

STRESS CONDITIONS WITHIN SIMPLE SHEAR TEST SPECIMEN

By

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On the basis of the assumption that the stresses within the simple shear test specimen are completely uniform, the two methods for interpreting the stresses within the specimen are described. The first method consists in assuming that the simple shear stress conditions are represented by the pure shear stress conditions. The second method is based on the assumption that the directions of the principal stress axes during simple shear are known. The differences between the two methods are discussed, and the undrained shear strength values of normally consolidated clays measured in the simple shear test are compared with those measured in the triaxial compression test on an equal basis.

INTRODUCTION

Although the simple shear test has some advantages over other tests, the stress conditions within the test specimen are not known: The average values of normal and shear stresses acting on the horizontal plane are only measured and the stresses are not known for any other plane within the specimen. From a practical point of view which enhance the utility value of the simple shear test, it is necessary to determine the entire state of stress within the test specimen, and which is possible by assuming that the stresses within the specimen are completely uniform.

There are the two methods for interpreting the stresses within the simple shear test specimen on the basis of the assumption that the stresses and strains are

uniform throughout the test specimen, as shown in Fig.1. The first method⁽¹⁾ which is denoted as A-method in Fig.1 consists in assuming that the simple shear specimen is considered to be an element of soil subjected to pure shear. The second method⁽²⁾ denoted as B-method in Fig.1 is based on the assumption that the directions of the principal stress axes during simple shear are expressed by the following relation proposed by Oda and Konishi⁽³⁾.

$$\tau_h/\sigma_n = \kappa \cdot \tan\psi \quad (1)$$

where σ_n and τ_h are the measured average values of normal and shear stresses on the horizontal plane, ψ is the inclination angle of the maximum principal stress axis to the vertical direction and κ is a

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material constant which is expressed by the following equation^{(2), (4), (5)}.

$$\kappa = 1 - K_o = \sin \phi_{CV} = \frac{2 \sin \phi_{\mu}}{1 + \sin \phi_{\mu}} \quad (2)$$

where K_o is the coefficient of earth pressure at rest, ϕ_{CV} is the friction angle at the critical void ratio state and ϕ_{μ} is the interparticle friction angle.

Although the above two methods have been severally utilized for interpreting the behaviour of soil in the simple shear apparatus without especial consideration, there are considerable differences between the two methods, and it is necessary to clarify the differences. In this paper, the differences between the above two methods are discussed, furthermore the undrained strength values of normally consolidated clays measured in the simple shear test are compared with those measured in the triaxial compression test based on the two methods.

TWO METHODS FOR INTERPRETING THE STRESSES WITHIN THE SIMPLE SHEAR TEST SPECIMEN

In the simple shear apparatus, the initial stress state when only the normal stress, σ_n , is applied on the horizontal plane is at rest as is well known as K_o -consolidation, and at this state the vertical and

horizontal stresses, σ_v and σ_h , which are equal to the maximum and minimum principal stresses, σ_1 and σ_3 , are expressed as follows:

$$\left. \begin{aligned} \sigma_v &= \sigma_1 = \sigma_n \\ \sigma_h &= \sigma_3 = K_o \sigma_n \end{aligned} \right\} \quad (3)$$

In the A-method, it is assumed that the simple shear stress conditions are represented by the pure shear stress conditions. The principal stress changes in pure shear, $\Delta \sigma_1$ and $\Delta \sigma_3$, are of equal magnitude but opposite sign⁽¹⁾, so that for the case of the increase of shear stress, τ_h , under the constant normal stress, σ_n , the Mohr's stress circles during shear are the cocentric circles, as shown in Fig.2. Therefore, the horizontal stress, σ_h , is a constant value of $K_o \sigma_n$ during shear if K_o -value is constant. The maximum shear stress, τ_{max} , and the maximum and minimum principal stresses, σ_1 and σ_3 , are expressed from the geometry as follows:

