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Online Signal Frequency Analysis using Power Series Type Wavelet Transform

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***Abstract*-- A new approach to estimate the power system frequency and signal components using wavelet transform is proposed in this paper. The wavelet function is expressed by the power series terms and is able to be performed by the z-transform, which can shorten the estimating time of the signal frequency compared with the general convolution integral. The fundamental frequency (60Hz) or the harmonics frequency (300Hz) of the signal is quickly estimated. The on-line frequency estimation by the proposed method is successfully achieved in the experiment using the DSP system. The application using Induction motor system and the rectifier system**

***Index Terms*—wavelet transform, time-frequency analysis, online estimation, component extraction**

I. Introduction

Recently it is reported that the developments of power electronics make the harmful influence of harmonics or a noise. Because the electric power systems increasingly complicate, wavelet analysis attracts attention as the new method[1]-[5]. The wavelet analysis is able to calculate a time-frequency analysis and acquire the frequency information on a signal. In general, the wavelet analysis has problems to apply it online to a power system, which are various amounts of calculation due to an integral, accuracy and so on. The Gabor function or several functions are widely used for the wavelet transform. Those are required the long calculation time for the convolution integral and it is difficult to calculate on line. Thus we introduced the new wavelet function to realize the online calculation of signal information[1]. It can be easily transformed to discrete time system by the z-transform, by which the amount of calculation decreases. The proposed method is able to detect frequency, phase and amplitude of the detected signal. Furthermore, the certain frequency component of the signal can be calculated using the obtained information.

In this paper, the online frequency estimation and the component extraction are confirmed applying it to the experimental system using a DSP. The effectiveness of the proposed method and influences of parameters of the wavelet function are investigated and the good results are obtained in the experiment.

II. Wavelet Analysis

A. Wavelet Transform

In order to acquire the local information on the observed signal $s(t)$, a core function, called a mother wavelet function, is defined. It is expanded, reduced or carried out its parallel time shifting. The wavelet transform can be derived from a mother wavelet function by introducing scaling factor a and time shifting factor τ.

$$\varphi_{a,\tau}(t) = \frac{1}{\sqrt{a}}\varphi\left(\frac{t-\tau}{a}\right) \tag{1}$$

Mother wavelet function $\varphi(t)$ is defined as an analytical function which satisfies the admissibility condition expressed in (2).

$$C_\varphi = \int_{-\infty}^{\infty}\left(\frac{|\psi(\omega)|^2}{\omega}\right)d\omega < \infty \tag{2}$$

$$\text{i.e.}\quad \psi(\omega)\big|_{\omega=0} = \int_{-\infty}^{\infty}\varphi(t)dt = 0 \tag{3}$$

It presents the band pass frequency property of $\varphi(t)$ where $\Psi(\omega)$ is the Fourier transform of $\varphi(t)$.

The wavelet transform of $s(t)$ by the wavelet function is defined as (4).

$$W_\varphi(a,\tau) = \int_{-\infty}^{\infty} s(t)\overline{\varphi_{a,\tau}(t)}dt \tag{4}$$

Replacing $\overline{\varphi_{a,\tau}(t)} = \eta_a(\tau - t)$, (5) is derived.

$$W_\eta(a,\tau) = \int_{-\infty}^{\infty} s(t)\eta_a(\tau - t)dt \tag{5}$$

$\overline{\varphi(t)}$ is the complex conjugate of $\varphi(t)$. If the scale factor a increases, a frequency band of the wavelet function spread and the band becomes narrow if a decrease.

Since the wavelet transform expressed by (5) is a linear function, it can be calculated by a convolution integral. Therefore the discrete time system of the convolution integral can be expressed by (6).

$$\begin{aligned} W(a,k) &= \{\eta_a(n)s(k-n) + \cdots + \eta_a(1)s(k-1) + \eta_a(0)s(k)\}T \\ &= \sum_{l=k-n}^{k} \eta_a(k-l)\,s(l)\,T \end{aligned} \tag{6}$$

Fig. 1 shows the convolution integral wavelet transform. A large amount of the data ($n > 100$) and a long calculation time are required to compute.

