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Wall Friction and Local Heat Transfer in Oscillatory Flow (Two-dimensional Unsteady Laminar Boundary Layer on a Flat Plate)*

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Wall friction and local heat transfer in an oscillating laminar boundary layer over a flat plate with an unheated starting length are theoretically analyzed. In the analysis, the power series solutions are derived for velocity and temprature distributions in three frequency regions, that is, low frequency ($\omega^*<1$), intermediate frequency ($1<\omega^*<1/K$), and high frequency ($\omega^*>1/K$) respectively. As the result, the frequency response of a hot film sensor is clarified theoretically. Especially it is shown that the amplitude of heat transfer oscillation with an unheated starting length increases with frequency at low and intermediate regions, and the phase angle is in advance of free stream oscillation at these frequency regions.

1. Introduction

It is important to measure the skin friction on body surfaces in a stream, for investigating the flow mechanism and estimating the friction loss. Recently the fluctuating skin friction measurement is required to inquire into the mechanism of an unsteady flow, and it seems that the hot film method is the most suitable one to measure a fluctuating skin friction of all existing techniques. In this method the frequency response of the probe must be known, but the hot film method has been established by Brown(1) and Bellhouse etc.(2) only for measurement of time-mean wall skin friction. Lighthill(3) studied the effect of a fluctuating oncoming stream on the skin friction and heat transfer on a two-dimensional body; and Mori and Tokuda (4) made a theoretical and experimental study of heat transfer from an oscillating cylinder. But these studies are not enough, because the hot film probe for skin friction measurement has a flat heated surface and is required to be located at any portion of body surfaces, and then the leading edge of hot film is different from that of wall surface. Therefore to clarify the frequency response of hot film, the relation between wall friction and heat transfer with an unheated starting length should be examined.

In this paper, the fluctuating velocity distribution and the fluctuating temperature distribution in an oscillating incompressible laminar boundary layer over a flat plate with an unheated starting length are theoretically analyzed. In the analysis, introducing new non-dimensional variables including both the unheated starting length and the fluctuating frequency, the equation of motion and the energy equation are solved by the power series solution method in the cases of low, intermediate and high frequencies respectively. As a result, the amplitude and the phase angle of skin friction fluctuation and those of heat transfer oscillation are clarified.

2. Governing equation

Fig. 1 shows a model of the hot film probe on a flat plate for skin friction measurement. The heated element is located at the distance x_0 from the leading edge of a plate which is immersed in a two-dimensional incompressible laminar flow, and the element length is very short. The sinusoidal oscillation is superimposed on the steady stream U_0. Then the free stream velocity U is expressed as follows:

$$U = U_0(1 + \varepsilon e^{i\omega t}) \qquad (1)$$

where it is assumed that U_0 is constant and ε is small compared with unity.

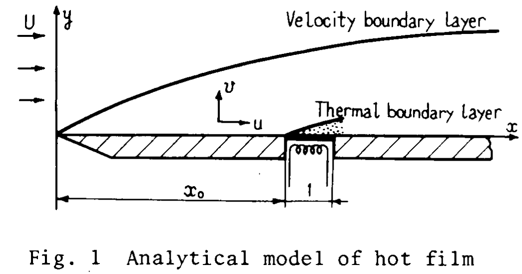

Fig. 1 Analytical model of hot film probe and co-ordinate system

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In the boundary layer along the flat plate, the equation of continuity, the equation of motion for streamwise velocity, and the energy equation are as follows:

$$\partial u/\partial x + \partial v/\partial y = 0 \qquad (2a)$$

$$\partial u/\partial t+u(\partial u/\partial x)+v(\partial u/\partial y)=\partial U/\partial t+\nu(\partial^2 u/\partial y^2) \qquad (2b)$$

$$\partial T/\partial t + u(\partial T/\partial x)+v(\partial T/\partial y)=\kappa(\partial^2 T/\partial y^2) \qquad (2c)$$

$$u=v=0,\ T=T_w(x_0\leq x\leq x_0+1),\ T=T_\infty(x<x_0, x>x_0+1)$$

$$\text{at } y=0$$

$$u=U,\ T=T_\infty \text{ as } y\to\infty$$

where T_w is the heated element temperature and T_∞ the free stream temperature. If ε is small compared with unity, u, v and T may be expressed approximately as follows:

$$\left.\begin{aligned} u(x,y,t)&=u_0(x,y)+\varepsilon e^{i\omega t}u_1(x,y,\omega)\\ v(x,y,t)&=v_0(x,y)+\varepsilon e^{i\omega t}v_1(x,y,\omega)\\ T(x,y,t)&=T_0(x,y)+\varepsilon e^{i\omega t}T_1(x,y,\omega)\end{aligned}\right\} \quad (3)$$

