Nonlinear Vibrations of Inextensional Beams

by

Kazuo TAKAHASHI* and Yutaka NOGUCHI**

In this paper, the nonlinear vibrations of inextensional beams are reported based upon the simplified equation of motion derived by Yoshimura and Uemura. This problem is analyzed by the application of Galerkin's method, in which the deflection of the beam is assumed from a series of the product of the normal mode of the corresponding linear beam, and an unspecified function of the time resulting in nonlinear ordinary differential equation is solved by the harmonic balance method. Numerical results are presented for the nonlinear free vibrations of the clamped-free beam and the free-free beam.

1. INTRODUCTION

In the previous researches on the nonlinear vibrations of beams, the nonlinearities were characterized either by nonlinear geometry (i. e., the usual linear approximation $\kappa=\frac{\partial^2 y}{\partial x^2}$, for curvature is replaced by. $\kappa=\frac{\partial \varphi}{\partial x}$, the nonlinear exact form, because of the presence of large deformations), or by an axial tension which depends on lateral displacement. In the case of beams having immovable ends such as a hinged-hinged beam, clamped-hinged beam or clamped-clamped beam, the nonlinearity is caused by an axial tension and the small slope assumption which reduces the exact expression for the beam curvature to linear one is adopted. Such a beam is considered by Woinowsky-Krieger etc.[1–3] The governing nonlinear equation of motion is given by the well-known one, the nonlinearity shows a hardening spring.

On the contrary to those, beams having free-free ends, clamped-free ends or hinged-roller ends are assumed to be free to move in the axial direction. The effect of large curvature is the nonlinear term in this problem. Under the assumption of the inextensional motion, taking into the rotatory inertia into account, free vibrations of a beam with hinged-roller ends were treated by means of a Rayleigh method by Takahashi.[4] Yoshimura and Uemura[5] derived nonlinear equations of motion considering both the axial force and the nonlinear curvature by using vector analysis of strain, and solved nonlinear free vibrations of hinged-roller beam and clamped-free beam under the assumption of inextensional motion.

*Department of Civil Engineering
**Department of Structural Engineering

Woodall[6] considered a similar problem by using a Galerkin method and a perturbation method and a finite difference method, and Atluri[7] investigated the nonlinear elasticity term arising from moderately large curvatures, and nonlinear inertia. In these analyses, the nonlinearity shows a weakly softening spring. On the other hand, Wagner[8] formulated a similar inextensional problem considering both longitudinal and transverse inertias by using a Hamilton's principle and solved nonlinear free vibrations of a clamped-free beam and a free-free beam. Kato[9] solved nonlinear free vibrations of a clamped-free beam with Wagner's equation of motion. Aravamudan and Murthy[10] used the same method and considered both contributions of the nonlinear curvature and the axial tension. According to the Wagner type approach, the influence of the large amplitude on the frequency is similar to that of a hardening spring and this result is not coincide with the results of Takahashi etc. Wagner type approach is assumed to be erroneous because the subsidiary condition is not taken into account correctly in the formulation of the basic equation.

The nonlinearity due to the nonlinear curvature is smaller than that due to an axial tension. The small parameter method such as a perturbation method is employed as to a method of solution assuming that the system is single degree-of-freedom. The effects of change of the vibration mode, the strain distribution and the higher order term of the deflection are still not investigated.

In this paper, by using the simplified equation of motion derived by Yoshimura and Uemura, nonlinear vibrations of a clamped-free beam and a free-free beam are reported. A multiple degree-of-freedom approach is applied to this problem by using a Galerkin method and the harmonic balance method.

2. METHOD OF SOLUTION

The nonlinear equations for the undamped slender beam, which consider nonlinear curvature and axial tension and rotatory inertia derived by Yoshimura and Uemura[5], will be given as follows:

$$\frac{\partial T}{\partial x}-(1+\varepsilon)N\kappa+\left(X-\rho A\frac{\partial^2 u}{\partial t^2}\right)\frac{1+u'}{1+\varepsilon}+\left(Z-\rho A\frac{\partial^2 y}{\partial t^2}\right)\frac{y'}{1+\varepsilon}=0\,, \quad (1)$$

$$\frac{\partial N}{\partial x}+(1+\varepsilon)T\kappa-\left(X-\rho A\frac{\partial^2 u}{\partial t^2}\right)\frac{y'}{1+\varepsilon}+\left(Z-\rho A\frac{\partial^2 y}{\partial t^2}\right)\frac{1+u'}{1+\varepsilon}=0\,, \quad (2)$$