$$\tau_{max} = \sqrt{\frac{\sigma_n^2 (1 - K_o)^2}{4} + \tau_h^2} \quad (4)$$

$$\sigma_1 = \frac{1}{2} (1 + K_o) \sigma_n + \sqrt{\frac{\sigma_n^2 (1 - K_o)^2}{4} + \tau_h^2} \quad (5)$$

$$\sigma_3 = \frac{1}{2} (1 + K_o) \sigma_n$$

Assumption

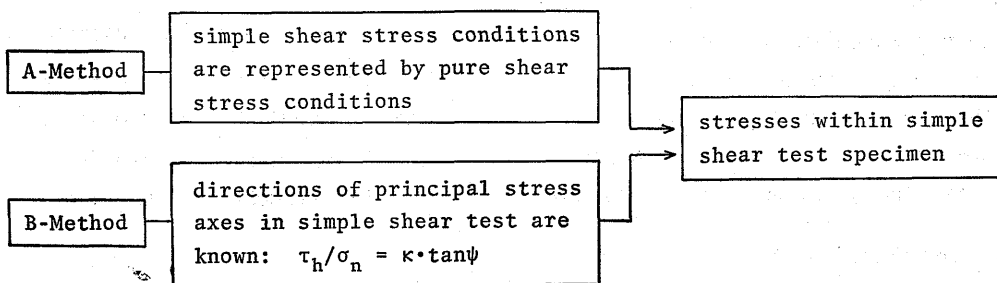


Fig. 1 Two methods for interpreting the stresses within simple shear test specimen

$$- \sqrt{\frac{\sigma_n^2 (1-K_0)^2}{4} + \tau_h^2} \quad (6)$$

And the inclination angle of the maximum principal stress axis to the vertical direction, ψ , is

$$\tan\psi = \frac{\sqrt{\frac{(1-K_0)^2}{4} + (\tau_h/\sigma_n)^2} - \frac{1-K_0}{2}}{(\tau_h/\sigma_n)} \quad (7)$$

In the B-method, ψ during simple shear is expressed by Eq. (1), and the principal stresses, σ_1 and σ_3 , are expressed as follows⁽²⁾ :

$$\sigma_1 = \frac{(1-K_0)\sigma_n^2 + \tau_h^2}{(1-K_0)\sigma_n} \quad (8)$$

$$\sigma_3 = K_0\sigma_n \quad (9)$$

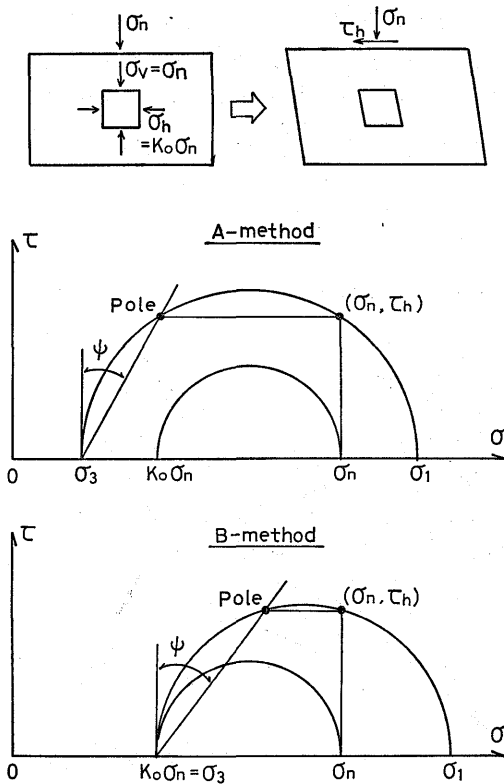


Fig. 2 Difference of Mohr's stress circles during simple shear based on the two methods