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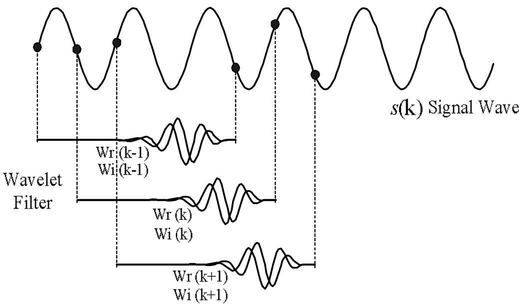

Fig. 1 Convolution integral of wavelet transform

B. Proposed Wavelet Function

A prototype of the proposed wavelet function, which consists of the power series expression, is described in (7) and the filter function $\eta_a(t)$ is derived as (8) and (9).

$$\varphi(t)=\left\{\frac{\sigma^7}{240}t^7+\frac{\sigma^8}{960}t^8+\frac{(C^2+1)\sigma^9}{17280}t^9\right\}\exp(\sigma t+j\omega_0 t)u(t) \tag{7}$$

$$\begin{aligned}\eta(t)&=\overline{\varphi(-t)}\\ &=\left\{-\frac{\sigma^7}{240}t^7+\frac{\sigma^8}{960}t^8-\frac{(C^2+1)\sigma^9}{17280}t^9\right\}\exp(-\sigma t+j\omega_0 t)u(-t)\end{aligned} \tag{8}$$

$$\eta_a(t)=\frac{1}{\sqrt{a}}\eta\left(\frac{t}{a}\right) \tag{9}$$

$$a=\frac{\omega_0}{2\pi f_0} \tag{10}$$

The where ω_0 and a band factor C are constant and $\sigma=\omega_0/C$. The parameters are determined so as to satisfy the admissible condition in (2). $u(t)$ is the unit step function. f_0 is the filter frequency, on which the estimated frequency is depended. Fig.2 shows an example of the real and imaginary parts of the filter function $\eta_a(t)$ at f_0 = 300 Hz.

C. Characteristics of the Proposed Wavelet Function

Parameters of the wavelet filter function are determined as the following conditions. Those satisfy the admissible condition.

$$\omega_0=2\pi,\ \sigma=\frac{\omega_0}{C},\ a=\frac{\omega_0}{2\pi f_0}=\frac{1}{f_0} \tag{11}$$

Fig. 3 and Fig. 4 illustrate the frequency spectrums of the proposed filter functions $\eta_a(t)$ on each condition. Characteristics of the wavelet filter function by changing f_0 and C are the following two points.

- The scale factor a is reciprocal of f_0. Frequency of the maximum spectrum becomes f_0. Thus the spectrum density depends on the filter frequency f_0 as shown in Fig. 3.
- The band factor C makes the band of spectrum fixed. If the C becomes small, the band of spectrum enlarges as shown in Fig.4. When the analysis range of the signal is focus on, the C must be increased.

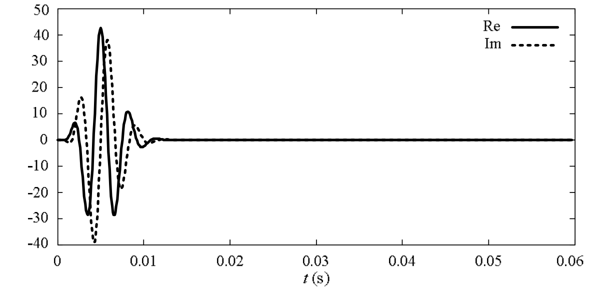

Fig. 2 Wavelet filter function (f_0 = 300Hz, $\omega_0=\sigma=2\pi$, C=1)

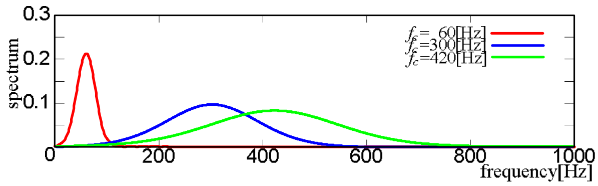

Fig.3 Frequency spectrum density ($\omega_0=\sigma=2\pi$, C=1)

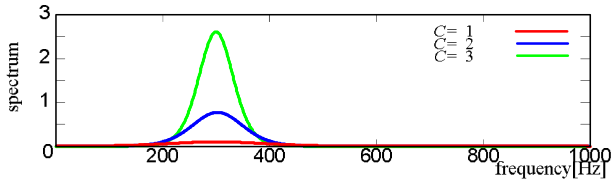

Fig.4 Frequency spectrum density (f_0=300Hz, $\omega_0=\sigma=2\pi$)

