621.914.5:621.833

Paper No. 198-21

RESEARCH ON THE TAPERED HOBS*

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The hobs are theoretically reconsidered on their peripheral taper and their rough cutting mechanism. The taper angle under a fixed length of hob varies according to the helix angle of gear and the setting angle and the cutting feed direction of hob. A method of determining the optimum taper shape is described on a calculation example.

Additionally, in order to relieve the cutting load concentrated on the side edge of end tooth, a new type side taper hob, whose tooth thickness is gradually reduced in the rough cutting teeth, is proposed. Design example is given and satisfactory result is obtained in a hobbing with the trial hob.

1. Introduction

Many large-sized hobs larger than module 20 ~ 30 mm and outside diameter 300 mm are manufactured by means of shortening their necessary length for hobbing, because their material homogeneity is unreliable and their heat treatment is difficult, and because the material and the production costs are high. In the shortened hobs, a tooth of either hob end works exclusively and wears unusually, so that such a hob is tapered on the peripheral surface toward the concentrating end and the cutting load is distributed on the unweared teeth.

The shape and angle of the taper are generally designed by the tool makers, therefore in using the tapered hob all teeth don't always work uniformly; e.g. the cutting load concentrates on the teeth of both the end and the intersection of the cylinder and the taper, or on the only tooth of the intersection. The users want to choose the proper hobbing feed according to the gear materials, their hardness and the stiffness of the machinery in use, and want to choose freely the conventional or the climb hobbing. They also want to generate gears having arbitrary number of teeth and various helix angles with the same tapered hob and know a cutting mechanism of the tapered hob after pre-milling or multi-passing. But very few works have been reported on the relation between the hobbing condition and the hob taper.

In hobbing with shortened hobs, the cutting load concentrates not only on the top edge but on the both side edges. Because of side-edge concentrations, the hob axial vibration and the gear torsional one, which are very harmful, will be caused. The cutting load concentration on the side edge has not been analyzed and no counter plans for them have been made.

In this paper, a new method of analyzing the hobbing mechanism of the tapered hob is shown and the relation between the hobbing condition and the peripheral taper is made clear. Additionally a method of preventing the concentration of cutting load on the side edges is proposed first, where the tooth thickness of hob is decreased by means of the double lead.

Table 1 Symbols and hobbing test

	Symbols	Hobbing test
Common	m : Module	* 12
	α_c : Pressure angle	* 20°
	Γ : Setting angle	22.1804
	a : Center distance	2305.2250
	Δa : Addendum shift	* 0
Gear	z : Number of teeth	* 320
	β : Helix angle	* 30° (RH)
	R_k : Outside radius	2229.0250
	R_o : Pitch circle radius	2217.0250
	R_r : Root circle radius	2200.2250
	R_g : Base circle radius	2043.8554
	h_k, h_f : Tooth hight coefficient	* 1.0, 1.4
	x : Addendum modification coefficient	* 0
Hob	r_k : Outside radius	* 105
	r_o : Pitch circle radius	88.2
	r_e : r_o - (dedendum)	76.2
	r_n : Outside radius of Nth tooth	
	z_w : Number of threads	* 2
	G_n : Number of gashes	* 10
	γ : Lead angle	7.8195°
	γ_e : Lead angle on r_e circle	9.0606°
	δ : Rake angle	* 3°
	ω ξ : Phase angle	ω = 1.1356°
	$\bar{\beta}$: Helix angle of gash	* 0°
	Δt : Tooth distance on axis	7.6106
	$\Delta\theta$: Tooth division angle	36°
	N : Tooth number	
Cutting	$+f$: Conventional cut feed	} * 1.5
	$-f$: Climb cut feed	
	Θ : Revolution angle of gear	
	θ : Revolution angle of hob	
	k : Revolution ratio between gear and hob	6.2504×10^{-3}

Notes; 1. Symbols are detailed in reference (1).
2. When * marked values are given, other values can be calculated with these ones.
3. Δa is length extending longer than regular value of the center distance in two times or three times hobbing.

* Received 14th December, 1979.

