# Analysis of Rotationally Symmetrical Antenna with Wider Beamwidth 

Daisuke YAGYU*. Kazumasa TANAKA, Takafumi FUJIMOT() and Mitsuo TAGUCHI<br>Faculty of Engineering, Nagasaki University<br>1-14 Bunkyo-machi, Nagasaki-shi, 852-8131, Japan

## 1.Introduction

The microstrip antema(MSA) is widely used for small-sized terminal of mobile communication system, as it is thin and light-weighted and it has larger gain as compared with wirc antemas. In some applications such as Global positioning system or Iridium satellite communication system which use a group of low-altitude orbiting satellites, the wider bermwidth is desired for the antemm mounted on a terminal. In order to widen beamwidth of MSA, the conformal MSA on a cylindrical conductor[1] or a spherical body[2] were proposed and analyzed. The authors has proposed an oblate spheroidal antenna(OSA) in order to widen its beamwidth and has shown the analytical method[3]. In this paper, this antemna is numerically and experimentally analy\%ed. OSA consists a lincarly polari\%ed circular patch MSA and upper and lower oblate spheroidal conductors. The substrate of MSA is truncated at the edge of patch. In the numerical analysis, the electromagnetic fields within the antenna cavity are expanded by the modal functions and their unknown expansion coefficients are determined by the electric and magnetic field integral equations on the antenna surface.

## 2.Formulation

Fig. 1 shows the analytical model and its coordinate system. The cylindrical coordinate system $(\rho, \phi, z)$ is used within the MSA, and the oblate spheroidal coordinate system ( $u, v, \phi$ ) on the outer surface of conductor. The radius of patch is $a$ and the height of conductor is $c$. The cross section of oblate spheroidal conductor is the same as the patch. The antenna is excited between the upper and lower conductors at the feed point $\rho=b, \phi=0$. The thickness and the dielectric constant of dielectric substratc are $D$ and $\epsilon_{1}=\epsilon_{r} \epsilon_{0}$, respectively. $\epsilon_{0}$ and $\mu_{0}$ are the dielectric constant and the permeability in free space, respectively. $S_{a}$ and $S_{c}$ denote the aperture of substrate and the conductor surface, respectively.

The thickness of substrate is assumed to be much smaller than the wavelength. Then the electromagnetic fields within the substrate do not vary with the perpendicular direction to the patch, and can be obtained by the cavity model. By applying the equivalence principle to this antemna, the auxiliary problems in the internal and external regions of antema are obtained[4]. Within the substrate, the electromagnetic fields are expanded by the modal functions. The cquivalent electric and magnetic currents $J_{a}$ and $M$ on $S_{a}$ are equal to the tangential component of magnetic and electric fields within the substrate, respectively;

$$
\begin{equation*}
J_{a}(a, \phi)=-\frac{j k_{1}}{\omega \mu_{0}} \sum_{n=0}^{N}\left\{B_{n} J_{n}^{\prime}\left(k_{1} a\right)+\frac{j \omega \mu_{0} I_{0}}{2\left(1+\delta_{n}\right)} J_{n}\left(k_{1} b\right) N_{n}^{\prime}\left(k_{1} a\right)\right\} \cos (n \phi) \boldsymbol{i}_{z} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
M(a, \phi)=\sum_{n=0}^{N}\left\{B_{n} J_{n}\left(k_{i} a\right)+\frac{j \omega \mu_{0} I_{0}}{2\left(1+\delta_{n}\right)} J_{n}\left(k_{1} b\right) N_{n}\left(k_{1} a\right)\right\} \cos (n, \phi) \boldsymbol{i}_{\dot{\phi}} \tag{2}
\end{equation*}
$$

$J_{n}$ and $N_{n}$ are Bessel and Neumann functions of order $n$ and the prime denotes the derivative with respeect to the argument. $I_{0}$ is the total current at the feed point. $\delta_{n}$ is defined as $\delta_{n}=1$ for $n=0$ and $\delta_{n}=0$ for $n \neq 0$.

The electric current $J_{c}$ on $S_{c}$ is assumed as follows:
$\boldsymbol{J}_{v}(u, v, \phi)=\left\{\begin{array}{r}\sum_{m=0}^{M} \sum_{n=0}^{N} A_{m r n} \cos (m v) \cos (n \phi) \boldsymbol{i}_{v}+\sum_{m=0}^{M} \sum_{n=1}^{N} C_{m n n} \cos (m v) \sin (n \phi) \boldsymbol{i}_{\phi} \\ \sum_{m=0}^{M} \sum_{n=0}^{N} A_{m n} \cos (m v) \cos (n \phi) \boldsymbol{i}_{v}-\sum_{m=0}^{M} \sum_{n=1}^{N} C_{m n} \cos (m v) \sin (n \phi) \boldsymbol{i}_{\dot{\phi}} \\ ; v \leq 0\end{array}\right.$
where, $\left\{B_{n}\right\},\left\{A_{m n}\right\},\left\{C_{m n}\right\}$ are unknown cocfficients to be detcrmined from the integral equations on the antema surface.

In the external region of antenna, the electromagnetic fields are expressed in terms of the equivalent clectric and magnetic currents $J_{a}$ and $M$ on $S_{a}$ and the electric current $J_{c}$ on $S_{c}$.