D. z-transform of the wavelet function

The proposed function includes the power series function and can be easily transformed the discrete time system by the z-transform and can shorten the calculation time in practice. The wavelet transform in the discrete time system is expressed in (12).

$$W_\eta(z)=a^{-\frac{1}{2}}T_S\,s(z)\eta(z) \tag{12}$$

where $s(z)$ and $\eta(z)$ are the z-transform of the discrete signal $s(nT)$ and the discrete function $\eta(nT/a)$ respectively. The z-transform $\eta(z)$ becomes as the following polynomial form.

$$\eta(z)=\sum_{n=0}^{\infty}\eta(^{nT_S}/_a)z^{-n}=\frac{\sum_{m=1}^{9}\delta_m z^{-m}}{1+\sum_{m=0}^{10}\lambda_m z^{-m}} \tag{13}$$

$$\lambda_m=k_m\left[\exp\left\{-\frac{\omega_0 T_S(1-jC)}{aC}\right\}\right]^m \tag{14}$$

$$\begin{aligned}\delta_m=&\left\{p_m\left(\frac{\omega_0 T_S}{aC}\right)^7+q_m\left(\frac{\omega_0 T_S}{aC}\right)^8+r_m(C^2+1)\left(\frac{\omega_0 T_S}{aC}\right)^9\right\}\\ &\times\left[\exp\left\{-\frac{\omega_0 T_S(1-jC)}{aC}\right\}\right]^m\end{aligned} \tag{15}$$

δ_m and λ_m are coefficients related to ω_0, a, C and the sampling period T_S. k_m, p_m, q_m and r_m are constant coefficients. (16) is derived from (12) and (13).

$$W_\eta(z) = a^{-\frac{1}{2}} T_S \left(\sum_{m=1}^{9} \delta_m z^{-m} s(z) \right) - \left(\sum_{m=0}^{10} \lambda_m z^{-m} W_\eta(z) \right) \quad (16)$$

By digitizing (16), the following equation is obtained.

$$W_\eta(k) = a^{-\frac{1}{2}} T_S \left(\sum_{m=1}^{9} \delta_m s(k-m) \right) - \left(\sum_{m=0}^{10} \lambda_m W_\eta(k-m) \right) \quad (17)$$

(17) enables fast and recursive computation for the convolution integral. The sampling time T_s must be selected to the suitable value in obedience to the focused frequency. In practical, T_s is determined to be held from a cycle of the analysis signal to two cycles of it.

E. Estimation of frequency

The instantaneous phase and frequency can be estimated by the real part $W_R(k)$ and the imaginary part $W_I(k)$ of $W_\eta(k)$. The instantaneous phase $\hat{\theta}(k)$ of the signal is obtained by (18).

$$\hat{\theta}(k) = \tan^{-1}\{W_I(k)/W_R(k)\} \quad (18)$$

where $W_R(k) \equiv \mathrm{Re}(W_\eta(k))$, $W_I(k) \equiv \mathrm{Im}(W_\eta(k))$.

The estimated frequency can be calculated by the following equation using the $\hat{\theta}(k)$ and $\hat{\theta}(k-1)$ at the sampling time kT_S and $(k-1)T_S$.

$$\hat{f}(k) = \frac{1}{2\pi} \cdot \frac{\hat{\theta}(k) - \hat{\theta}(k-1)}{T_S} \quad (19)$$

F. Extraction of components

Relationships between the signal $s(t)$ and the wavelet transform $W_\eta(k)$ about amplitude and phase are derived. Here we assume that the signal $s(t)$ is the sinusoidal wave.

$$s(t) = A\sin(\omega t + \alpha) \quad (20)$$

Applying (20) to (5) on conditions of $f = f_0$ and $\tau = 0$, (21) is derived.

$$W_\eta = j\frac{A}{2\sqrt{f_0}} \cdot \frac{63C^3}{2(1-2Cj)^2} \cdot e^{j\alpha} + j\frac{21AC^3}{4\pi\sqrt{f_0}} \cdot e^{j\alpha} = W_1 + W_2 \quad (21)$$

It forms $|W_1| \ll |W_2|$ in (21), hence W_η approximately becomes (22).

$$W_\eta = j\frac{21AC^3}{4\pi\sqrt{f_0}} \cdot e^{j\alpha} \quad (22)$$

(20) expresses the relationships of amplitude and phase between the signal and wavelet transform on voluntary phase α. Therefore, the extracted component of the signal $\hat{s}(k)$ is calculated by (23) and (24).