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2. Cutting-load concentration and its distribution on the hob teeth

The symbols used in this paper are listed in Table 1 and their + or − conditions are shown in Table 2. The analysis will be developed when a right helical gear is machined with a right threaded hob and the cutting mechanism of side edges will be analyzed with respect to the left side edge.

Table 2 Definitions of ± in symbols

Symbols	±
Δa	According to ± of addendum shift
β , $\bar{\beta}$	Right hand : +
γ , γ_e	Right hand : +
δ , ω	According to ± of rake angle
Γ	According to ± of ($\beta - \gamma$)
Θ	Right hand rev. of x axis : +
θ	Downward from horizotal plane : +
N	Right side from oth tooth : +
f	Conventional cut : +

2.1 Hob length required in the hobbing types

According to the helical hand of gear trace and the feed direction (conventional or climb), there are four types of hobbing with the right hand thread hob as shown in Fig.1 (a)~(d). There are two types even in the conventional feed. One is the case (a) where the hob teeth generate respectively an unfinished tooth space from the tooth root to the tip, and the other is the case (c) which is reverse to (a). In case (a), the generation of the involute tooth surface begins at the N_oth

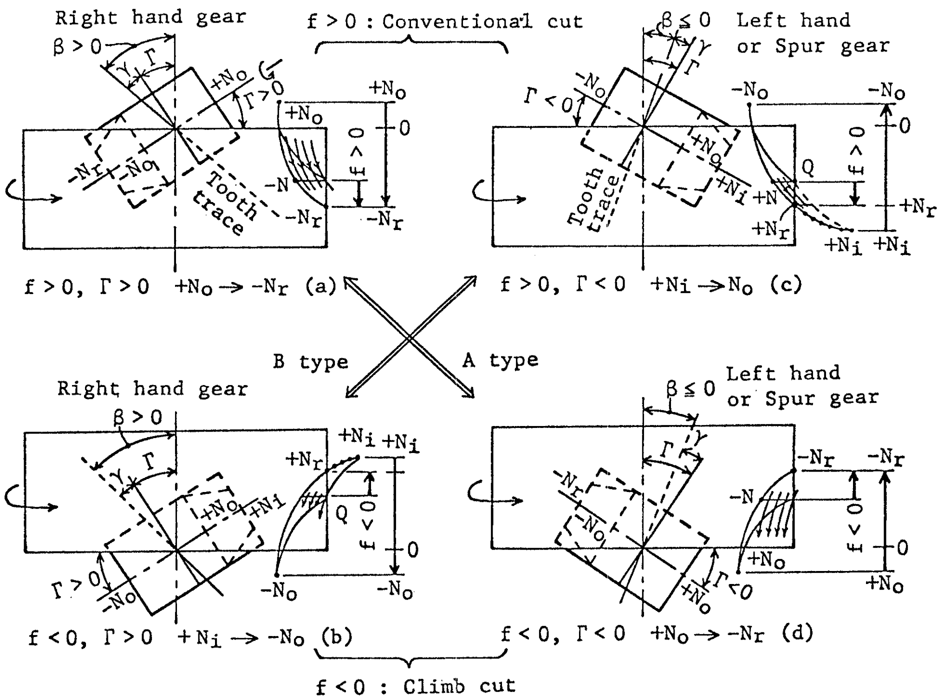
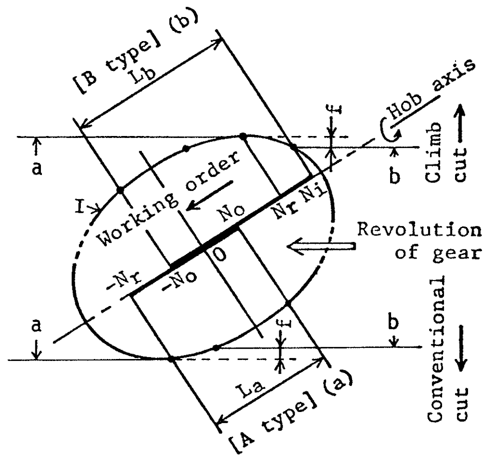


Fig.1 Hobbing state depending upon feed f and setting angle Γ (A type and B type)



a : Bottom land end after this hobbing
b : Bottom land end before this hobbing

Fig.2 Cutting zone of A type and B type

tooth and finishes at the $-N_o$th tooth through the 0th and then the cutting finishes at the $-N_r$th which cuts deepest in the tooth space. In case (c), the N_ith tooth begins to work at the corners Q in Fig.1(b) and (c) and the cutting progresses in an order of $N_i \to N_r \to N_o \to 0 \to -N_o$th tooth. The climb cut is also the same as the conventional one except that the shape of the unfinished tooth space is different.