Substituting Eq.(3) into Eq.(2), and equating the same order of ε, sets of equations are obtained. The zeroth-order equations are;

$$\partial u_0/\partial x + \partial v_0/\partial y = 0 \qquad (4a)$$

$$u_0(\partial u_0/\partial x)+v_0(\partial u_0/\partial y)=\nu(\partial^2 u_0/\partial y^2) \qquad (4b)$$

$$u_0(\partial T_0/\partial x)+v_0(\partial T_0/\partial y)=\kappa(\partial^2 T_0/\partial y^2) \qquad (4c)$$

$$u_0=v_0=0,\ T_0=T_w(x_0\leq x\leq x_0+1),\ T_0=T_\infty(x<x_0,$$

$$x>x_0+1) \text{ at } y=0$$

$$u_0=U_0,\ T_0=T_\infty \text{ as } y\to\infty$$

The first order equations are;

$$\partial u_1/\partial x + \partial v_1/\partial y = 0 \qquad (5a)$$

$$i\omega u_1+u_0(\partial u_1/\partial x)+u_1(\partial u_0/\partial x)+v_0(\partial u_1/\partial y)$$

$$+v_1(\partial u_0/\partial y)=i\omega U_0+\nu(\partial^2 u_1/\partial y^2) \qquad (5b)$$

$$i\omega T_1+u_0(\partial T_1/\partial x)+u_1(\partial T_0/\partial x)+v_0(\partial T_1/\partial y)$$

$$+v_1(\partial T_0/\partial y)=\kappa(\partial^2 T_1/\partial y^2) \qquad (5c)$$

$$u_1=v_1=0,\ T_1=0 \text{ at } y=0$$

$$u_1=U_0,\ T_1=0 \text{ as } y\to\infty$$

To solve Eqs.(4) and (5), the stream function ψ and the non-dimensional temperature θ are used in the following form.

$$\psi(x,y,t)=\psi_0(x,y)+\varepsilon e^{i\omega t}\psi_1(x,y,\omega) \qquad (6a)$$

$$\theta(x,y,t)=\theta_0(x,y)+\varepsilon e^{i\omega t}\,\Xi\,(x,y,\omega) \qquad (6b)$$

ψ_0 and θ_0 are given as the well known steady state solutions.

$$\psi_0 = \sqrt{\nu x U_0}\ f(\eta) \qquad (7)$$

$$\theta_0 = 1-(1/\Gamma(4/3))\int_0^{\eta_T} e^{-\eta_T^3}\, d\eta_T \qquad (8)$$

On the other hand, the equations with respect to ψ_1 and Ξ are derived from Eqs. (5) and (6) in the following sections.

3. Analysis of velocity distribution

3.1 Case of low frequency ($\omega^*<1$)

The unsteady part of the stream function is similar to Eq.(7), that is, $\psi_1=\sqrt{\nu x U_0}\ \Phi(\xi,\eta)$, and if the non-dimensional frequency parameter ω^* is smaller than unity, Φ may be written in the following power series,

$$\Phi(\xi,\eta) = \sum_{n=0}^{\infty} \xi^n \Phi_n(\eta) \qquad (9)$$

Substituting Eq.(9) into (5), and equating the same order of ξ, the following equation is obtained.

$$2\Phi_n'''+f\Phi_n''-2nf'\Phi_n'+(2n+1)f''\Phi_n=2\Phi_{n-1}'+A_n \qquad (10)$$

where $\Phi_{-1}=0,\ A_1=-2,\ A_n=0(n=0,2,3,\ldots.)$

$$\Phi_n=\Phi_n'=0(n=0,1,2,3,\ldots.) \text{ at } \eta=0$$

$$\Phi_0'=1,\ \Phi_n'=0(n=1,2,3,\ldots.) \text{ as } \eta\to\infty$$

The solutions of Eq.(10) have been already obtained for the lower power terms in the literatures(5,6), but in this paper the power series solution is obtained up to the 7th power so as to examine the truncational error, as listed in Table 1. For comparison the value obtained by Nakagawa etc.(7) is also listed in Table 1. Both values are almost the same.

Table 1. The values of $\Phi_n''(0)$ at low frequency

n	Present theory	Nakagawa (7)
0	0.4980	0.4981
1	0.8480	0.8485
2	-0.4785	-0.4697
3	0.3765	0.3677
4	-0.2785	-0.2695
5	0.1845	0.1769
6	-0.1100	-0.1045
7	0.0595	——

3.2 Case of high frequency ($\omega^*>1$)

If the frequency paramter is larger than unity, the unsteady component of the stream function is defined as follows as indicated by Illingworth(6).

$$\psi_1=U_0\sqrt{\nu/i\omega}\,\Phi(\alpha,\beta); \qquad (11)$$

$$\alpha=1/\sqrt{i\omega x/U_0}\ ,\ \beta=y\sqrt{i\omega/y}$$

Substituting Eq.(11) into Eq.(5b), the following equation is obtained.

$$2\Phi_{\beta\beta\beta}+(\alpha f-\alpha^2\beta f')\Phi_{\beta\beta}+(\alpha^3\beta f''-2)\Phi_\beta +\alpha^3 f'\Phi_{\alpha\beta}-\alpha^4 f''\Phi_\alpha+2=0 \quad (12)$$

$$\Phi=\Phi_\beta=0 \text{ at } \beta=0,\ \Phi_\beta=1 \text{ as } \beta\to\infty$$

where the subscripts α and β denote the differentiation with respect to α and β respectively.

