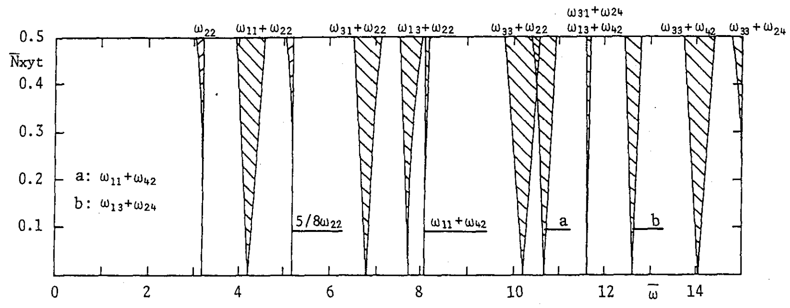


Fig. 9 Unstable regions of the square plate: case II, type 1, $\bar{N}_{xyo}=0.0$ and $\mu=1.0$.

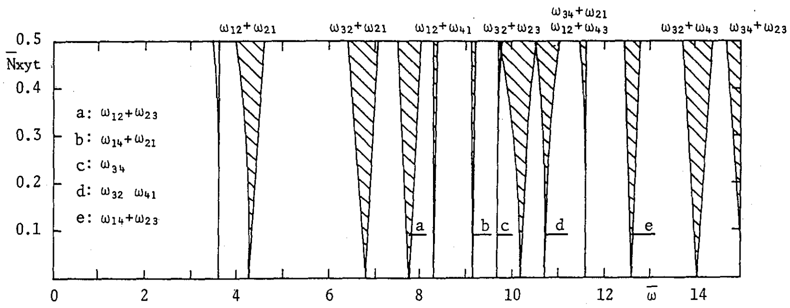


Fig. 10 Unstable regions of the square plate: case II, type 2, $\bar{N}_{xyo}=0.0$ and $\mu=1.0$.

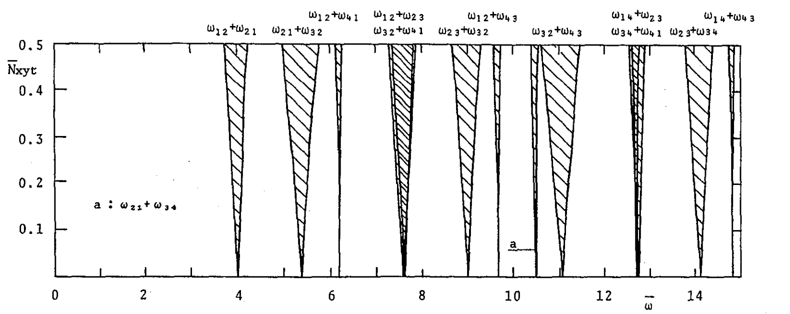


Fig. 11 Unstable regions of the rectangular plate: case III, type 1, $\bar{N}_{xyo}=0.0$ and $\mu=1.5$.

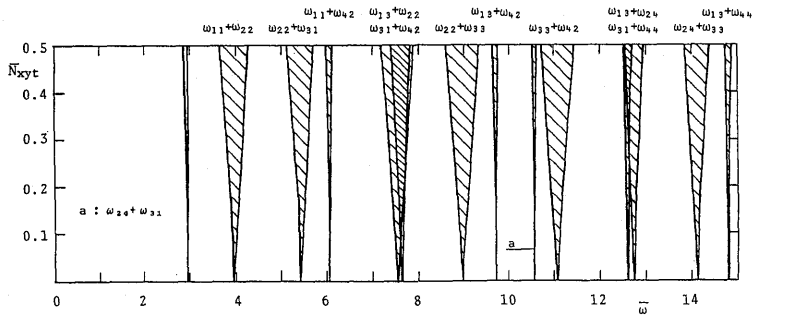


Fig. 12 Unstable regions of the rectangular plate: case III, type 2, $\bar{N}_{xyo}=0.0$ and $\mu=1.5$.