The hobs must be tapered toward the N_rth tooth which cuts lowest in the conventional cut (cuts highest in the climb cut). This tooth is either a preceding one (case (b) or (c) in Fig.1) which enters the tooth space first or a succeeding one (case (a) or (d)). When the former is called B type and the latter A type, these can be classified as follows

$$\text{A type} : f * \Gamma > 0, \quad \text{B type} : f * \Gamma < 0 \quad \cdots (1)$$

where + or − sign of f and Γ is taken as shown in Table 2. An oval curve I in Fig.2 is an intersection of the gear addendum cylinder and that of the hob as seen through the gear. The hob is on the other side of paper and the inside of the curve is concave on this side. The upper half of the oval shows case (b) of B type and the lower half shows case (a) of A type in Fig.1. As is obvious from Fig.2, the necessary hob length for hobbing is constant from N_o to $-N_r$ in A type. B type hob needs a length from N_i to $-N_o$, whose length is longer from N_i to $-N_r$ than A type, and this length varies with feed f.

The hob length L_a or L_b requred for A or B type will be calculated by using N_o, N_r, N_i. From the previous paper(1),

$$N_o = m(\cos^2\beta/\tan\alpha_c + \tan\alpha_c + 0.25\pi)/\cos\gamma/\Delta t \quad \cdots\cdots\cdots\cdots (2)$$

where the obtained N_o counts as one fraction of more than 0.5 inclusive and cuts away the rest.

H in Fig.3 is an ellipse which is drawn by the top center of the Nth tooth with outside radius r_n and G is the addendum cylinder of the gear. The Nth tooth of the hob comes in or out of the gear blank at point P in A type or in B. The cut length Z at point P is given by the following iterative calculation.

$$\left.\begin{array}{l} x_1 = a - R_k \quad \text{(starting value)} \\ y = \pm\sqrt{r_n^2 - x_1^2}\sin\Gamma \\ \quad \text{(equation of ellipse H,} \\ \quad\ + : \text{A type,} \quad - : \text{B type)} \\ X = y + \Delta t N\cos\Gamma \\ Y = \sqrt{R_k^2 - X^2} \\ \quad \text{(equation of circle G)} \\ x_2 = a - Y \\ |x_1 - x_2| < \varepsilon \qquad \text{no} \quad x_1 = x_2 \\ \quad (\varepsilon = \text{allowable error}) \\ Z = \sqrt{r_n^2 - x_2^2}\cos\Gamma - \Delta t N\sin\Gamma \qquad \text{yes} \end{array}\right\} \cdot\cdot (3)$$

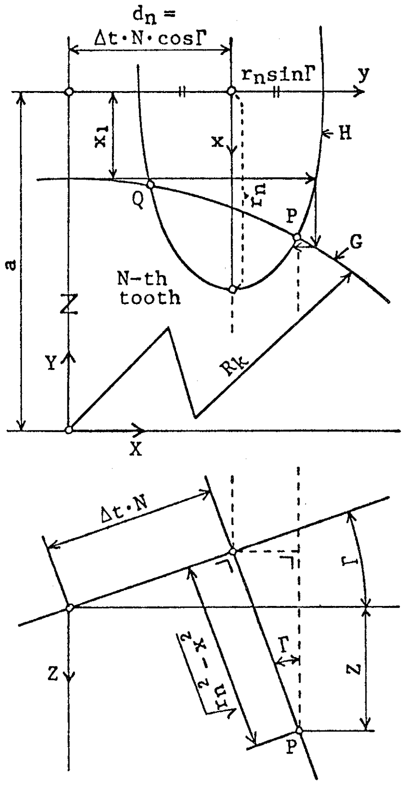


Fig.3 Length of cut down

Putting $r_n = r_k$ and $N = N \pm 1$, Z is obtained for every N through Eq.(3). N which gives Z_{max} is N_r. When Z is calculated by using N which is added one by one to N_r, a tooth which exists just before the Nth tooth satisfying $Z_{max} - Z > f$ first is N_i. Without the peripheral taper, L_a and L_b are respectively