As the thermal boundary layer is very thin compared with the velocity boundary layer in the present case, it is assumed that the velocity distribution is linear in the thermal boundary layer. Therefore f' in Eq.(12) is expressed as follows.

$$f'=f''(0)\cdot\eta=f''(0)\cdot\alpha\cdot\beta;\ f''(0)=0.33206$$

Expanding Φ into a power series with respect to α, that is, $\Phi=\sum_{n=0}^{\infty}\alpha^n\Phi_n(\beta)$ and equating the same order of α, the following analytical solutions are obtained.

$$\Phi_0=\beta+e^{-\beta}-1,\ \Phi_1=\Phi_2=\Phi_4=\Phi_5=\Phi_7=\Phi_8=\dots=0$$

$$\Phi_3=(f''(0)/16)[4\beta^2-13+(13-13\beta+5\beta^2+2\beta^3/3)e^{-\beta}]$$

$$\Phi_6=(f''(0)^2/64)[32\beta^3/3+142\beta-5563/8+e^{-\beta}(\beta^6/18+17\beta^5/15+259\beta^4/24+743\beta^3/12+1867\beta^2/8+4427\beta/8+5563/8)] \quad (13)$$

and the values of the second derivatives of Φ_n at $\beta=0$ are as follows.

$$\Phi_{0\beta\beta}(0)=1,\ \Phi_{3\beta\beta}(0)=5f''(0)/16,$$
$$\Phi_{6\beta\beta}(0)=443f''(0)^2/512, \quad (14)$$
$$\Phi_{9\beta\beta}(0)=2794f''(0)^3/4096$$
$$\Phi_{1\beta\beta}(0)=\Phi_{2\beta\beta}(0)=\Phi_{4\beta\beta}(0)=\Phi_{5\beta\beta}(0)=\Phi_{7\beta\beta}(0)=\Phi_{8\beta\beta}(0)=0$$

3.3 Wall skin friction fluctuation

The wall shear stress τ consists of two parts, that is, the steady mean term τ_0 and the unsteady term τ_1.

$$\tau=\tau_0+\varepsilon e^{i\omega t}\tau_1 \quad (15)$$

As the result of velocity distribution analysis, the following expressions are derived.

$$\tau_1/\tau_0=(1/f''(0))\sum_{n=0}^{\infty}\xi^n\Phi_n''(0) \quad (\omega^*<1)\ (16a)$$

$$\tau_1/\tau_0=(1/f''(0))\sum_{n=0}^{\infty}\alpha^{n-1}\Phi_{n\beta\beta}(0) \quad (\omega^*>1)\ (16b)$$

where $\tau_0=f''(0)\cdot\rho U_0^2\sqrt{\nu/U_0x}$

4. Analysis of temperature distribution

The length of a heated element is very short and the unheated starting length exists as shown in Fig. 1. Then the thermal bounlary layer thickness on the heated element is very thin compared with the velocity boundary layer thickness. Therefore it is assumed that the velocity distribution in the thermal boundary layer is linear.

$$u=(\tau_0/\rho\nu)y+e^{i\omega t}(\tau_1/\rho\nu)y \quad (17)$$

Since $v_0=v_1=0$, substituting Eq.(17) into (5c), the following energy equation is derived.

$$\frac{i\omega}{\tau_0/\rho\nu}\Xi+y^+\frac{\partial\Xi}{\partial x^+}+\left(\frac{\tau_1}{\tau_0}\right)y^+\frac{\partial\theta_0}{\partial x^+}=\frac{1}{P_r}\frac{\partial^2\Xi}{\partial y^{+2}} \quad (18)$$

$$x^+=\int_{x_0}^{x}\sqrt{\tau_0/\rho}/\nu\,dx,\ y^+=y\sqrt{\tau_0/\rho}/\nu$$

4.1 Case of low frequency ($\omega^*<1$)

If the frequency parameter is smaller than unity, it is justified to change the independent variables x^+,y^+ into the following new variables.

$$\xi_T=(i\omega x^+\rho\nu/\tau_0)\cdot(P_r/9x^+)^{1/3}$$
$$\eta_T=y^+(P_r/9x^+)^{1/3} \quad (19)$$

Using the variables defined above, Eq.(18) becomes as follows.

$$\Xi_{\eta_T\eta_T}+3\eta_T^2\Xi_{\eta_T}-6\xi_T\eta_T\Xi_{\xi_T}-9\xi_T\Xi+3\eta_T^2(\tau_1/\tau_0)\theta_{0\eta_T}=0 \quad (20)$$

$$\Xi=0 \text{ at } \eta_T=0,\ \Xi=0 \text{ as } \eta_T\to\infty$$

The subscripts η_T and ξ_T denote the differentiation with respect to η_T and ξ_T respectively. Since Eq.(16a) which should be substituted into Eq.(20) is a function of ξ, the relation between ξ and ξ_T must be known. If we put $\xi_T=K\cdot\xi$ in this relation, K is written as follows.