The minimum principal stress, σ_3 , is therefore determined by only the normal stress, σ_n , independent of the shear stress, τ_h . For the case that the values of σ_n and K_0 are constant during shear, the minimum principal stress, σ_3 , is constant but the horizontal stress, σ_h , changes, as shown in Fig.2. The maximum shear stress, τ_{max} , is expressed as follows:

$$\tau_{max} = \frac{1}{2} (\sigma_1 - \sigma_3) = \frac{(1-K_0)^2 \sigma_n^2 + \tau_h^2}{2(1-K_0)\sigma_n} \quad (10)$$

Fig. 3 shows Eq. (1), Eq. (7) and the experimental results on Leighton Buzzard Sand ($\phi_{CV}=35^\circ$) by Cole⁽⁶⁾ for the case of $K_0=1-\sin\phi_{CV}=0.426$. It is seen from this figure that Eq. (1) which forms the basis of the B-method agrees well with the

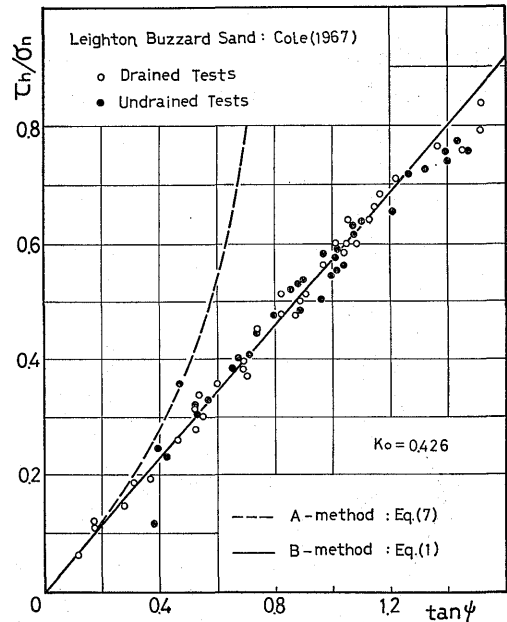


Fig. 3 Relationship between $\tan\psi$ and τ_h/σ_n during simple shear based on the two methods, and experimental results of a sand

experimental results of sand and the difference between the A-method and the B-method becomes large with increasing the value of stress ratio, τ_h/σ_n , on the horizontal plane.

The magnitudes of the maximum and minimum principal stresses, σ_1 and σ_3 , and its directions during shear are shown in Fig.4 for the case of $K_0=0.5$ and $\sigma_n = 200$ kN/m². In the A-method, as the value of stress ratio, τ_h/σ_n , increases the value of the minimum principal stress, σ_3 , decreases, and the value is negative when τ_h/σ_n exceeds $\sqrt{K_0}$, that is, the tensile stresses produce on some planes within the test specimen under the compressive stress condition. On the other hand, σ_3 is always a positive constant value during shear and any tensile stresses do not produce within the test specimen, in the B-method.

Now, the coefficient of earth pressure at rest, K_0 , depends on the internal friction

angle in effective stress, ϕ' , and K_0 -value decreases with increasing the value of ϕ' as is known from Jáký's following empirical equation⁽⁷⁾.

$$K_0 = 1 - \sin\phi' \tag{11}$$

Using Eq. (11), when the value of τ_h/σ_n is the following value, the minimum principal stress, σ_3 , is zero in the A-method.

$$(\tau_h/\sigma_n)_{\sigma_3=0} = \sqrt{K_0} = \sqrt{1 - \sin\phi'} \tag{12}$$

Consequently, as the shear strength of the test specimen is larger, the tensile stresses will produce within the specimen at the smaller value of τ_h/σ_n , in the A-methods. Let consider a case that the value of the stress ratio, $(\tau_h/\sigma_n)_f$, on the horizontal plane at failure in the simple shear test is 0.7, which is usually measured on dense sand (c.f. Fig. 3). Assuming $(\tau_h/\sigma_n)_f = \tan\phi$, then $\phi = 35^\circ$ and $K_0 = 1 - \sin\phi = 0.426$. And the value of τ_h/σ_n at state of $\sigma_3 = 0$ is from Eq. (12) as follows: $(\tau_h/\sigma_n)_{\sigma_3=0} = \sqrt{K_0} = 0.653 < (\tau_h/\sigma_n)_f = 0.7$