$$\frac{\partial M}{\partial x}+N+G-\rho I\frac{\partial^2 \varphi}{\partial t^2}=0\,, \quad (3)$$

where N, T and M are the axial force, the shear force and the bending moment at a point x, ρ is is the mass density per unit length, A the cross-sectional area of the beam, I the cross-sectional moment of inertia, ε normal strain, κ curvature, u and y the deflections of any point x in the axial and transverse directions, ϕ the angle between the tangent to the median line and the x axis, and t the time, and x is space co-ordinate along the axis of a beam after deformation, and X, Y and G are external forces.

By the aid of the vector analysis of strain, the exact curvature and strain of the neutral axis are given by

$$\kappa=\left\{\frac{\partial^2 y}{\partial x^2}\left(1+\frac{\partial u}{\partial x}\right)-\frac{\partial^2 y}{\partial x^2}\frac{\partial y}{\partial x}\right\}\Big/\left\{\left(1+\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial y}{\partial x}\right)^2\right\}^{3/2},$$

$$\varepsilon=\left\{\left(1+\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial y}{\partial x}\right)^2\right\}^{1/2}-1. \quad (4)$$

The assumptions used to further develop the equations of motion are

(1) The beam material is homogeneous, isotropic and obeys Hooke's law.
(2) The cross-section of the beam is uniform.
(3) A cross-section, originally plane, remains plane and normal to the deformed axis.
(4) The deformation in the cross-sectional plane is negligible.
(5) External and internal damping are neglected.
(6) The beam is initially straight.
(7) Rotatory inertia is negligible.
(8) Elongation of the middle plane is negligible.
(9) No external tangential force or moment acts on the beam.

The axial force T and the bending moment M are given by

$$T=EA\varepsilon,\ M=EI\kappa. \tag{5}$$

Using the above mentioned assumptions, Eq. 4 can be rewritten as follows:

$$\kappa=\frac{\partial^2 y}{\partial x^2}\Big/\sqrt{1-(\partial y/\partial x)^2},\ \varepsilon=0. \tag{6}$$

Combining Eqs. (1), (2), (3), (5) and (6), the resulting equation is differentiated with respect to the x co-ordinate, we obtain

$$\rho A\frac{\partial^2 y}{\partial t^2}+EI\left[\frac{\partial^4 y}{\partial x^4}+\left\{\left(\frac{\partial^2 y}{\partial x^2}\right)^3+2\frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial x^2}\frac{\partial^3 y}{\partial x^3}\right\}\Big/\left\{1-\left(\frac{\partial y}{\partial x}\right)^2\right\}+2\left(\frac{\partial y}{\partial x}\right)^2\left(\frac{\partial^2 y}{\partial x^2}\right)^3\Big/\left\{1-\left(\frac{\partial y}{\partial x}\right)^2\right\}\right]=p_0\cos\Omega t. \tag{7}$$

Adopting the nonlinear terms up to an order of five, the following equation for the present problem is given by

$$\rho A\frac{\partial^2 y}{\partial t^2}+EI\left\{\frac{\partial^4 y}{\partial x^4}+\left(\frac{\partial^2 y}{\partial x^2}\right)^3+2\frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial x^2}\frac{\partial^3 y}{\partial x^3}+2\left(\frac{\partial y}{\partial x}\right)^3\frac{\partial^2 y}{\partial x^2}\frac{\partial^3 y}{\partial x^8}+3\left(\frac{\partial^2 y}{\partial x^2}\right)^3\left(\frac{\partial y}{\partial x}\right)^2\right\}=p_0\cos\Omega t. \tag{8}$$

A normal mode solution is assumed as follows:

$$y=\ell\sum_{i=1}^{\infty}Xi(x)Ti(t), \tag{9}$$

where ℓ is the length of the beam, Ti (t) an unknown function of the time, and Xi (x) a space variable satifying the geometric boundary conditions of the beam, which denotes the associated linear problem. Therefore, it follows that

$$EI\frac{d^4X_i}{dx^4}-\rho A\omega_i{}^2X_i=0, \tag{10}$$

where ω_i is the natural linear frequency of i-th mode. From Eq. (8), the following equation can be obtained with the help of Eq. (10) as