$$K=[1-(x_0/x)^{3/4}]^{2/3}P_r^{1/2}(4/9f''(0))^{2/3} \quad (21)$$

It is clear from Eq.(21) that K is a parameter of unheated starting length (x_0/x), and in general is a function of x. But K must be kept constant to solve Eq.(20) numerically. Fig. 2 shows the value of K versus x_0/x for $P_r=0.72$. If the unheated starting length x_0 is zero, the value of K is 1.08867, and K decreases to zero as x_0/x approaches unity. Assuming that K is constant, the variable ξ_T in Eq.(20) is changed into ξ.

$$\Xi_{\eta_T\eta_T}+3\eta_T^2\Xi_{\eta_T}-6\xi\eta_T\Xi_\xi-9K\xi\Xi+3\eta_T^2(\tau_1/\tau_0)\theta_{0\eta_T}=0 \quad (22)$$

Expanding Ξ into a power series with respect to ξ, that is, $\Xi=\sum_{n=0}^{\infty}\xi^n\Xi_n(\eta_T)$, and equating the same order of ξ, the following equation is derived.

$$\Xi_n''+3\eta_T^2\Xi_n'-6n\eta_T\Xi_n-9K\Xi_{n-1}-3\eta_T^2(e^{-\eta_T^3}/\Gamma(4/3))(\Phi_n''(0)/f''(0))=0 \quad (23)$$

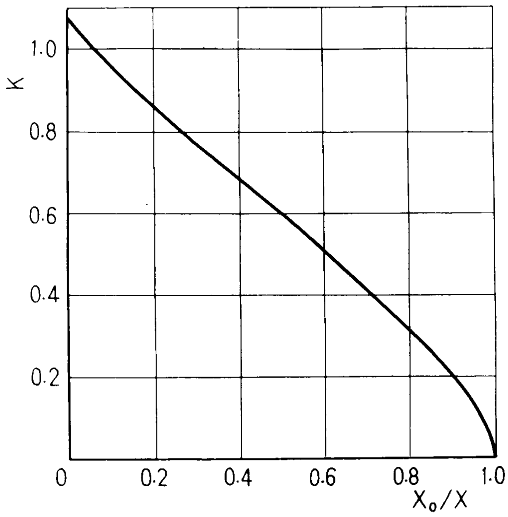


Fig. 2 Variation of unheated starting length parameter K with x_0/x (P_r=0.72)

where $\Xi_{-1}=0$, the prime denotes the differentiation with respect to η_T and the boundary conditions are;

$$\Xi_n=0 \text{ at } \eta_T=0, \text{ and } \Xi_n=0 \text{ as } \eta_T \to \infty$$

Eq.(23) is solved numerically for some different values of K, and the values of temperature gradient at the wall are listed in Table 2.

4.2 Case of intermediate frequency ($1<\omega^*<1/K$)

The energy equation (20) is originally applicable to the intermediate frequency region. In this case Eq.(16b) is substituted into the term (τ_1/τ_0) of Eq.(20) or (22). As it will be mentioned later, it is most suitable to take into account the terms up to n=3 in Eq.(16b). Therefore it is valid to expand Ξ into the following power series; $\Xi=\sum_{n=0}^{\infty}\xi^{\frac{n}{2}-1}\,\Xi_n(\eta_T)$.

The equation for Ξ_n is derived by equating the same power of ξ.

$$\Xi_n''+3\eta_T^2\,\Xi_n'-3(n-2)\eta_T\,\Xi_n-9K\,\Xi_{n-2}$$
$$-3\eta_T^2(e^{-\eta_T^3}/\Gamma(4/3))(\Phi_{3-n,\beta\beta}(0)/f''(0))=0 \qquad (24)$$
$$\Xi_{-2}=\Xi_{-1}=0$$

Eq.(24) is solved numerically in the same manner as the case of low frequency, and the values of $\Xi_n'(0)$ are listed in Table 3.

4.3 Case of high frequency ($\omega^*>1/K$)

If the frequency parameter is larger than 1/K, the variables x^+, y^+ in Eq.(18) are changed into the following variables.

$$\alpha_T=1/\sqrt{\xi_T}, \quad \beta_T=\eta_T/\alpha_T \qquad (25)$$

Eq.(18) yields the following.

$$\Xi_{\beta_T\beta_T}-9\,\Xi+3\beta_T\alpha_T^4\,\Xi_{\alpha_T}+3\beta_T\alpha_T^4(\tau_1/\tau_0)\theta_{0\alpha_T}=0 \qquad (26)$$

