Comparison of Numerical Solutions of Hollow Cylindrical Dipole Antennas

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1. Introduction

Authors have calculated the center-fed hollow cylindrical dipole antenna and the center-fed solid dipole antenna by using Pocklingotn's integral equation and reported the assumption of current distribution near antenna end affects the numerical solution [1].   Pocklington's integral equation is derived from the boundary condition that the tangential component of electric field vanishes on the antenna axis or its side surface.   The piecewise sinusoidal function is used as the current expansion function on the side surface and the weighting function.   If the current distribution is expanded so as to satisfy the edge condition at the antenna end, the numerical results rapidly converges for the case of antenna length is from 0.3 to 1.5 λ (λ: wavelength) and the its radius is equal or less than 0.005 λ.   The numerical result obtained by Pocklington's integral equation applied on the antenna axis agrees well with that applied on the antenna surface.

In this paper, the current distributions of center-fed hollow cylindrical dipole antennas are calculated by using WIPL-D [2] and AWAS [3].   In AWAS (Analysis of wire antennas and scatterers) produced by A.R.Djordjevic et al., the current distribution is assumed on the antenna surface and Pocklington's integral equation for the unknown current is derived from the boundary condition that the tangential component of electric field vanishes on the antenna axis.   The polynomial is used as the current expansion function.   The weighting function is the unit pulse function.   The relative error of feed point current and the root mean square error of currents calculated by both simulators are compared [4]-[7].   As the criterion for accuracy of calculated results, the numerical solutions of Pocklington's integral equation on the antenna surface with the surface current expansion functions satisfying the edge condition at the antenna ends are adopted [1].   Since the current distribution is determined from the boundary condition of the tangential component of electric field on the antenna axis, the accurate current should produce zero electric field on the antenna axis.   Therefore the tangential components of electric field on the antenna axis due to the calculated currents are calculated and compared.

2. Current expansion function and weighting function

Figure 1 shows the center-fed hollow cylindrical dipole antenna for numerical calculation.   In WIPL-D and AWAS simulators, the antenna is divided into N straight segments.   The current on the segment m is expanded by the polynomials of order n_m ;

$$I_m\left(s\right) = \sum_{i=0}^{n_m} I_{mi}\left(\frac{s}{h_m}\right)^i, \quad 0 \le s \le h_m: \qquad m = 1, 2, \ldots, N \tag{1}$$

where s is the local coordinate along the segment. h_m is the length of the segment m. $\left\{I_{mi}\right\}$ are the unknown coefficients to be determined by Pocklington's integral equation.

As the criterion for accuracy of calculated data, the numerical solutions of Pocklington's integral equation derived from the boundary condition of electric field on the antenna surface are adopted.   As the current expansion functions, the sum of piecewise sinusoidal expansion functions and the functions satisfying the edge

condition at antenna ends are used, that is [1];

$$I_s(z) = \begin{cases} I_2^s (z/\Delta)^{1/2}, & 0 \le z \le \Delta \\ \sum_{i=2}^{M} I_i^s f_i(z), & \Delta \le z \le L - \Delta \\ I_M^s \{(L-z)/\Delta\}^{1/2}, & L - \Delta \le z \le L \end{cases} \tag{2}$$

$$\Delta = L/M \tag{3}$$

$$f_i(z) = \frac{\sin k(\Delta - |z - z_i|)}{\sin k\Delta}, \quad \begin{cases} z_2 \le z \le z_3 & for \ i = 2 \\ z_{i-1} \le z \le z_{i+1} & for \ 3 \le i \le M - 1 \\ z_{M-1} \le z \le z_M & for \ i = M \end{cases} \tag{4}$$

$$z_i = \Delta(i-1) = L(i-1)/M \tag{5}$$

The piecewise sinusoidal function $f_i(z)$ is also used as the weighting function. The antenna is excited by the magnetic frill current at the feed point.　The ratio of outer and inner radii of magnetic frill is 2.984.