$$T_n+\alpha_n{}^2T_n+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\beta^n{}_{klm}T_kT_lT_m+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{q=1}^{\infty}\sum_{s=1}^{\infty}\gamma^n{}_{klmqs}T_kT_lT_m T_qT_s=p\delta_n\cos\omega\tau \tag{11}$$

where

$$\alpha_n=(\lambda_n/\lambda_1)^2,\ \beta^n{}_{klm}=\int_0^1\left(2\frac{dX_k}{d\xi}\frac{d^2X_2}{d\xi^2}\frac{dX^2{}_m}{d\xi^2}+\frac{d^2X_,}{d\xi^2}\frac{d^2X_l}{d\xi^2}\frac{d^2X_m}{d\xi^2}\right)X_n d\xi/(\lambda_1{}^4\varepsilon_n)$$

$$\gamma^n{}_{klmqs}=\int_0^1\left(2\frac{dX_k}{d\xi}\frac{dX_l}{d\xi}\frac{dX_m}{d\xi}\frac{d^2X_q}{d\xi^2}\frac{d^3X_s}{d\xi^3}+3\frac{d^2X_k}{d\xi^2}\frac{d^2X_l}{d\xi^2}\frac{d^2X_m}{d\xi^2}\frac{dX_q}{d\xi}\frac{dX_s}{d\xi}\right)X_n d\xi/(\lambda_1{}^4\varepsilon_n),\ \delta_n=\int_0^1X_n d\xi/(\lambda_1{}^4\varepsilon_n),\ \varepsilon_n=\int_0^1X_n{}^2d\xi,\ \tau=\omega_1t,\ \bar{\omega}=\Omega/\omega_1,\ p=p_o\ell^3/EI,$$

$$\lambda_1=\ell\sqrt[4]{\rho A\omega_1{}^2/EI},\ \xi=x/\ell$$

Since there is no known exact solution of Eq. (11), the harmonic balance method is used here. For Duffing type of Eq. (11), a solution of the form is assumed as

$$T_n=\sum_{s=1,3}^{\infty}b^s{}_n\cos s\bar{\omega}\tau, \tag{12}$$

where $b^s{}_n$ is an amplitude component and the $\bar{\omega}$ frequency ratio.

Substituting Eq. (12) into Eq. (11) and aplly-ing the harmonic balance method, a set of nonlinear algebraic equations will be obtained and solved by the Newton Raphson method.

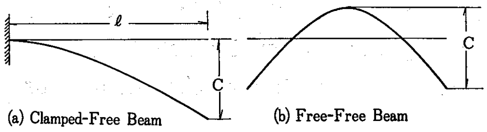

Fig. 1 Coordinate system and definition of amplitude

3. NONLINEAR FREE VIBRATIONS

The equation for the first second modes can be obtained from Eq. (11) in the following form in the cases of the clamped-free beam and the free-free beam as shown in Fig. 1

$$T_n+\alpha_n{}^2T_n+\sum_{k=1}^{2}\sum_{l=1}^{2}\sum_{m=1}^{2}\beta^n{}_{klm}T_kT_lT_m+\sum_{k=1}^{2}\sum_{l=1}^{2}\sum_{m=1}^{2}\sum_{q=1}^{2}\sum_{s=1}^{2}\gamma^n{}_{klmqs}T_kT_lT_mT_qT_s=\bar{P}\delta_n cos\bar{\omega}\tau. \tag{13}$$

Table 1 Coefficients of α_n, $\beta^n{}_{klm}$ and δ_n of Eq. (13)

	n	α_n	β^n_{111}	β^n_{112}	β^n_{122}	β^n_{222}	δ_n
(a)	1	1.00000	-0.40891	-4.80430	$-4.38875_{10}{}^{1}$	$-0.40619_{10}{}^{1}$	0.12667
	2	6.26689	0.66595	-6.35774	$-4.74362_{10}{}^{1}$	$-1.35675_{10}{}^{1}$	-0.07020
(b)	1	1.00000	-1.98812	$-1.01391_{10}{}^{2}$	$-4.34299_{10}{}^{3}$	$-0.88648_{10}{}^{4}$	0.0
	2	5.40392	0.61022	$-0.16222_{10}{}^{2}$	$-0.32177_{10}{}^{3}$	$-0.73117_{10}{}^{4}$	0.0