Table 2. The values of $\Xi_n'(0)$ at low frequency

n \ K	1.08867	0.5	0.1	0.02	0
0	-0.55957	-0.55957	-0.55957	-0.55957	-0.55957
1	0.12789	-0.25052	-0.50764	-0.55906	-0.57192
2	0.22603	0.36899	0.22729	0.24062	0.23049
3	-0.35167	-0.27277	-0.16133	-0.14466	-0.14105
4	0.31113	0.16552	0.99515×10^{-1}	0.87095×10^{-1}	0.85207×10^{-1}
5	-0.21242	-0.90739×10^{-1}	-0.53299×10^{-1}	0.48860×10^{-1}	-0.47848×10^{-1}
6	0.12252	0.46005×10^{-1}	0.27481×10^{-1}	0.25238×10^{-1}	0.24723×10^{-1}
7	-0.62327×10^{-1}	-0.21707×10^{-1}	-0.13104×10^{-1}	-0.12043×10^{-1}	-0.11799×10^{-1}

Table 3. The values of $\Xi_n'(0)$ at intermediate frequency

n \ K	1.08867	0.5	0.1	0.02	0
0	-0.34744	-0.34744	-0.34744	-0.34744	-0.34744
1	0.	0.	0.	0.	0.
2	0.63427	0.29128	0.58256×10^{-1}	0.11651×10^{-1}	0.
3	-0.84289	-0.84289	-0.84289	-0.84289	-0.84289
4	-0.67305	-0.14195	-0.56778×10^{-2}	-0.22711×10^{-3}	0.
5	0.92411	0.42439	0.84877×10^{-1}	0.16975×10^{-1}	0.
6	0.51719	0.60091×10^{-1}	0.40073×10^{-3}	0.32058×10^{-5}	0.
7	-0.67372	-0.14209	-0.56835×10^{-2}	-0.22734×10^{-3}	0.

where the boundary condition requires that $\Xi=0$ at $\beta_T=0$, and $\Xi=0$ as $\beta_T\to\infty$. Eq.(16b) is substituted into Eq.(26), and $\theta_{0\alpha_T}$ is approximated by the following power series;

$$\theta_{0\alpha_T}=-\beta_T/\Gamma(4/3)\cdot(1-\alpha_T{}^3\beta_T{}^3+\alpha_T{}^6\beta_T{}^6-\cdots\cdots)$$

Since Eq.(16b) is a function of α, the variable α_T in Eq.(26) is changed into using the relation $\alpha_T=\alpha/\sqrt{K}$. By expanding Ξ into a power series, that is, $\Xi=\sum_{n=0}^{\infty}\alpha^n\Xi_n(\beta_T)$, and by substituting that into Eq.(26), a differential equation with respect to Ξ_n is derived. This equation is solved analytically as follows.

$$\Xi_{3\beta_T}(0)=(-2/9f''(0))/K^2\Gamma(4/3)$$

$$\Xi_{6\beta_T}(0)=(35/162K^3f''(0)-5/72)/K^2\Gamma(4/3)$$

$$\Xi_{9\beta_T}(0)=(-595/5832K^3f''(0)-25/1296\sqrt{K^3}-443f''(0)/2304)/K^2\Gamma(4/3) \qquad (28)$$

4.4 Heat transfer fluctuation

The local Nusselt number is defined by $N_u=-x(\partial\theta/\partial y)_{y=0}$. N_u consists of two parts which are a steady mean term and an unsteady term.

$$N_u=N_{u0}+\varepsilon e^{i\omega t}N_{u1} \qquad (29)$$

The steady term is derived from Eq.(8).

$$N_{u0}=1/\Gamma(4/3)\cdot(f''(0)P_r/12)^{1/3}\cdot\sqrt{R_{ex}}[1-(x_0/x)^{3/4}]^{-1/3} \qquad (30)$$

And the unsteady term is expressed in the following equations.

$$N_{u1}/N_{u0}=-\Gamma(4/3)\sum_{n=0}^{\infty}\xi^n\Xi_n'(0) \quad (\omega^*<1) \qquad (31a)$$

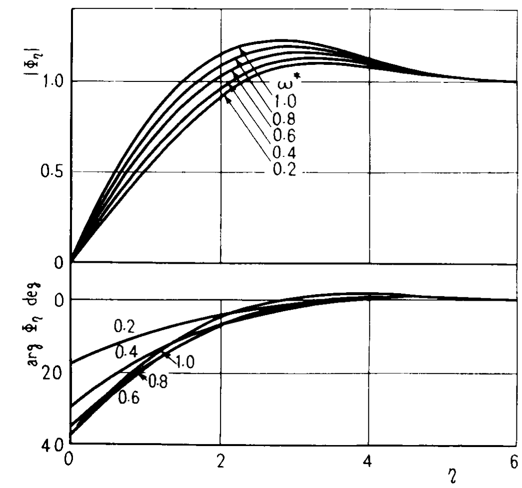
$$N_{u1}/N_{u0}=-\Gamma(4/3)\sum_{n=0}^{\infty}\alpha^{n/2-1}\Xi_n'(0) \qquad (31b)$$
$(1<\omega^*<1/K)$

$$N_{u1}/N_{u0}=-\Gamma(4/3)\sqrt{K}\sum_{n=0}^{\infty}\alpha^{n-1}\Xi_{n\beta_T}(0) \qquad (31c)$$
$(\omega^*>1/K)$