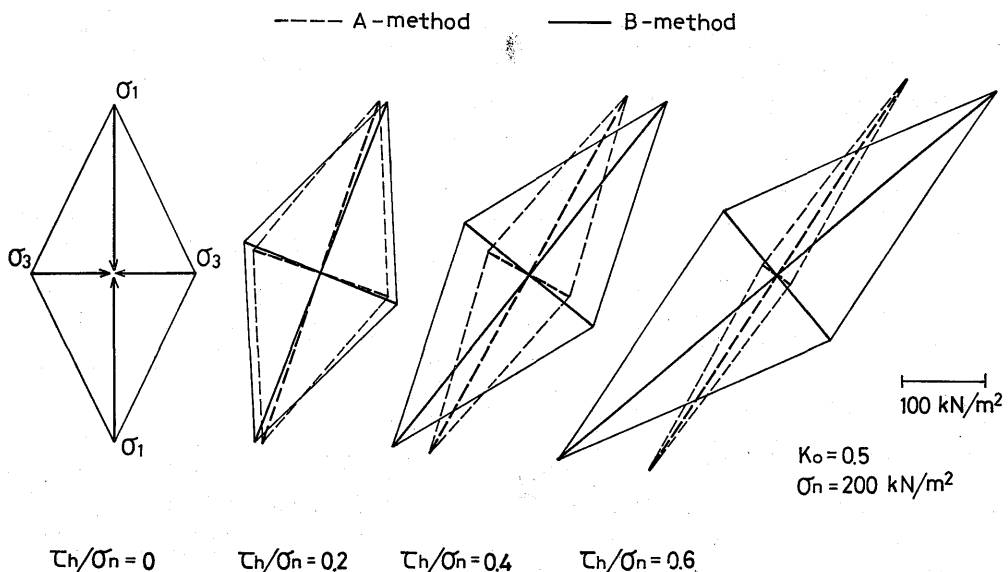


Fig 4 Magnitudes of σ_1 and σ_3 , and its directions during simple shear based on the two methods

Therefore, the tensile stresses must produce within the test specimen before the failure under the condition that the stresses and strains are uniform throughout the specimen, in the A-method. Is it true?

COMPARISON OF THE UNDRAINED STRENGTH VALUES OF NORMALLY CONSOLIDATED CLAYS MEASURED IN SIMPLE SHEAR AND TRIAXIAL COMPRESSION TESTS

The stress conditions in the simple shear test may provide those of some types of field loading conditions, so that from a practical point of view it is important to know the relation between the strength values measured in the simple shear test and the those measured in other types of tests. In that case, it is necessary to compare those on an equal basis because the measured shear stress, τ_h , on the horizontal plane in the simple shear test is not the maximum shear stress, nor is it the shear stress on the failure plane⁽¹⁾. If we can determine the

principal stresses, σ_1 and σ_3 , in the simple shear test, it is possible to compare the undrained strength values of normally consolidated clays measured in the simple shear test with those measured in the triaxial compression test on an equal basis. The undrained strength values of normally consolidated clays are usually expressed in terms of c_u/p , where c_u is one-half of the compressive strength, or principal stress difference at failure, and p is the consolidation pressure. The consolidation pressure, p , is the initial vertical stress, σ_n , in the simple shear test, so that the value of c_u/p in this test is expressed as follows:

In the A-method, from Eq.(4)

$$(c_u/p)_{SS} = (\tau_{max}/\sigma_n)_f$$

$$= \sqrt{(\tau_h/\sigma_n)_f^2 + \frac{(1-K_o)^2}{4}} \tag{13}$$

In the B-method, from Eq.(10)

$$(c_u/p)_{SS} = (\tau_{max}/\sigma_n)_f$$

$$= \frac{(\tau_h/\sigma_n)_f^2 + (1-K_o)^2}{2(1-K_o)} \tag{14}$$

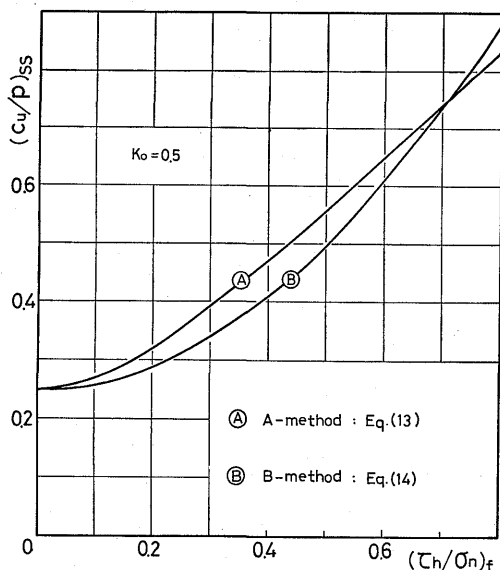


Fig. 5 Relationship between $(c_u/p)_{SS}$ and $(\tau_h/\sigma_n)_f$ in simple shear test based on the two methods

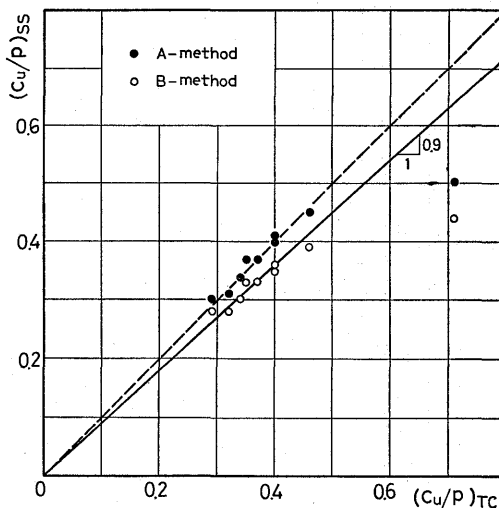


Fig. 6 Relationship between calculated values of $(c_u/p)_{SS}$ based on the two methods and measured values of $(c_u/p)_{TC}$

Eq. (13) and Eq. (14) are shown in Fig.5, for the case of $K_0=0.5$.

Table 1 shows the experimental results of normally consolidated clays in the simple shear and triaxial compression tests ⁽¹⁾, ⁽²⁾, ⁽³⁾. The values of $(c_u/p)_{ss}$ in the simple shear test can be calculated by Eq. (13) or Eq. (14) using the values of K_0 and $(\tau_h/\sigma_n)_f$, and those values are also shown in Table 1. The relationships between the calculated values of $(c_u/p)_{ss}$ based on the two methods and the measured values of $(c_u/p)_{TC}$ which is the undrained strength values measured in the triaxial compression test are shown in Fig.6. It is known from Fig. 6 that there are the following relations between $(c_u/p)_{ss}$ and $(c_u/p)_{TC}$.

In the A-method

$$(c_u/p)_{ss} \approx (c_u/p)_{TC} \quad (15)$$

In the B-method

$$(c_u/p)_{ss} \approx 0.9(c_u/p)_{TC} \quad (16)$$

It should be noted that $(c_u/p)_{ss}$ is approximately equal to $(c_u/p)_{TC}$ in the A-method, but which not always verifies that the A-method is correct because the

simple shear specimen may satisfy the plane strain condition, whereas the triaxial test specimen is not so.