(a)clamped-free beam (b)free-free beam

Table 2 Coefficients of $\gamma^n{}_{klmqs}$ of Eq. (13)

	n	γ^n_{11111}	γ^n_{11112}	γ^n_{11122}	γ^n_{11222}	γ^n_{12222}	γ^n_{22222}
(a)	1	0.66595	-7.02443	$-0.80148_{10}{}^{2}$	$-2.91688_{10}{}^{2}$	$-0.64398_{10}{}^{3}$	$-0.44638_{10}{}^{3}$
	2	0.18923	-9.79669	$-1.29555_{10}{}^{2}$	$-7.27552_{10}{}^{2}$	$-1.42793_{10}{}^{3}$	$-1.29853_{10}{}^{3}$
(b)	1	-8.01996	$-6.50858_{10}{}^{2}$	$-4.23366_{10}{}^{4}$	$-2.75735_{10}{}^{5}$	$-0.39535_{10}{}^{6}$	$-7.38284_{10}{}^{6}$
	2	1.22059	$-1.05752_{10}{}^{2}$	$-0.51372_{10}{}^{4}$	$-1.41518_{10}{}^{5}$	$-0.75206_{10}{}^{6}$	$-4.11565_{10}{}^{6}$

(a)clamped-free beam (b)free-free beam

Coefficients α_n, $\beta^n{}_{klm}$, $\gamma^n{}_{klmqs}$ and δ_n are summarized in Tables 1 and 2. In these cases, the value c in Fig. 1 is taken to unity for each mode. Employing the first second harmonics as to time variable, Eq. (12) can be expressed as follows:

$$T_n=a_n cos\bar{\omega}\tau+b_n cos3\bar{\omega}\tau. \tag{14}$$

Substituting Eq. (14) into Eq. (13) and using the harmonic balance method, the following four nonlinear coupled equations are obtained as

$$(\alpha^2{}_n-\omega^2)a_n+\beta^n{}_{111}f_{111}+\beta^n{}_{112}f_{112}+\beta^n{}_{122}f_{122}+\beta^n{}_{222}f_{222}+\gamma^n{}_{11111}f_{11111}+\gamma^n{}_{11112}f_{11112}+\gamma^n{}_{11122}f_{11122}+\gamma^n{}_{11222}f_{11222}+\gamma^n{}_{12222}f_{12222}+\gamma^n{}_{22222}f_{22222}=\delta_n\bar{p},\ (\alpha^2{}_n-9\omega^2)b_n+\beta^n{}_{111}g_{111}+\beta^n{}_{112}g_{112}+\beta^n{}_{122}g_{122}+\beta^n{}_{222}g_{222}+\gamma^n{}_{11111}g_{11111}+\gamma^n{}_{11112}g_{11112}+\gamma^n{}_{11122}g_{11122}+\gamma^n{}_{11222}g_{11222}+\gamma^n{}_{12222}g_{12222}+\gamma^n{}_{22222}g_{22222}=0, \tag{15}$$

where

$$f_{ijk}=\frac{1}{4}\{3a_ia_ja_k+2(a_ib_jb_k+b_ia_jb_k+b_ib_ja_k)+a_ia_jb_k+a_ib_ja_k+b_ia_ja_k\},$$