$$\hat{A} = |W_\eta| \frac{4\pi\sqrt{f_0}}{21C^3} \quad (23)$$

$$\hat{s}(k) = \hat{A}\sin(\hat{\theta}(k) - \frac{\pi}{2}) \quad (24)$$

(24) can be calculated at each sampling time of the wavelet transform calculation. If you need more detail information of the signal during the sampling time, an interpolation is required. The interpolation of the signal is calculated in (25) and (26) by using the estimated values.

$$\hat{\theta}(k + \Delta t) = \hat{\theta}(k) + 2\pi\hat{f} \cdot \Delta t \quad (25)$$

$$\hat{s}(k + \Delta t) = \hat{A}\sin(\hat{\theta}(k + \Delta t) - \frac{\pi}{2}) \quad (26)$$

where Δt ($\Delta t < T_S$) is the interpolation interval.

III. Experimental Results

A. Experimental system

Experiments of the online estimation on each condition are implemented with the proposal system using a DSP (TMS320C6713). The examined signal $s(t)$, which includes fundamental component of 60Hz, 5th order harmonics and 7th order harmonics, is computed by the DSP as the virtual sinusoidal current.

$$s(t) = \sin\omega t + 0.3\sin 5\omega t + 0.1\sin 7\omega t \quad (27)$$

B. Frequency estimation

Figs. 5-8 show the experimental results of the frequency estimation on several conditions. In Fig.5 on C = 2, a very good estimation result can be obtained. Fig.6 and Fig.7 show the results in case that the signal frequency f is different from the filter frequency f_0 on the same condition of Fig.5. Even if the f_0 is not the same as the signal frequency, the frequency can be estimated.

Figs. 8 shows the results of the harmonics frequency (f_0 = 300Hz) estimation on several conditions. On C = 2, a good estimation result can be obtained.

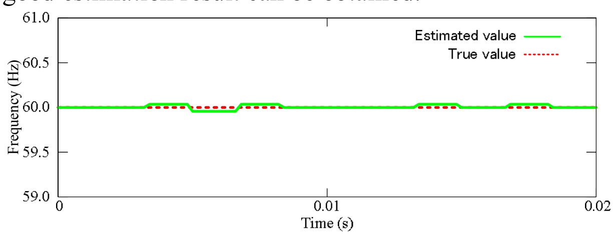

Fig.5 Frequency estimation result at f = 60Hz (T_S = 1ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, C=2)

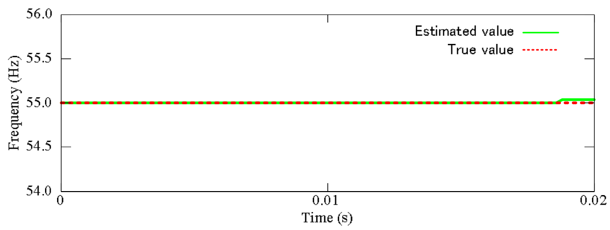

Fig.6 Frequency estimation result at f = 55Hz (T_S = 1ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, C=2)

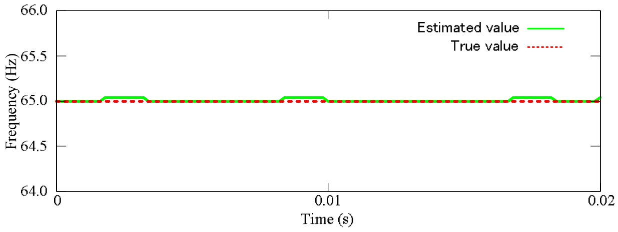

Fig.7 Frequency estimation result at $f = 65$Hz
($T_S = 1$ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

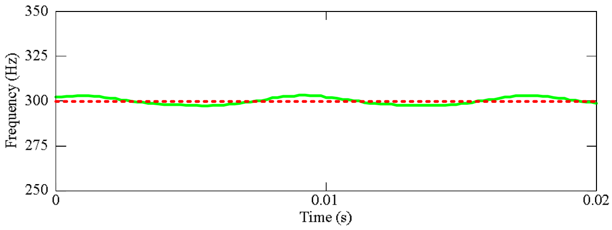

Fig.8 Frequency estimation result at $f = 300$Hz
($T_S = 0.333$ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

C. Extraction of components

Fig. 9 and Fig.10 show the results of the extraction the fundamental (60Hz) component without interpolation. The extracted data of the signal $\hat{s}(k)$ is calculated at every sampling period. Because the sampling time is 1ms, the extracted signal becomes rough waveform.