$$\left.\begin{array}{l} \text{A type} : L_a = (|N_o| + |N_r|)\Delta t \\ \text{B type} : L_b = (|N_o| + |N_i|)\Delta t \end{array}\right\} \cdots\cdots\cdots (4)$$

For example, the following are obtained respectively $N_o = 5$, $N_r = -46$, $N_i = 48$, so that $L_a = 389$ mm for A and $L_b = 404$ mm for B type are given under $\Delta t = 7.6106$ mm. Z_{max} is equal to 167.479 mm when $N_r = -46$.

2.2 Typical shapes of peripheral taper and their cutting behaviors

When the allowable hob length is shorter than the one calculated through Eq.(4), the hob must be tapered on its peripheral surface. As shown in Fig.4, $\pm N_o$ region of the hob should be made in the shape of cylinder, because this portion is necessary to generate the involute tooth surface. In case of a taper angle ζ less than 4 degrees, a single taper in Fig.4 "a" is suitable, but if ζ is from 4 to 7 degrees, a double taper "b" should be adopted so as to prevent the cutting load concentration on some teeth which are at an intersection of cylinder and taper. If the allowable hob length is much shorter, then it is unavoidable that a part of the second taper exists in the generating zone.

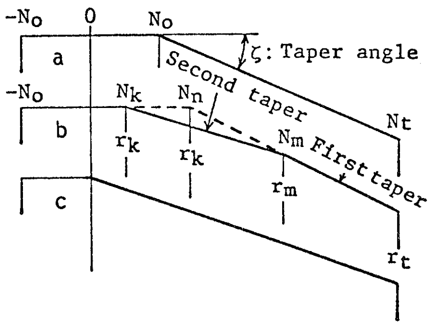


Fig.4 Typical examples of the peripheral taper

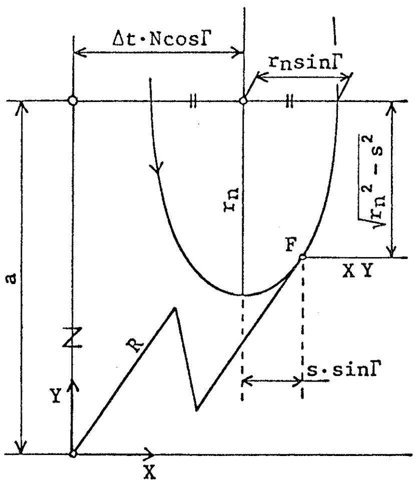


Fig.5 Depth of cut (R)

It is desirable to make the corner portion with a triple-stepped or rounded taper, but such machining is technically so difficult that the corner cut is applied, which will be explained later. In case of a rough machining or if a deformed tooth root is tolerated, "C" type is effective. The theoretical explanation will be developed on the double taper of case "b".

In case that the taper shapes and their dimensions are defined by the tooth numbers N_n, N_k, N_m, N_t and their radii r_k, r_m, r_t, a radius r_n of any tooth number N is

$$\left.\begin{array}{l} N \leq N_k : r_n = r_k \\ N_k < N \leq N_m : r_n = r_k - (r_k - r_m)(N - N_k)/(N_m - N_k) \\ N_m < N \leq N_t : r_n = r_m - (r_m - r_t)(N - N_m)/(N_t - N_m) \end{array}\right\} \cdots\cdots\cdots\cdots\cdots (5)$$