$$
\begin{aligned}
g_{ijk} = {} & \frac{1}{4}\{a_i a_j a_k + 3b_i b_j b_k + 2(a_i a_j b_k + \\
& a_i b_j a_k + b_i a_j a_k)\}, \\
f_{ijklm} = {} & \frac{1}{16}\{10a_i a_j a_k a_l a_m + 6b_i b_j b_k b_l b_m \\
& +5(a_i a_j a_k a_l b_m + a_i a_j a_k b_l a_m + a_i a_j b_k a_l \\
& a_m + a_i b_j a_k a_l a_m + b_i a_j a_k a_l a_m) + 6(a_i a_j \\
& a_k b_l b_m + a_i a_j b_k a_l b_m + a_i b_j a_k a_l b_m + b_i a_j \\
& a_k a_l b_m + a_i a_j b_k b_l a_m + a_i b_j a_k b_l a_m + b_i \\
& a_j a_k b_l a_m + a_i b_j b_k a_l a_m + b_i a_j b_k a_l a_m + b_i \\
& b_j a_k a_l a_m) + 3(b_i b_j b_k a_l a_m + b_i b_j a_k b_l a_m \\
& + b_i a_j b_k b_l a_m + a_i b_j b_k b_l a_m + b_i b_j a_k a_l \\
& b_m + b_i a_j b_k a_l b_m + a_i b_j b_k a_l b_m + a_i b_j b_k \\
& a_l b_m + b_i a_j a_k b_l b_m + a_i b_j a_k b_l b_m + a_i a_j \\
& b_k b_l b_m) + 6(b_i b_j b_k b_l a_m + b_i b_j b_k a_l b_m \\
& + b_i b_j a_k b_l b_m + b_i a_j b_k b_l b_m + a_i b_j b_k b_l \\
& b_m)\}, \\
g_{ijklm} = {} & \frac{1}{16}\{5a_i a_j a_k a_l a_m + 10b_i b_j b_k b_l \\
& b_m + 6(a_i a_j a_k a_l b_m + a_i a_j a_k b_l a_m + a_i a_j b_k \\
& a_l a_m + a_i b_j a_k a_l a_m + b_i a_j a_k a_l a_m) + 3(a_i a_j \\
& a_k b_l b_m + a_i a_j b_k a_l b_m + a_i b_j a_k a_l b_m + b_i \\
& a_j a_k a_l b_m + a_i a_j b_k b_l a_m + a_i b_j a_k b_l a_m + \\
& b_i a_j a_k b_l a_m) + 6(b_i b_j b_k a_l a_m + b_i b_j a_k b_l \\
& a_m + b_i a_j b_k b_l a_m + a_i b_j b_k b_l a_m + b_i b_j a_k \\
& a_l b_m + b_i a_j b_k a_l b_m + a_i b_j b_k a_l b_m + b_i a_j \\
& a_k b_l b_m + a_i b_j a_k b_l b_m + a_i a_j b_k b_l b_m)\}.
\end{aligned}
$$

The nondimensional amplitude is defined by the following equation when the time $\bar{\omega}\tau$ is equal to $n\pi(n=0,1,2,\cdots\cdots)$

$$c/\ell = a_1 + a_2 + b_1 + b_2 . \qquad (16)$$

(1) Clamped-Free Beam

Frequency ratio $\bar{\omega}$ versus various amplitude ratio c/ℓ of the first natural frequency when the nonlinear term of the five order is neglected is shown in Table 3 (a). As the nonlinear restoring force $\beta^1{}_{111}$ of the first natural frequency is negative as shown in Table 1 (a), it is evident from Table 3 (a) that the influence of the large amplitude is similar to that of a softening spring. The frequency ratio decreases with increase of the amplitude ratio. The frequency ratio in Table 3 (b) shows the frequency ratio of the corresponding solution of the single degree-of-

Table 3 Frequency ratios of the clamped-free beam

c/ℓ	(a)	(b)	(c)	(d)	(e)	(f)	(g)
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9985	0.9985	0.9985	0.9938	0.9980	0.9977	1.0015
0.2	0.9939	0.9939	0.9937	0.9752	0.9918	0.9908	1.0061
0.3	0.9862	0.9861	0.9854	0.9432	0.9815	0.9793	1.0138
0.4	0.9756	0.9752	0.9729	0.8965	0.9669	0.9632	1.0245
0.5	0.9620	0.9610	0.9554	0.8327	0.9478	0.9425	1.0383

(a) present solution(2 degree of freedom, 2 term solution)

(b) present solution(1 degree of freedom), perturbation method

(c) present solution(2 degree of freedom, 2 term solution), higher order solution

(d) energy method by T.Takahashi

(e) Galerkin method by Yoshimura and Uemura

(f) Galerkin method by Bauer

(h) perturbation method by Kato

freedom obtained by a perturbation method. As the magnitude of the nonlinear term in this problem is small, it will be seen from Table 3 (a) and (b) that the difference between the multiple degree-of-freedom solution and the single degree-of-freedom is very small. This means that the effect of the change of the normal mode due to amplitude on frequency is small.

The frequency ratio in Table 3 (c) shows the frequency ratio which considers the nonlinear term up to an order of five. The solution of Table 3 (c) becomes slightly smaller than that of Table 3 (a). That is, the effect of the order of five is greater that of the change of the mode.

Table 3 (d), (e), (f) and (g) show results obtained by various basic equations and methods of solution, i. e., (d) shows the Takahashi's solution obtained by the energy method which neglects the axial displacement and uses an approximate curvature $\kappa=\frac{\partial^2 y}{\partial x^2}\left\{1-\frac{3}{2}\left(\frac{\partial y}{\partial x}\right)^2\right\}$, (e) shows Yoshimura and Uemura's result whose basic equation is the same one up to an order of three of Eq. (9) and solved by a Galerkin method. However, the space function is not coincide with the beam function. (f) shows the result obtained by Bauer' basic equation which neglects the axial displacement and inertia force. The last column (g) shows the result obtained by Wagner's basic equation. The nonlinear effect shows a hardening spring type in this case only. This discrepancy arises from a incorrect variational problem.