The magnetic field integral equation is formulated from the continuity condition on the tangential component of magnetic field on $S_{n}$;

$$
\begin{align*}
\boldsymbol{i}_{\tilde{n}} \cdot & {\left[\frac{1}{2 \pi} n \times f_{S_{a}}\left\{-j \omega \epsilon_{0} M \psi+J_{a} \times \nabla^{\prime} \psi-\frac{1}{j \omega \mu_{0}}\left(\nabla^{(2)^{\prime}} \cdot M\right) \nabla^{\prime} \psi\right\} d S^{\prime}\right.} \\
& \left.+\frac{1}{2 \pi} \boldsymbol{n} \times \int_{S_{c}}\left(J_{a} \times \nabla^{\prime}, \psi\right) d S^{\prime}\right] \\
= & -\frac{j k_{i}}{\omega \mu \mu_{0}} \sum_{n=0}^{N}\left\{B_{n} J_{n n}^{\prime}\left(k_{1} \rho\right)+\frac{j \omega \mu_{0} I_{0}}{2\left(1+\delta_{n}\right)} J_{n}\left(k_{1} b\right) N_{n}^{\prime}\left(k_{1} \rho\right)\right\} \cos (n, \phi) \tag{4}
\end{align*}
$$

where, $\nabla^{(2)^{\prime}}$. denotes the surface divergence. $\boldsymbol{n}$ is the unit vector normal to the antenna surface. $\psi$ is Green`s function. The electric field integral equation is derived from the boundary condition that the tangential component of electric field vanishes on $S_{c}$;

$$
\begin{align*}
\frac{1}{2 \pi} \boldsymbol{n} \times & {\left[\int_{S_{a}}\left\{-j \omega \mu_{0} J_{a} \psi-M \times \nabla^{\prime} \psi-\frac{1}{j \omega \epsilon_{0}}\left(\nabla^{(2)^{\prime}} \cdot J_{a}\right) \nabla^{\prime} \psi\right\} d S^{\prime}\right.} \\
& \left.+f_{S_{c}}\left\{-j \omega \mu_{0} J_{c} \psi-\frac{1}{j \omega \epsilon_{0}}\left(\nabla^{(2)^{\prime}} \cdot J_{c}\right) \nabla^{\prime} \psi\right\} d S^{\prime}\right]=0 \tag{5}
\end{align*}
$$

The equivalent electric and magnetic currents on $S_{a}$ and the electric current on $S_{c}$ arc calculated by applying Galcrkin's method to these integral ecpuations. The input, impedance is defined as a half of the voltage at feed point divided by the total current according to the definition of input impedance in the measurement.

## 3.Results and Discussion

Fig. 2 shows the calculated and measured input impedances of OSA. The radius of patch $a$ is 15 mm , and the feed point is located at $b=10.0 \mathrm{~mm}$. The thickness $D$ and
the relative dielectric constant $\epsilon_{r}$ of substrate is 1.528 mm and 2.15 , respectively. In the mumerical calculation, the ratio of height to radius of oblate spheroidal conductor is chosen as c:/a=0.05 and 0.95 . The number of expansion mode is chosen as $M=N=3$. In the measurement, the circular patch MSA with the substrate truncated at the edge of patch is used. This antenna is mounted on the ground plane of $14 \mathrm{~cm} \times 14 \mathrm{~cm}$ and coaxially fed from this plane. Relatively good agreement is observed for between the calculated ( $c / a=0.05$ ) and the measured $(c / a=0)$ results.

Fig. 3 shows the calculated radiation pattcrns of OSA in E- and H-plancs. In E-plane, the beamwidth becomes wider as the height of oblate spheroidal conductor becomes higher. The vertical component of current on $S_{0}$ becomes larger as the height of oblate spheroidal conductor becomes highor. Therefore, the intensity of radiation field at the low angle of elevation in E-plane becomes larger. In H-plane, however, little difference is found on radiation patterns with $c / a=0.05$ and 0.95 .

## 4. Conclusion

The circular patch MSA inserted in the center of oblate spheroidal conductor has been numerically and experimentally analyzed. The electromagnetic fields within the MSA are expressed as the summation of modal functions in the cylindrical coordinate system. Their unknown cocfficients and the equivalent clectric and magnetic currents on antenna surface are determined by the magnetic field integral equation on the aperture of substrate aud the electric field integral equation on the conductor surface. This antemma has wider beamwidth in E-plane as compared with circular patch MSA. Also, the bandwidth becomes wider and the resonant frequency becomes lower as the height of oblate spheroidal conductor becomes higher.

## References

[1] K.-M.Luk et al.: "Analysis of the cylindrical-rectangular patch antenna": IEEE Trans., AP-37: 2, pp.143-147, Feb. 1989
[2] K.-M.Luk et al.: "Patch antemnas on sphorical body", Proc. IEE, 138, 1, pp.103-108, Feb. 1991
[3] D.Yagyu et al.:"Analysis of microstrip antenna with wider beamwidth", Proc. ICARSM, Voronczh, Russia, vol. 1, pp.6269, May 1997
[4] C.A.Balanis:"Antenna theory: $\Lambda$ nalysis and design:; pp.447-454, John Wiley \& Sons; 1982


Fig. 1: Analytical model


Fig. 2: Input Impedances
$a=15 \mathrm{~mm}, b=10.0 \mathrm{~mm}, D=1.528 \mathrm{~mm}, \epsilon_{r}=2.15$


Fig. 3: Radiation patterns