Figs. 11-14 show the results of the extraction of the fundamental (60Hz) component with interpolation. The sampling time T_S of the wavelet transform is 1ms and the period of the interpolation Δt is 100μs. Fig. 11 shows the original signal $s(t)$, the extracted fundamental component $s_1(t)$ and the harmonics $s_h(t)$. The harmonics component $s_h(t)$ is calculated by the following equation.

$$s_{uh}(t) = s_u(t) - s_{u1}(t) \quad (28)$$

Figs. 12-14 show the estimated values respectively. The estimated phase has the lead of π/2. The phase of the extracted signal coincides with the phase of the original one giving the phase delay as shown in (24) or (26). It is confirmed that each value can be estimated correctly and the component can be obtained accurately.

Secondly, Figs. 15-18 show the results of the extraction of the harmonics (300Hz) component with interpolation. The sampling time T_S of the wavelet transform is 0.6ms and the period of the interpolation Δt is 100μs. It is confirmed that the value can be estimated correctly and the component can be obtained accurately the same as the 60Hz case.

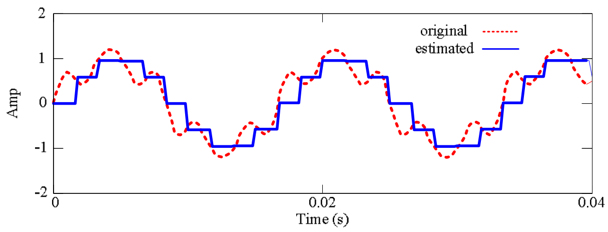

Fig.9 Component extraction result without interpolation
($T_S = 1$ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

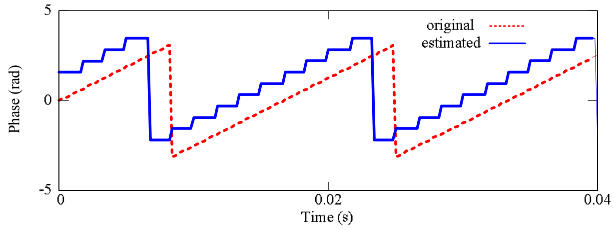

Fig.10 Phase of component extraction without interpolation
($T_S = 1$ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

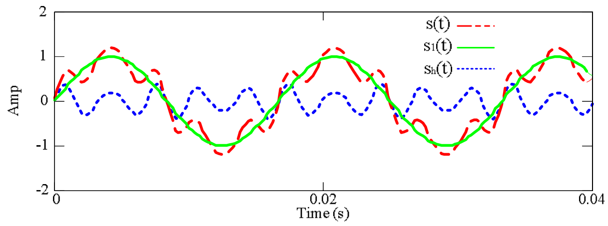

Fig.11 Component extraction result with interpolation
($T_S = 1$ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

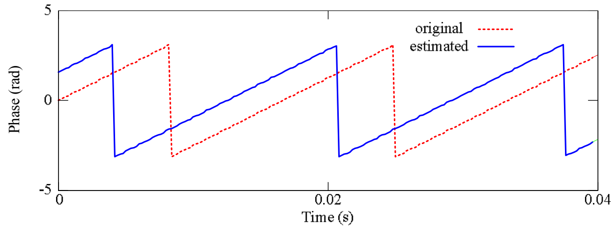

Fig.12 Phase of component extraction with interpolation
($T_S = 1$ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

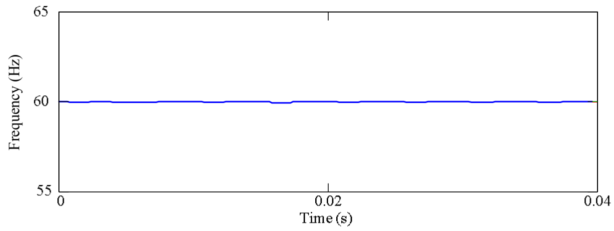

Fig.13 Estimated frequency of component extraction
($T_S = 1$ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

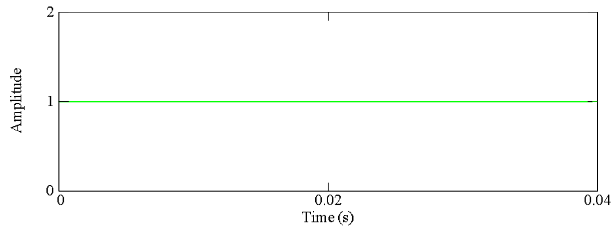

Fig.14 Estimated amplitude of component extraction
($T_S = 1$ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, $C=2$)