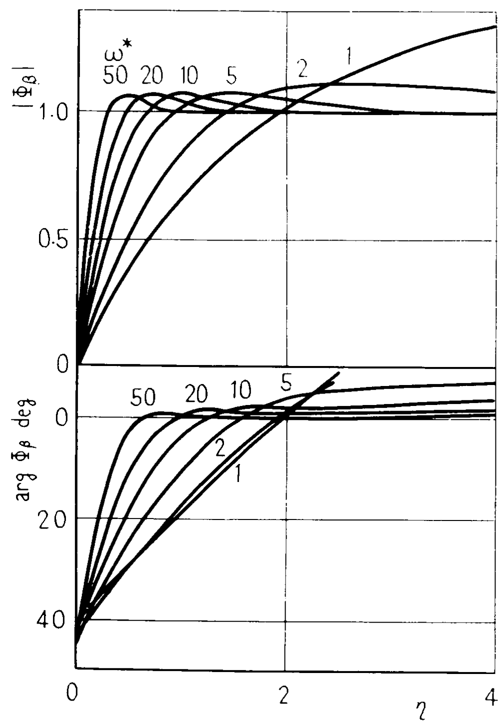
5. Numerical results and discussion

The typical profiles of the amplitude and the phase angle of velocity fluctuation at several different values of frequency parameter ω^* are shown in Figs. 3(a) and (b) for low and high frequency regions respectively. At low frequency, similar results have been already obtained in literatures(8,9,10). In this paper, solution are obtained up to the 7th power in low frequency and up to 9th power in high frequency, so that these solutions are applicable in a wider frequency region and also near the region where the frequency parameter is unity. The maximum amplitude appears near the wall at any frequency, and the position comes closer to the wall with an increasing frequency. And it can be seen that the phase angle near the wall is in advance of the free stream fluctuation. The phase advance increases with frequency, and an asymptotic phase advance 45° at the wall is attained. And the asymptotic solution at very large frequency is identical with Stoke's profile.

In Figs. 4(a) and (b), the amplitude and the phase angle of wall skin friction fluctuation are shown. In these figures the solid lines indicate the low frequency solution, the broken lines the high frequency solution, and the numbers on the line are the values of exponent in Eq.



(a) Case of low frequency ($\omega^*<1$)

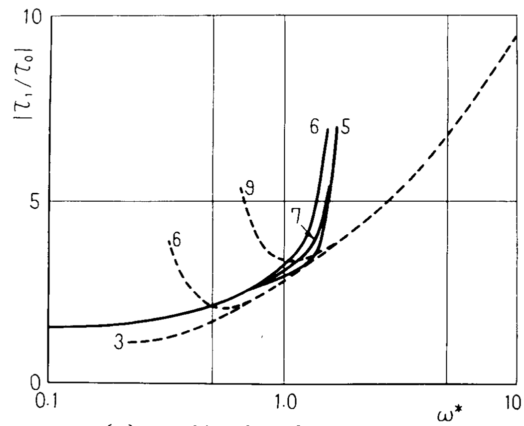


(b) Case of high frequency ($\omega^*>1$)

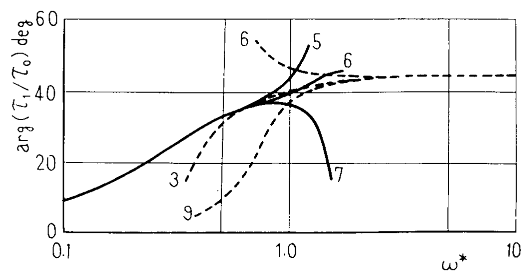
Fig. 3 Amplitude and phase angle of velocity fluctuation

(16a)or(16b). The amplitude of skin friction fluctuation increases monotonously with frequency. The phase angle is in advance of the free stream fluctuation and an asymptotic phase advance 45° is attained at very large frequency. The low frequency solution Eq.(16a) does not always agree with the high frequency solution Eq.(16b) near the region where the frequency parameter is unity, but the solution truncated at 5th power for low frequency agrees well with the one truncated at third power for high frequency in that region.

The profiles of amplitude of the temperature fluctuation at some different values of frequency parameter are shown in Figs. 5(a), (b) and (c) which are the cases of low, intermediate and high frequencies respectively. The unheated starting length parameter K is selected to be 0.5 in these examples. The temperature fluctuation