3. Numerical results and discussion

The relative error of feed point current $I(L/2)$ is defined by

$$\mathrm{Re}\,lative \ error = \frac{I(L/2) - I_s(L/2)}{I_s(L/2)} \tag{6}$$

The root mean square error of current $I(z)$ is defined as follows:

$$Root \ mean \ square \ error = \frac{\left\{\int_0^L |I(z) - I_s(z)|^2 \, dz\right\}^{1/2}}{\left\{\int_0^L |I_s(z)|^2 \, dz\right\}^{1/2}}, \tag{7}$$

where $I_s(z)$ denotes the current distribution given by the reference solution.　The feed point current of reference solution $I_s(L/2)$ converges, as M becomes more than 50.　Therefore, the number of segments of reference data are set to $M=50$ for $L \le 0.3\lambda$　and $M=100$ for $L \ge 0.4\lambda$.

Figure 2 shows the comparison between the relative error of the feed point currents by WIPL-D and AWAS. Figure 3 shows the root mean square error of current calculated by using WIPL-D and AWAS.　In these figures, the length of dipole L is from 0.1λ to 1.0λ and its radius a is from 0.0002λ to 0.01λ (λ denotes wavelength).　A dipole is divided into two segments ($N=2$).　The order of current expansion polynomial is $n = n_1 = n_2 = 4$ or 6. When $a \le 0.002\lambda$, $0.3\lambda \le L \le 1.0\lambda$ and $n=6$, the relative error of both simulators is less than 10%.　When $a \le 0.005\lambda$, $0.3\lambda \le L \le 1.0\lambda$ and $n=6$, the root mean square error is less than 5% for WIPL-D and less than 10% for AWAS.　As the length of dipole becomes shorter, the root mean square error becomes serious for thicker dipole antenna.

Figure 4 show the examples of calculated current distribution.　The length of dipole is $L = 0.4\lambda$ and its radius is $a = 0.01\lambda$.　The order of current expansion polynomial is $n = n_1 = n_2 = 4$.　Figure 5 show the tangential component of electric fields on the antenna axis calculated by the current distributions in Figure 4.

The tangential component of electric field obtained from the current distribution by both simulators are almost same except in the vicinity of feed point.

4. Conclusion

The current distribution on the center-fed hollow cylindrical dipole antenna has been calculated by using WIPL-D and AWAS. The relative error of feed point current and the root mean square error of current are compared. As the criterion for comparison, Pocklington's integral equation for the surface current formulated from the boundary condition on the antenna surface is numerically solved. The current of this reference solution is expanded by the piecewise sinusoidal function and the function satisfying the edge condition at antenna ends.

The root mean square errors of calculated current by WIPL-D and AWAS simulators are less than 10% when the length of dipole is from 0.3λ to 1.0λ, its radius is equal or less than 0.005λ and the order of polynomial is 6. Since the error may become considerably larger when the radius of dipole becomes larger than 0.005λ, these simulators may not apply thicker dipole antenna. As the length of dipole becomes shorter, the error becomes serious.