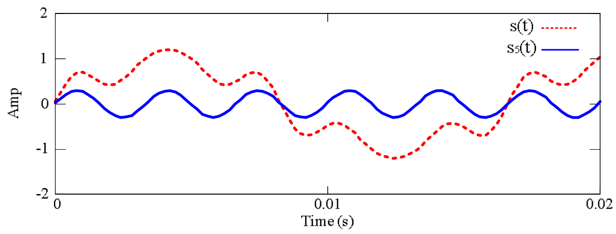

Fig.15 Component extraction result with interpolation
($T_S = 0.6$ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, $C=3$)

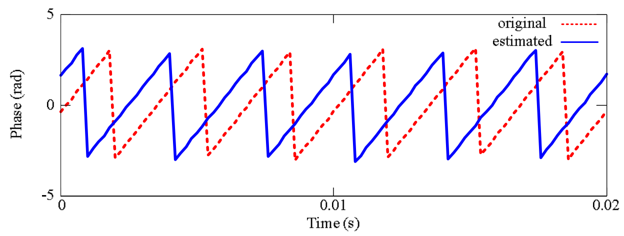

Fig.16 Phase of component extraction with interpolation (T_S = 0.6ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, C=3)

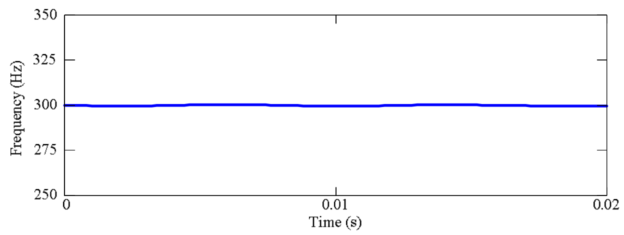

Fig.17 Estimated frequency of component extraction (T_S = 0.6ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, C=3)

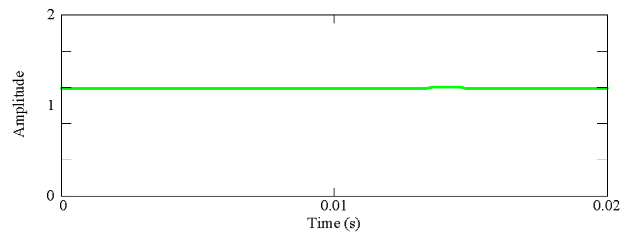

Fig.18 Estimated Amplitude of component extraction (T_S = 0.6ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, C=3)

IV. Application of the Proposed Method

A. Frequency estimation of the IM input current

In order to demonstrate the validity of the proposed method, the current supplied to the Induction Motor (IM) from the inverter is detected and its frequency is estimated by the proposed wavelet transform. The system configuration is shown in Fig. 19. The control of the IM is realized by the vector control. The speed of the IM is controlled by giving the constant speed command value.

The results are shown in Figs.20 and 21. The step change of the rotation speed command value is carried out for 1 second with 1200-1500-1200 rpm. As shown in Figs. 20 and 21, the frequency of the current on step change of the rotor speed command. Although the estimated results on transient have ripples due to the complex current changes, the online frequency estimation is realized in steady state.

B. Extraction of components of the rectifier current

To examine the proposed method for the practical signal, an input current of the three phase diode rectifier is utilized for the signal and the frequency estimation and the component extraction is carried out. Fig. 22 depicts the configuration of the experimental system. The u-phase current i_u is detected as the signal. For FFT analyze, the current i_u in Fig. 23 has the fundamental component (60Hz), 20% of the 5th order and 10% of the 7th order harmonics component.