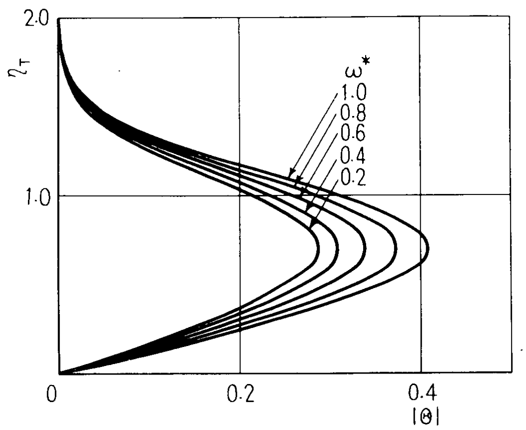


(a) Amplitude of wall skin friction fluctuation

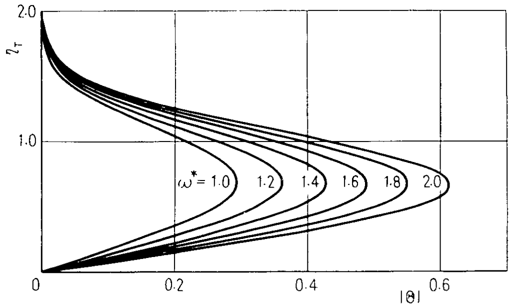


(b) Phase angle of wall skin friction fluctuation

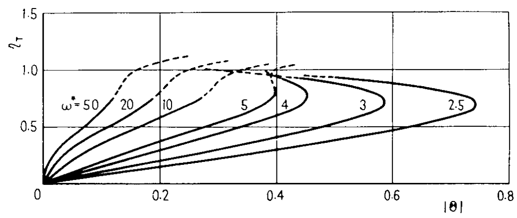
Fig. 4 Frequency characteristic of wall skin friction fluctuation



(a) Case of low frequency ($\omega^*<1$)



(b) Case of intermediate frequency ($1<\omega^*<1/K$)

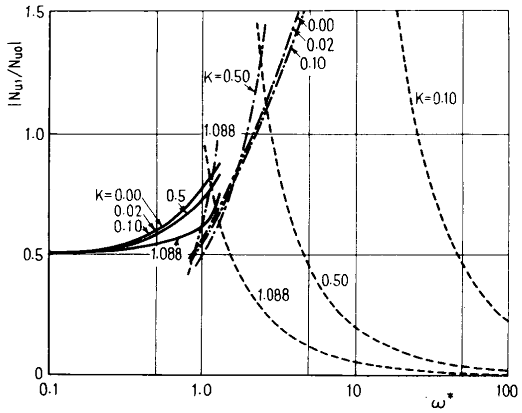


(c) Case of high frequency ($\omega^*>1/K$)

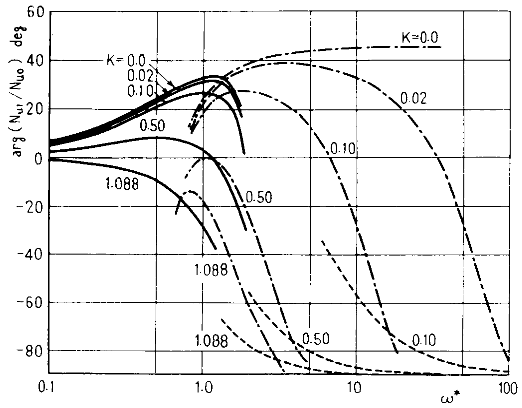
Fig. 5 Profiles of temperature fluctuation amplitude

appears only near the wall and the maximum amplitude appears near $\eta_T=0.7$ at any frequency. The upper edge of fluctuating region is nearly equal to that of the steady mean thermal boundary layer. These temperature profiles depend a little on the value of K. In Fig. 5(c), the solution by Eq.(27) is not valid at large value of η_T as shown by the broken line.

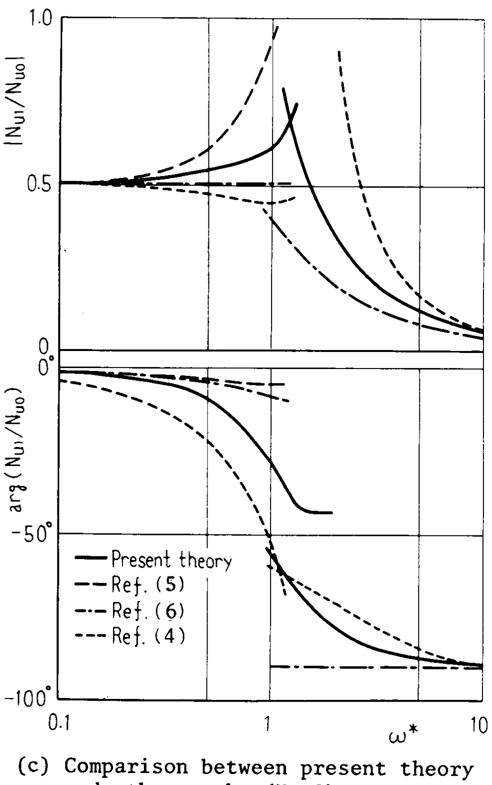
Figs. 6(a), (b) and (c) show the amplitude and the phase angle of heat transfer fluctuation. In Figs. 6(a) and (b), the solid lines are for the case of low frequency, the chain lines for that of intermediate frequency and the broken lines for that of high frequency. The parameter is the value of K. As the frequency increases, the amplitude increases at low and intermediate frequencies, but it decreases to zero rapidly at high frequency. And the



(a) Amplitude of heat transfer fluctuation



(b) Phase angle of heat transfer fluctuation



(c) Comparison between present theory and other works (Xo=0)

Fig. 6 Frequency characteristic of heat transfer fluctuation

amplitude tends to become larger as K becomes smaller. With regard to the phase angle of heat transfer fluctuation as shown in Fig. 6(b), it is influenced considerably by the unheated starting length parameter K. For example, if there is no unheated starting length which corresponds to K=1.08867, the phase lag becomes larger as the frequency increases and an asymptotic phase lag 90° is attained at very large frequency just as in the previous works. It is interesting to note that the phase advance appears at low frequency in a case with an unheated starting length. As the value of K becomes smaller, that is, the unheated starting length becomes longer compared with the heated element length, the phase advance becomes larger. In addition, in the case of K=0, an asymptotic phase advance 45° is attained at very large frequency, and this phase advance of heat transfer is identical with that of wall skin friction.