References
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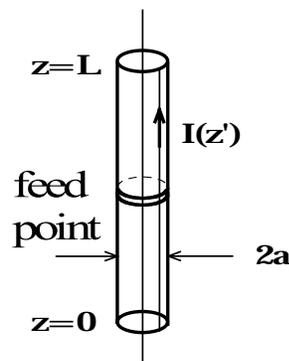
Figure 1 Hollow cylindrical dipole antenna.
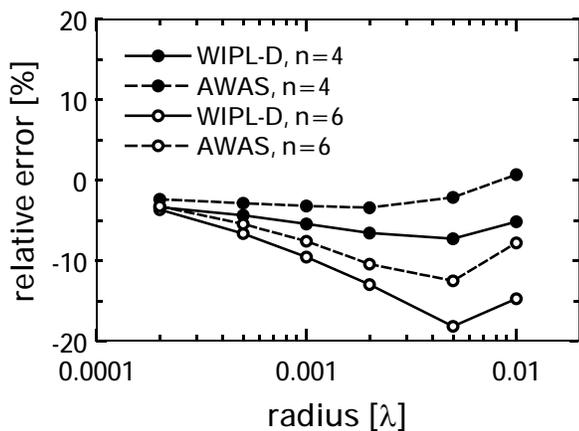
(a) Real part $L = 0.4\lambda$
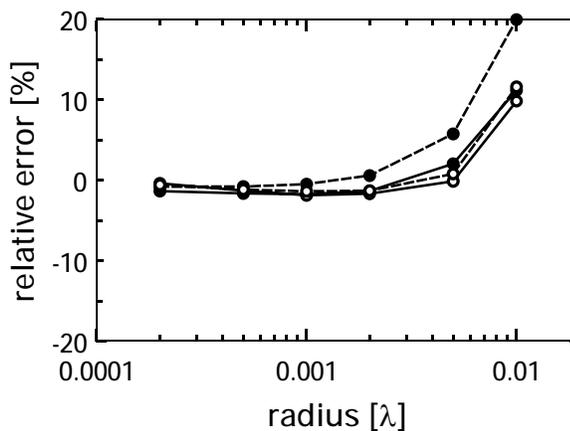
(b) Imaginary part $L = 0.4\lambda$
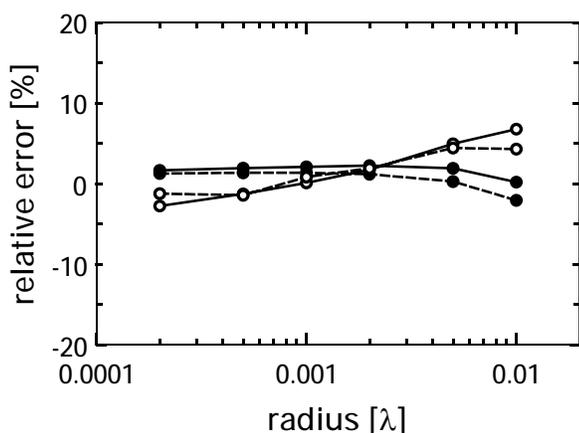
(c) Real part $L = 0.6\lambda$
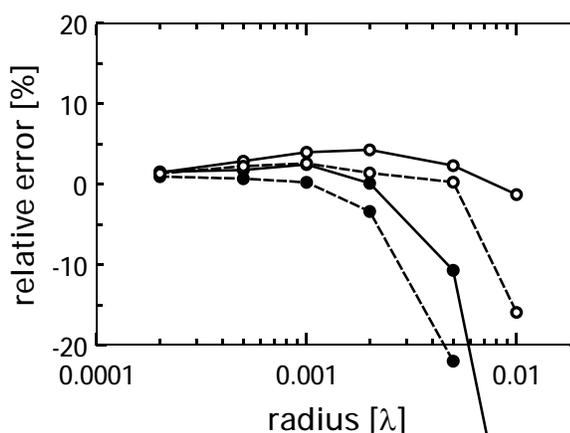
(d) Imaginary part $L = 0.6\lambda$
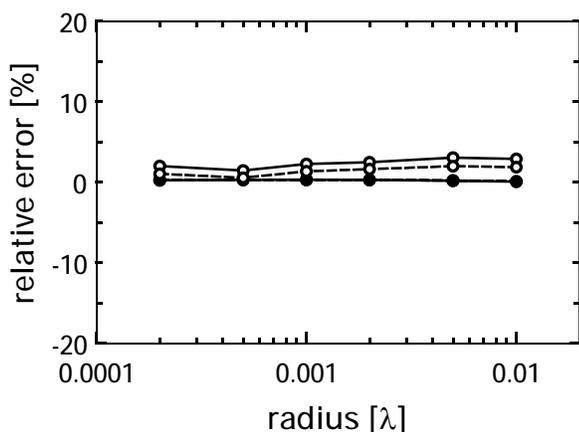
(e) Real part $L = 1.0\lambda$
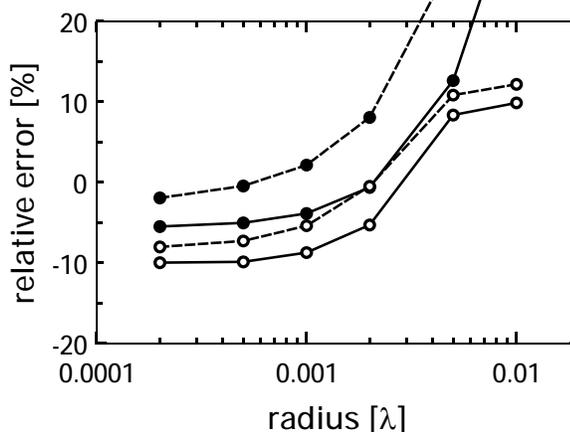
(f) Imaginary part $L = 1.0\lambda$