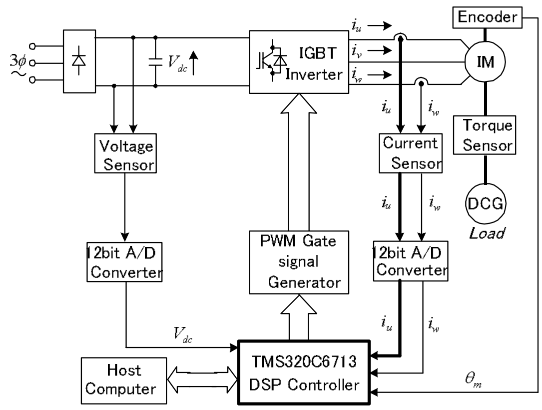

Fig.19 Experimental system with IM

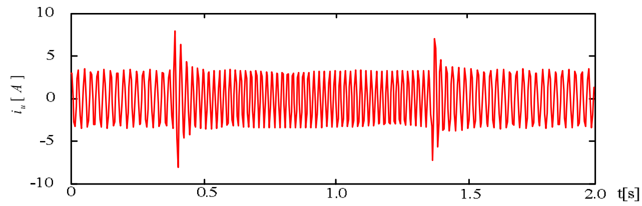

Fig.20 Waveform of the IM input current (analyzed signal)

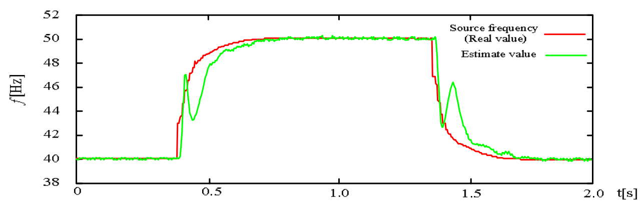

Fig.21 Estimated frequency of rectifier input current (T_S = 2ms, f_0=40Hz, $\omega_0 = \sigma = 2\pi$, C=1)

Figs. 24-27 show the results of the extraction of the fundamental (60Hz) component with interpolation. The waveforms of the input current i_u, the extracted fundamental current i_{u1} and the calculated harmonics current i_{uh} are shown in Fig. 24. The estimated values are shown in Figs. 25-27 respectively. Figs. 28-31 show the results of the extraction of the harmonics (300Hz) component. Though the estimated frequency and amplitude have a little oscillation in the case of f_0 =300Hz, the estimation and the extraction are sufficiently accomplished in both cases.

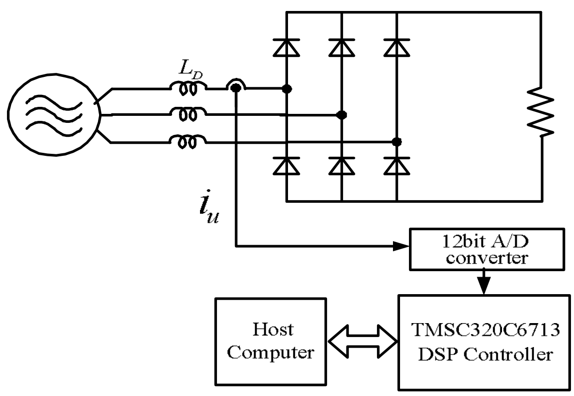

Fig.22 Experimental system with rectifier

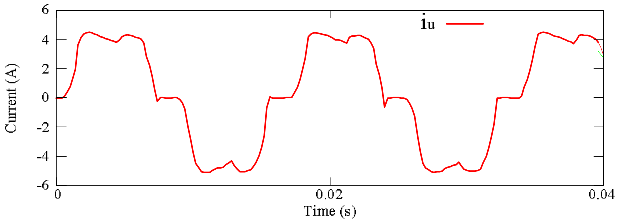

Fig.23 Waveform of the rectifier input current (analyzed signal)

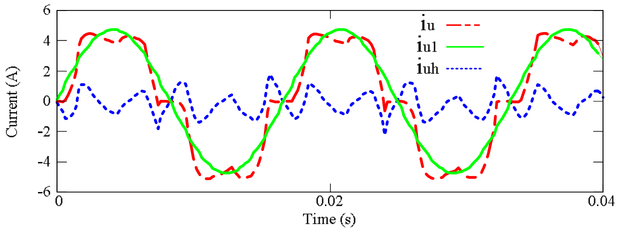

Fig.24 Current component extraction result (with interpolation)
(T_S = 1ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, C=2)

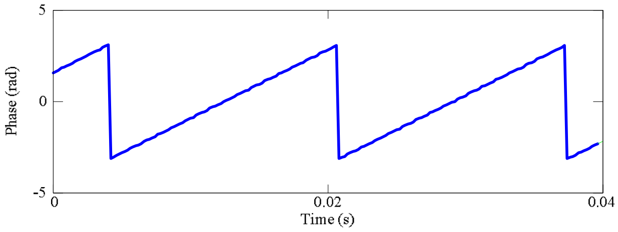

Fig.25 Phase of component extraction (with interpolation)
(T_S = 1ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, C=2)