Table 2 Natural frequency ω_{mn}, costants k_{11} and λ_{cr}: $\bar{N}_{xyo}=0.0$.

	case I (μ=1.5)				case II (μ=1.0)				case III (μ=1.5)			
m/n	1	2	3	4	1	2	3	4	1	2	3	4
1	1.0000	3.0769	6.5385	11.3846	1.0000	2.3966	4.4755	7.2309	1.0000	2.4506	4.6563	7.5965
2	1.9231	4.0000	7.4615	12.3077	1.8909	3.2707	5.3686	8.1433	1.5460	2.9613	5.1587	8.0974
3	3.4615	5.5385	9.0000	13.8462	3.5305	4.8486	6.9330	9.7114	2.4732	3.8386	6.0179	8.9511
4	5.6154	7.6923	11.1538	16.0000	5.8826	7.1474	9.2015	11.9666	3.7514	5.0711	7.2222	10.1410
k_{11}	1.4444				2.9333				5.2005			
λ_{cr}	1.1503				12.5892				11.4893			

shearing force normalized to the corresponding buckling shearing force, while the abscissa $\bar{\omega}=\Omega/\Omega_{11}$ is the exciting frequency mormalized to the lowest natural frequency. Further, the hatched portions represent the regions of various types of instability. The narrow regions of instability with $\bar{\omega}$ less than 0.1 at $\bar{N}_{xyt}=0.5$ are omitted in the figures. Normalized natural frequency ω_{mn}, and the constants k_{11} and λ_{cr} for each are given in Table 2.

Wide unstable regions of sum type combination resonances in the vicinity of $\omega_{mn}+\omega_{k\ell}$ are obtained as shown in Fig. 7 through Fig. 12. Combination resonances of types 1 and 2 for each boundary condition occur at the same frequency $\omega_{mn}+\omega_{k\ell}$ (type 1) $=\omega_{m'n'}+\omega_{k'\ell'}$ (type 2) in which $m+k=m'+K'$ and $n+\ell=n'+l'$. The present analysis using matrix partition is very useful to distinguish the kinds of unstable regions. As the diagonal elements of the coefficient matrix [G] are zero in the present case, the simple parametric resonance excited by the direct term is not obtained. Although the secondary unstable regions of the simple resonance such as ω_{mn} occur through the coupling terms, the widths of them are narrower than those of combination resonances. Therefore, combination resonances are important for the present problem. Unstable regions of the combination resonance are wide when wave numbers in the x and y directions are close, i.e., adjacent numbers, $k=m+1$ and $\ell=n+1$ ($m<k$, $n<\ell$) as can be seen in these figures. On the other hand, when wave numbers are farther apart, widths of combination resonances are narrow.

6. 2. Effect of Static Stress

Figs. 13 and 14 show the unstable regions of a square plate considering a static shearing force $\bar{N}_{xyo}=0.3$ for case I. The non-dimensional natural frequency ω_{mn}, and the constants k_{11} and λ_{cr} are summarized in Table 3. Simple parametric resonances with $2\omega_{mn}$ occur in conjunction with $\bar{N}_{xyo}$. This result corresponds to the fact that the coupling between the modes occurs through restoring force terms in equation (9). The static shearing force has an influence upon the unstable regions. The simple

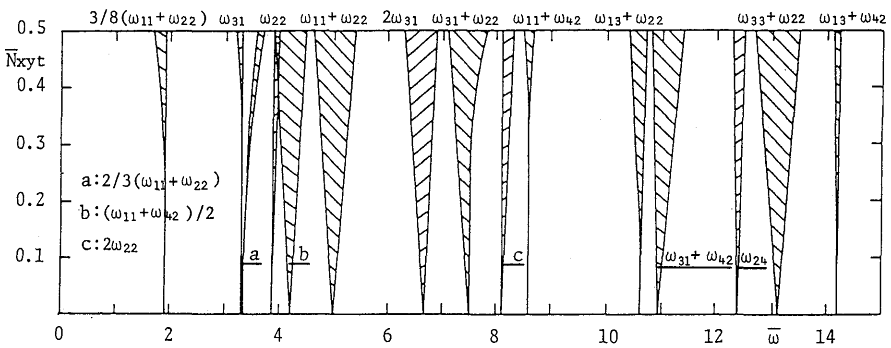


Fig. 13 Unstable regions of the rectangular plate: case I, type 1, $\bar{N}_{xyo}=0.3$ and $\mu=1.5$.

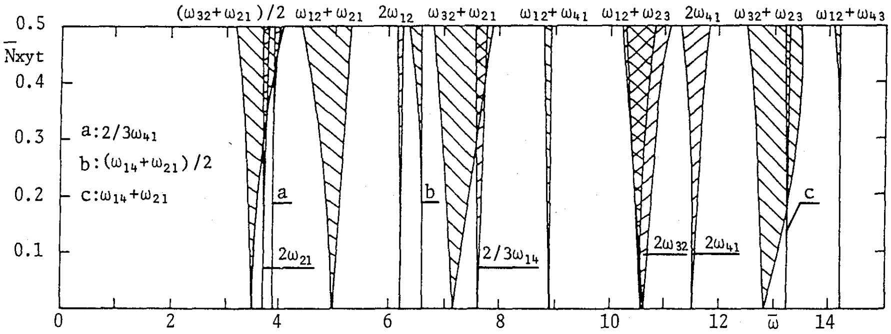


Fig. 14 Unstable regions of the rectangular plate: case I, type 2, $\bar{N}_{xyo}=0.3$ and $\mu=1.5$.