In Figs. 6(a) and (b), although the discontinuity and the disagreement of curves exist between two adjacent frequency regions where the frequency parameter is nearly equal to unity or 1/K, in the real phenomena it will be expected that there will be a continuous change. In the case without unheated starting length, the assumption required in the present theory may not be satisfied, but the result will be in good agreement with the results in literatures(4,5,6) as shown in Fig. 6(c). Therefore Eq.(31) will be applied for the case without unheated starting length as well as Eq.(30).

In Fig. 7, the influence of unheated starting length on the amplitude and the phase angle of heat transfer flucuation is indicated in the case of low frequency. It is clear that the amplitude becomes constant as K becomes smaller than 0.15, and the phase angle also becomes constant in the region where K is smaller than 0.02.

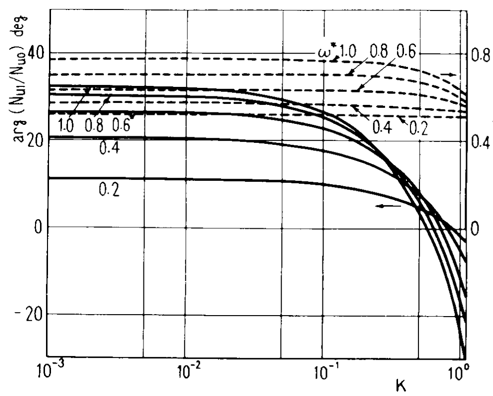


Fig. 7 Influence of unheated starting length on heat transfer fluctuation (case of low frequency)

6. Conclusions

The velocity distribution and the temperature distribution with unheated starting length in an oscillating laminar boundary layer over a flat plate were analyzed theoretically by means of boundary layer approximation, and the amplitude and the phase angle of wall skin friction fluctuation and heat transfer fluctuation were calculated. The results obtained in this paper are as follows.

(1) The amplitude and the phase advance of skin friction fluctuation increase with frequency, and an asymptotic phase advance 45° is attained at very large frequency.

(2) The maximum amplitude of temperature fluctuation appears near $\eta_T=0.7$ at any frequency, which corresponds to about one third of time mean thermal boundary layer thickness.

(3) If there is an unheated starting length, the amplitude of heat transfer fluctuation increases with frequency at low and intermediate frequencies, but decreases to zero rapidly at high frequency.

(4) In a case without unheated starting length, heat transfer is found to fluctuate always with phase lag, but the phase advance appears at low and intermediate frequencies if there is unheated starting length.

(5) When the unheated starting length parameter is very small ($K<0.02$), a phase advance 45° is attained asymptotically at very large frequency in heat transfer fluctuation as well as in wall skin friction flucuation.

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Nomenclature

f = Blusius's stream function

i = Imaginary unit

K = Unheated starting length parameter; Eq.(21)

N_u = Nusselt number

P_r = Prandtl number

T = Temperature

t = Time

U = Free stream velocity

u,v = x,y-components of velocity

x,y = Co-ordinates parallel and normal to the plate

$x^+ = \int_{x_0}^{x} \sqrt{\tau_0/\rho}/\nu dx$

$y^+ = y\sqrt{\tau_0/\rho}/\nu$

x_0 = Unheated starting length

$\alpha = 1/\sqrt{i\omega x/U_0}$; Eq.(11)

$\alpha_T = 1/\sqrt{\xi_T}$; Eq.(25)

$\beta = y\sqrt{i\omega/y}$; Eq.(11)

$\beta_T = \eta_T/\alpha_T$; Eq.(25)

$\Gamma(4/3)=0.8930$; Gamma-function

$\xi = i\omega x/U_0$

$\xi_T = (i\omega x^+\rho\nu/\tau_0)\cdot(P_r/9x^+)^{1/3}$; Eq.(19)

$\eta = y\sqrt{U_0/\nu x}$

$\eta_T = y^+(P_r/9x^+)^{1/3}$; Eq.(19)

$\theta = (T-T_\infty)/(T_w-T_\infty)$

$\Xi = (T_1-T_\infty)/(T_w-T_\infty)$

κ = Thermal diffusivity

ν = Kinematic viscosity

τ = Wall skin friction

ω = Angular frequency of oscillating free stream

$\omega^* = \omega x/U_0$; Non-dimensional frequency parameter

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