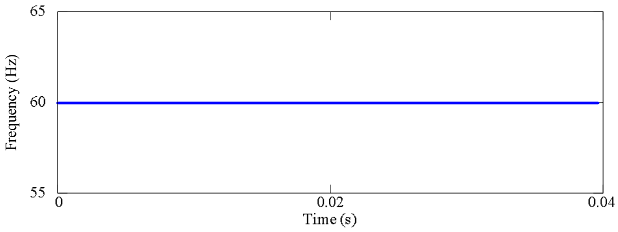

Fig.26 Estimated current frequency of component extraction
(T_S = 1ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, C=2)

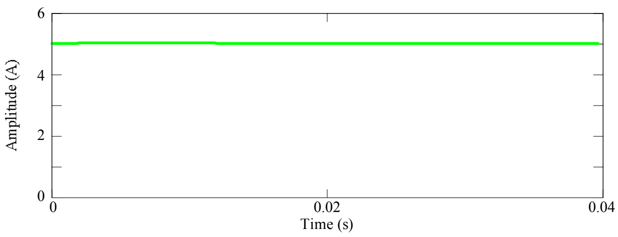

Fig.27 Estimated current amplitude of component extraction
(T_S = 1ms, f_0=60Hz, $\omega_0 = \sigma = 2\pi$, C=2)

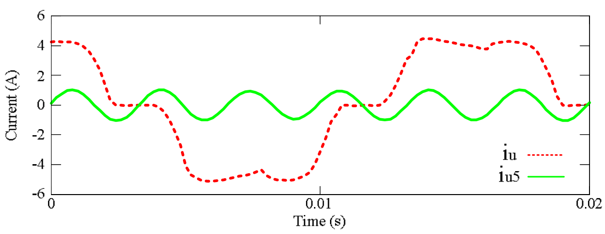

Fig.28 Current component extraction result (with interpolation)
(T_S = 0.333ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, C=3)

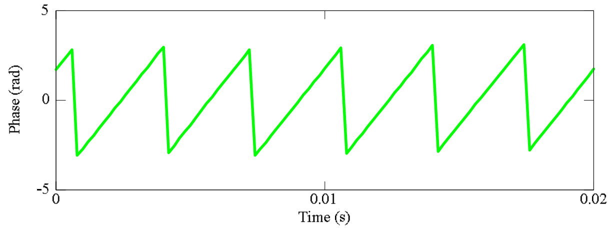

Fig.29 Phase of component extraction (with interpolation)
(T_S = 0.333ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, C=3)

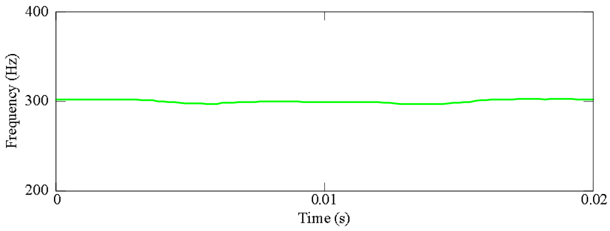

Fig.30 Estimated current frequency of component extraction
(T_S = 0.333ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, C=3)

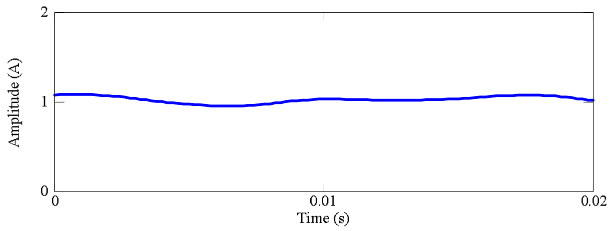

Fig.31 Estimated current amplitude of component extraction
(T_S = 0.333ms, f_0=300Hz, $\omega_0 = \sigma = 2\pi$, C=3)

V. CONCLUSIONS

The online frequency estimation using the proposed wavelet transform are introduced. The proposed wavelet function consists of the power series and can be transformed to the discrete time system by the z-transform, which can shorten the calculation time greatly compared with the general convolution integral.

The fundamental (60Hz) signal and the harmonics signal (300Hz) frequencies can be quickly estimated and the components of the signal can be extracted by the proposed method in the experiment using the DSP system. The application for the power system (the IM and the rectifier) is achieved by the experiment.

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