The cutting behavior by the peripheral edges of the tapered hob is analyzed. Figure 8(a) illustrates a tooth profile of the Nth tooth, whose outside radius r_k passes through its top center F and is viewed from its front. Figure 8(b) shows a state where the Nth tooth cuts deepest into the gear blank and a broken line in Fig.(c) is a tooth profile when its top center F reaches the plane h. This plane exists under the common normal of the gear and the hob by $f + h$ and is perpendicular to the gear axis. Figure 5 shows that Fig.(c) is viewed from over the gear and the cutting depth R of the Nth tooth on h plane can be calculated through X,Y co-ordinates of point F. From Figures 5 and 8

$$\left.\begin{array}{l} s = \Delta t N \tan\Gamma + (f + h)/\cos\Gamma \\ X = \Delta t N \cos\Gamma + s \cdot \sin\Gamma \\ Y = a - \sqrt{r_n^2 - s^2} \\ R = \sqrt{X^2 + Y^2} \end{array}\right\} \cdots\cdots\cdots\cdots (6)$$

If any N is put above Eq.(6) after a value of $f + h$ is set, r_n can be obtained through Eq.(5) and R through Eq.(6). Calculating R for various values of N, the minimum value of R will be obtained as R_b in Fig.6 and this position is on the unfinished bottom land before one revolution of gear. In case that the position of the bottom land is radius R_c by pre-cutting, R_b must be replaced by R_c, if R_b is longer than R_c. If R_{min} is selected for $f = 0$, the bottom land position R_a in Fig.6 and its tooth number N_a are obtained. $R_1, R_2, \cdots$ are calculated by using $N_a + 1$, $N_a + 2, \cdots$ until just before satisfying $R > R_b$. Their differences $t_a = R_1 - R_a$, $t_1 = R_2 - R_1, \cdots$ are the cutting thicknesses on this plane. In the calculation through Eq.(6) when s becomes longer than r_n or R longer than R_k, the tooth used in this calculation does't cut the gear blank and then the calculation must be continued by using the next tooth number.

To determine suitable shape and dimensions of hob taper, the single taper (Fig.4 "a") is first adopted in a gentle taper, where N_n is selected equal to N_o. The double taper (Fig.4 "b") will be adopted in a steep

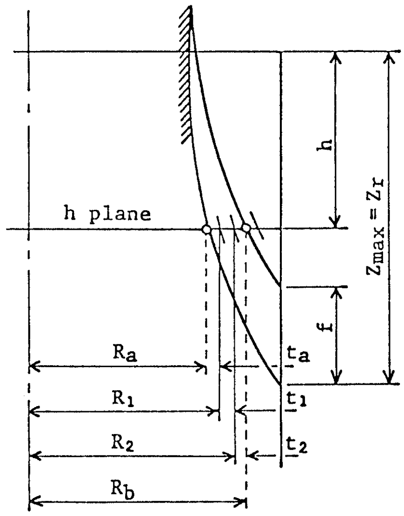


Fig.6 Cutting on *h* plane

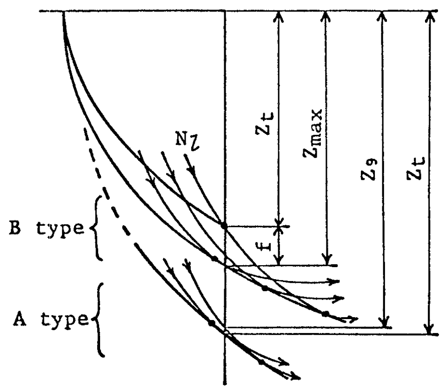


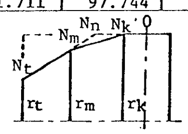
Fig.7 Search of the optimum taper

Table 3 Plan examples of the peripheral taper

h mm	Tooth number	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
0												-3								13
5												↓		13	[μm]		-3		13	
10												28	1				↓	80		
15												114					96	18		63
20												197					199		98	
25												286					286		182	
30												375					376		269	
35												466				54	362		358	
40										5	126	326			5	232	221		450	
45										185	253			62	185	253			546	
50					22					122			123	248	122				606	
55	-18			68	116						60	186	291				51	172	420	
60	↓		116	62				-12		124	253	153				119	243	279	↑	
65	37	163	2					↓	192	324	104			67	192	318	114		-5	
70	212	78						156	399	76		14	142	270	262					
75	380						203	368	151			222	354	55						
80	474	-18			41	174	424	104				220								
85	573	↓		127	263	360														
90	678	79	219	360	148															
93	745	220	360	306																