Table 3 Natural frequency ω_{mn}, costants k_{11} and λ_{cr}: $\bar{N}_{xyo}=0.3$.

	case I ($\mu=1.5$)			
m/n	1	2	3	4
1	0.9641	3.0901	6.5401	11.3965
2	1.8459	4.0552	7.4746	13.3103
3	3.3403	5.3087	9.0322	13.8580
4	5.7754	7.5723	11.2069	16.0176
k_{11}	1.4444			
λ_{cr}	7.1503			

resonances of small width in absence of the static shearing forces become of larger width.

6. 3. Effect of Damping

The undamped (h=0.0) and damped (h=0.02) regions of instability, in which the damping constant h is constant for all modes, are shown in Fig. 15. The effect of damping depends on the width of unstable regions. The narrower unstable region becomes stable in the presence of damping.

7. Conclusions

The sum type combination resonances are predominant for a rectangular plate subjected o the dynamic shearing force. The widths of unstable regions are broad when half-wave numbers of the modes of vibration in both x and y directions are close together independently of boundary conditions.

The static shearing force influences the kinds of unstable regions of a rectangular plate subjected to the dynamic shearing force. The simple resonances whose widths are narrow in the absence of the static shearing force become broad in width.

The effect of damping depends on the width of unstable regions. The narrower unstable region becomes stable in the presence of damping.

Appendix A: Coefficient Matrices

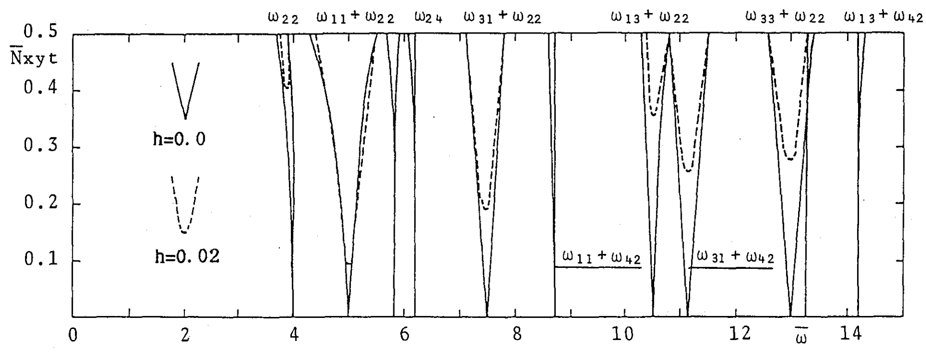


Fig. 15 Effect of damping on unstable regions: case I, type 1, $\bar{N}_{xyo}=0.0$ and $\mu=1.5$.

$[A]: a\{j+(i-1)N, n+(m-1)N\}=I^1{}_{mnij}$ (A−1)

$[B]: b\{j+(i-1)N, n+(m-1)N\}=\omega^2{}_{mn}I^1{}_{mnij}$ (A−2)

$[C]: c\{j+(i-1)N, n+(m-1)N\}=\chi I^2{}_{mnij}$ (A−3)

where $I^1{}_{mnij}=\int\int W_{mn}W_{ij}dxdy, I^2{}_{mnij}=\int\int \frac{\partial^2 W_{mn}}{\partial x \partial y} W_{ij}dxdy$ and $\chi=-2\frac{1}{\mu}\frac{\lambda_{cr}\pi^2}{k_{11}{}^4}$,

Appendix B: Vibration Analysis

The equation of motion describing free vibrations of a plate subjected to static stress is given by setting $\bar{N}_{xyt}=0$ in equation (9) as follows:

$[I]\{T\}+[F]\{T\}+\bar{N}_{xyo}[G]\{T\}=\{0\}$ (B−1)

The solution of the vibration problem is assumed in the form

$\{T\}=e^{i\bar{n}\tau}\{\bar{T}\}$ (B−2)

where $\bar{n}$ in the non-dimensional natural frequency. Equation (B−1) can be rewritten as

$([F]+\bar{N}_{xyo}[G]\{\bar{T}\}=\bar{n}^2\{\bar{T}\}$ (B−3)

The eigenvalue $\bar{n}^2$ and the corresponding eigenvector $\{\bar{T}\}$ can be obtained by using the scientific subroutine library of a digital computer.

If one puts $\bar{n}=0$ and $\bar{N}_{xyo}=1$, the equation for the buckling eigenvalue λ_{cr} is obtained.

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