Figure 2   Comparison between relative error of feed point currents by WIPL-D and AWAS.
Order of polynomial $N = 2,\ n = n_1 = n_2$
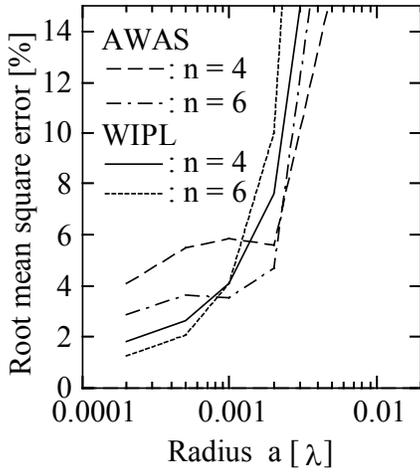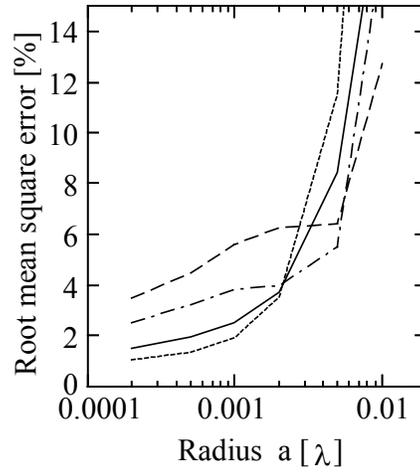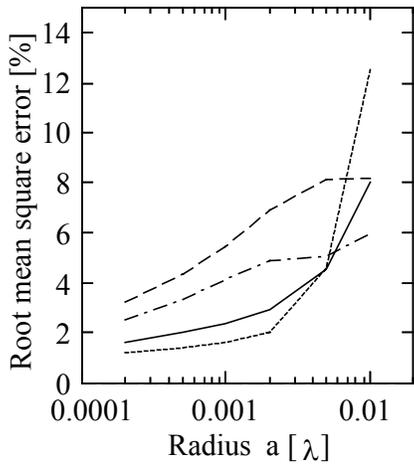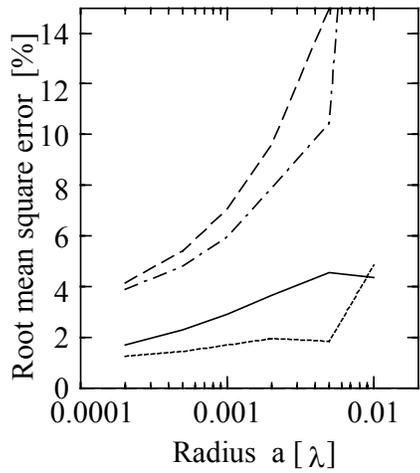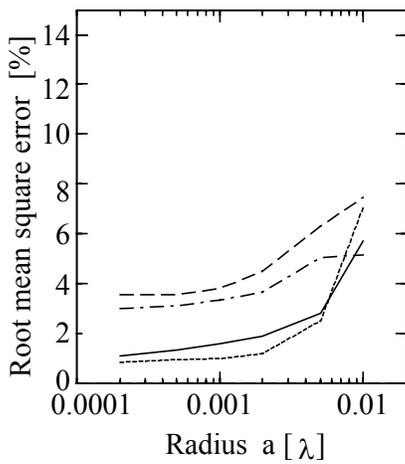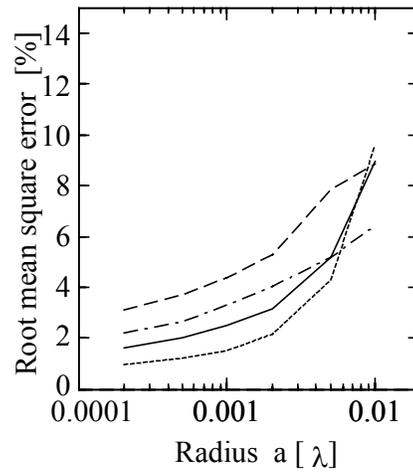
Figure 3   Root mean square error of current calculated by using WIPL-D and AWAS.