CONCLUDING REMARKS

Duncan and Dulop ⁽¹⁾ have showed by the finite element method analysis that the stresses in the simple shear specimen of soft clay are not uniform, and that progressive failure occurs in the simple shear test. Nevertheless, from a practical point of view, assuming that the stresses within the simple shear specimen are completely uniform, its test results will provide useful information. There are the two methods for interpreting the stresses within the simple shear specimen on the basis of the assumption, as described in this paper, but the considerable differences between the two methods exist. The A-method has been utilized widely for the study of liquefaction used the simple shear apparatus ⁽¹⁰⁾. On the other hand, it has been confirmed that the behaviour of sand in the simple shear test can be explained well by Eq.(1) which forms the basis of

Table 1 Experimental results of normally consolidated clays in simple shear and triaxial compression tests, and calculated values of $(c_u/p)_{ss}$ based on the two methods

Material or Site	K_0	$(\tau_h/\sigma_n)_f$	$(c_u/p)_{TC}$		$(c_u/p)_{ss}$	
					A-method:Eq.(13)	B-method:Eq.(14)
San Francisco Bay Mud	0.45	0.25	0.35	(1)	0.37	0.33
Manglerud Clay	0.51	0.18	0.29	(2)	0.30	0.28
Bangkok	0.45	0.42	0.71*	(3)	0.50	0.44
Kimola	0.45	0.36	0.46*	(3)	0.45	0.39
Drammen	0.49	0.32	0.40*	(3)	0.41	0.36
Sundland	0.48	0.30	0.40*	(3)	0.40	0.35
Vaterland	0.51	0.28	0.37*	(3)	0.37	0.33
Studenterlunden	0.51	0.19	0.32*	(3)	0.31	0.28
Drammen	0.49	0.22	0.34*	(3)	0.34	0.30

* K_0 -consolidation

(1) : Duncan, et al (1969), (2) : Bjerrum, et al (1965), (3) : Berre, et al (1973)

the B-method⁽⁹⁾,⁽¹¹⁾. And Ohara and Matsuda⁽¹²⁾ have adopted the B-method for interpreting the undrained shear strength of a saturated clay measured in the simple shear test. At the present situation, it is difficult to judge the propriety of either of the two methods, and further theoretical and experimental investigations will be necessary in future.

REFERENCES

- (1) Duncan, J.M. and Dunlop, P. (1969): Behavior of soils in simple shear test, Proc. 7th ICSMFE, Vol.1, p.101-109.
- (2) Ochiai, H. (1975): The behaviour of sands in direct shear tests, Jour. of JSSMFE, Vol.15, No.4, p.93-100, (in Japanese).
- (3) Oda, M. and Konishi, J. (1974): Rotation of principal stresses in granular material during simple shear, Soils and Foundations, Vol.14, No.4, p.39-53.
- (4) Ochiai, H. (1976): The coefficient of earth pressure at rest of sands, Jour. of JSSMFE, Vol.16, No.2, p.105-111, (in Japanese).
- (5) Ochiai, H. (1976): Discussin "On the relation $\tau/\sigma_n = \kappa \cdot \tan\phi$ in the simple shear test", Soils and Foundations, Vol.16, No.3, p.81-85.
- (6) Cole, E.R.L. (1967): The behaviour of soils in the simple shear apparatus, Ph. D. thesis, Cambridge University.
- (7) Jäky, J. (1948): Pressure in silos, Proc. 2nd ICSMFE, Vol.1, p.103-107.
- (8) Bjerrum, L. and Landva, A. (1965): Direct simple-shear test on a Norwegian quick clay, Geotechnique, Vol.16, No.1, p.1-20.
- (9) Berre, T. and Bjerrum, L. (1973): Shear strength of normally consolidated clays, Proc. 8th ICSMFE, Vol.1, p.39-49.
- (10) for example, Seed, H.B. and Peacock, W.H. (1971): Test procedures for measuring soil liquefaction characteristics, Proc. ASCE, Vol.97, No.SM8, p.1099-1119.
- (11) Oda, M. (1975): On the relation $\tau/\sigma_n = \kappa \cdot \tan\phi$ in simple shear test, Soils and Foundations, Vol.15, No.4, p.35-41.
- (12) Ohara, S. and Matsuda, H. (1978): Dynamic shear strength of saturated clay, Proc. of JSCE, No.274, p.69-78, (in Japanese).