Table 4 Normal mode of the clamped–free beam

ξ \ A	0.0	0.1	0.3	0.5
0.0	0.0000	0.0000	0.0000	0.0000
0.2	0.0639	0.0639	0.0645	0.0657
0.4	0.2299	0.2301	0.2314	0.2344
0.6	0.4611	0.4613	0.4629	0.4662
0.8	0.7255	0.7256	0.7266	0.7286
1.0	1.0000	1.0000	1.0000	1.0000

ξ \ c/ℓ	0.0	0.1	0.3	0.5
0.0	1.0000	1.0000	1.0000	1.0000
0.2	0.7254	0.7260	0.7304	0.7394
0.4	0.4611	0.4625	0.4738	0.5081
0.6	0.2299	0.2307	0.2385	0.2740
0.8	0.0639	0.0640	0.0656	0.0784
1.0	0.0000	0.0000	0.0000	0.0000

Table 5 Strain distribution of the clamped–free beam

The normal mode and strain distribution of the clamped-free beam are shown in Tables 4 and 5. It will be seen that these vary slightly from those of the linear one.

(2) Free-Free Beam

Frequency ratios of the free-free beam are shown in Table 6. Table 6 (a) shows the frequency ratio of the present solution which considers the nonlinear term up to an order of three, and (b) shows the ratio considering the contribution of an order of five, while (c) shows the ratio of the corresponding solution obtained by the single degree-of-freedom approach. The

Table 6 Frequency ratios of the free–free beam

c/ℓ	(a)	(b)	(c)
0.0	1.0000	1.0000	1.0000
0.1	0.9926	0.9923	0.9925
0.2	0.9706	0.9670	0.9702
0.3	0.9360	0.9179	0.9329
0.4	0.8907	0.8401	0.8807
0.5	0.8383	0.7200	0.8136

nonlinear term $\beta^{1}{}_{111}$ of the free-free beam is greater than that of the clamped-free beam as shown in Table 1 (a). The frequency ratios show that the free-free beam exhibits much change in frequency with amplitude than does the clamped-free beam. The difference between the multiple degree-of-freedom solution and the single degree-of-freedom solution comes out with increase of amplitude. This means that the mode of vibration which changes with amplitude modifies the frequency. The contribution the order of five can not be negligible as shown in Table 6 (b). The normal mode of the free-free beam is shown in Table 7.

Table 7 Normal mode of the free–free beam

ξ \ c/ℓ	0.0	0.1	0.2	0.3	0.4	0.5
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.5372	0.5373	0.5391	0.5433	0.5475	0.5568
0.2	0.0977	0.0987	0.1005	0.1063	0.1129	0.1228
0.3	-0.2720	-0.2718	-0.2712	-0.2695	-0.2670	-0.2672
0.4	-0.5202	-0.5213	-0.5233	-0.5307	-0.5372	-0.5489
0.5	-0.6078	-0.6096	-0.6130	-0.6240	-0.6368	-0.6530

4. CONCLUSIONS

In this paper, the nonlinear vibrations of the clamped-free beam and the free-free beam are analyzed. A summary of the results is as follows:

(1) Clamped-Free Beam

(a) The influence of the large amplitude on the frequency of the clamped-free beam is similar to that of a weakly softening spring.

(b) The influence of the change of the normal mode with the amplitude on the nonlinear frequency is small. The difference of the solutions between the multiple mode approach and the single mode approach is small. The solution of the multiple mode approach gives slightly smaller frequency ratio than that of the single mode approach.

(c) The effect of the order of five of the deflection modifies the frequency ratio obtained by the solution of the order of three.

(d) The previous solutions give different results. This reason is due to the occasion when the higher term is neglected or a incorrect variational problem.

(e) The normal mode and strain distribution vary with amplitude. These changes are cosiderably small.

(2) Free-Free Beam

(a) Frequency ratios of the free-free beam are much affected by the amplitude than that of the clamped-free beam.

(b) The frequency ratio must be calculated by the multiple mode approach which cosiders the higher order of the deflection.

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