Order of polynomial $N = 2, \ n = n_1 = n_2$
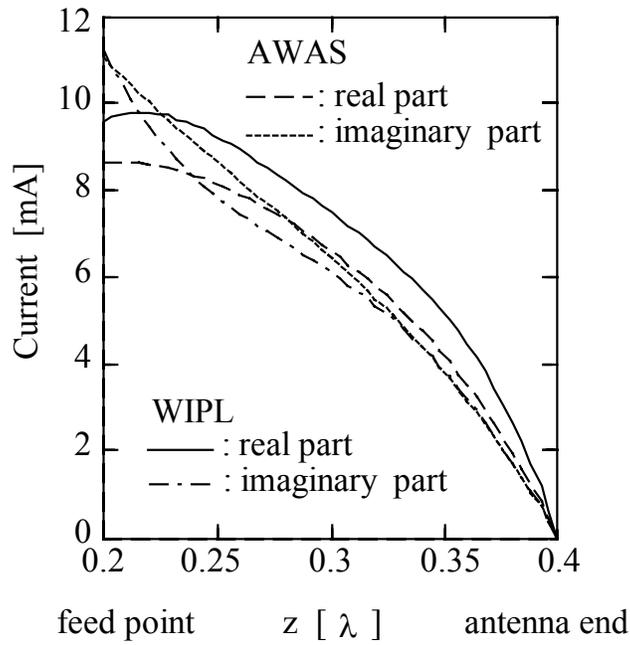
Figure 4   Current distributions calculated by WIPL-D and AWAS.
$L = 0.4\lambda, \ a = 0.01\lambda, \ N = 2, \ n = n_1 = n_2 = 4$
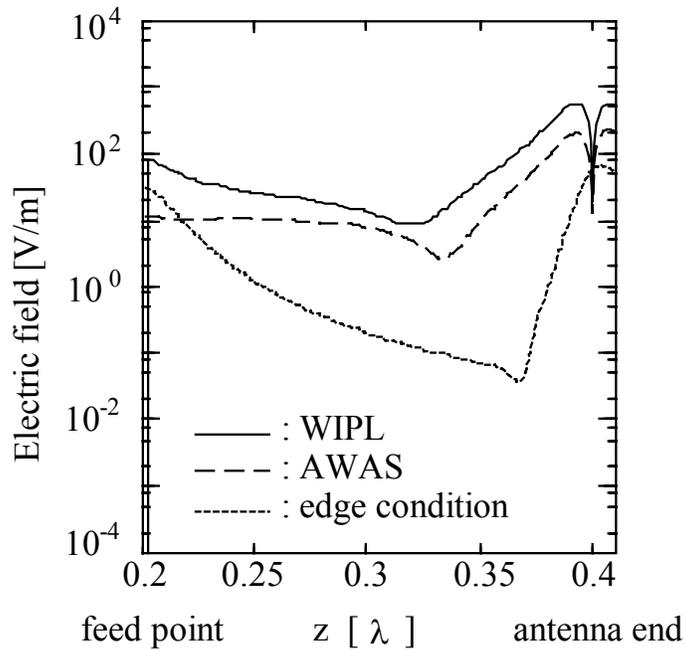
Figure 5   Tangential component of electric field on antenna axis (absolute value).
$L = 0.4\lambda, \ a = 0.01\lambda, \ N = 2, \ n = n_1 = n_2 = 4$