	N_t	N_m	N_n	N_k	r_t	r_m	Corner-cut
a	-18				105.		
b	-18		-5		91.711	(ζ7.65°)	No
c	-18	-12	-5	-3	91.711	97.844	No
d	-18	-12	-5	-3	91.711	97.744	Given

(a) Shortened and no tapered hob



(b) Single tapered hob

(c) Double tapered no corner-cut; given corner-cut →

(d) Double tapered and both corner-cut (0.1mm) taper. In such a case a tooth outside N_o by 2 or 3 teeth will be selected as N_n. By increasing the ζ-value gradually from 0 degree, ζ in case that the N_tth tooth cuts lowest can be found. Thus, the A type taper (A type in Fig.7) is determined by r_t obtained through such ζ. In case that $N_1, N_2, \cdots, N_8, N_t, N_9$ are the teeth before and behind N_tth tooth, their outside radii are

$$\left.\begin{aligned} &\Delta r = \Delta t \cdot \tan\zeta \\ &r_t = r_k - \Delta r (N_t - N_n) \\ &r_7 = r_t + 2\Delta r, \quad r_8 = r_t + \Delta r \\ &r_9 = r_t - \Delta r \end{aligned}\right\} \cdots\cdots\cdots (7)$$

After calculating Z_t and Z_9 through Eq.(3) which are respectively the cut lengths of the N_tth and N_9th teeth, r_t is computed by ζ by which Z_t changes just from $Z_t < Z_9$ to $Z_t > Z_9$ and the taper shape is determined by this r_t.

On the B type taper (Fig.7), Z_{max} should be selected from Z_t and $Z_8, Z_7, \cdots, Z_1$, which are obtained by using ζ larger than that of A type taper. Then r_t is found by ζ which is just before satisfying $Z_{max} - Z_t > f$. The cutting volume of such N_tth tooth is so small in both A and B types that practical r_t is determined by using the taper angle modified to 0.8～0.9 times the mathematically obtained ζ. If ζ becomes larger than 3 degrees, the second taper (Fig.4 "b") is applied to the teeth from N_k to N_m, which are several ones contained before and behind the corner tooth N_n. The optimum positions of N_k and N_m (second taper) are determined as follows. The cutting state diagram (Table 3) on all the teeth is drawn after calculating through Eqs.(5) and (6) varying h, and the positions of N_k and N_m must be respectively adjusted so that the cutting-load may be uniformly distributed on all the teeth.

Table 3 explains how the shapes and dimensions of the peripheral taper are planned for a given example. In case (a) in which only the shortening is applied without the taper, the concentration of the cutting length 30 mm and the maximum thickness 0.745 mm must occur on the −18th tooth at the shortened hob end. Case (b) illustrates a cutting behavior of a single taper, where ζ is modified equal to 7.65 degrees while the optimum taper angle 8.8 degrees (11.2° for B type) is numerically obtained under $N_n = -5$. In this case the cutting load concentrates on the −5th tooth at the corner. Case (c) shows a cutting behavior of the −3rd tooth in case that the second taper is applied between the −3rd and the −12th tooth. There are still considerable concentrations on the corner teeth, so that the radii of both corner teeth (−3rd and −12th teeth) are shortened by 0.1 mm as in case (d). This means that the cutting load is moderately distributed over the teeth existing before and behind the corners. Thus this corner-cut is a very effective way because of a good distribution of the cutting loads.

If the taper angle ζ becomes too large, twice or three times hobbing is also effective, whose cutting depth is 1/2 or 1/3 times the whole tooth depth and the taper angle ζ becomes about 1/2 or 1/3 times as large as ζ in one time hobbing. Then the optimum taper angle is calculated putting $a = a + \Delta a$ in Eq.(6), where a is the center distance and Δa is 1/2 or 1/3 times the whole depth. For example, ζ is 8.8° under one time hobbing, becomes 4.2° under two times and 2.3° under three times.

2.3 Side tapered hob and its cutting behavior

Point P in Fig.9 is an intersection of tooth profile and any circle with radius R on h plane, and P_x is normal to the tooth profile at P. A cutting behavior of side edge at P may be calculated through a length which

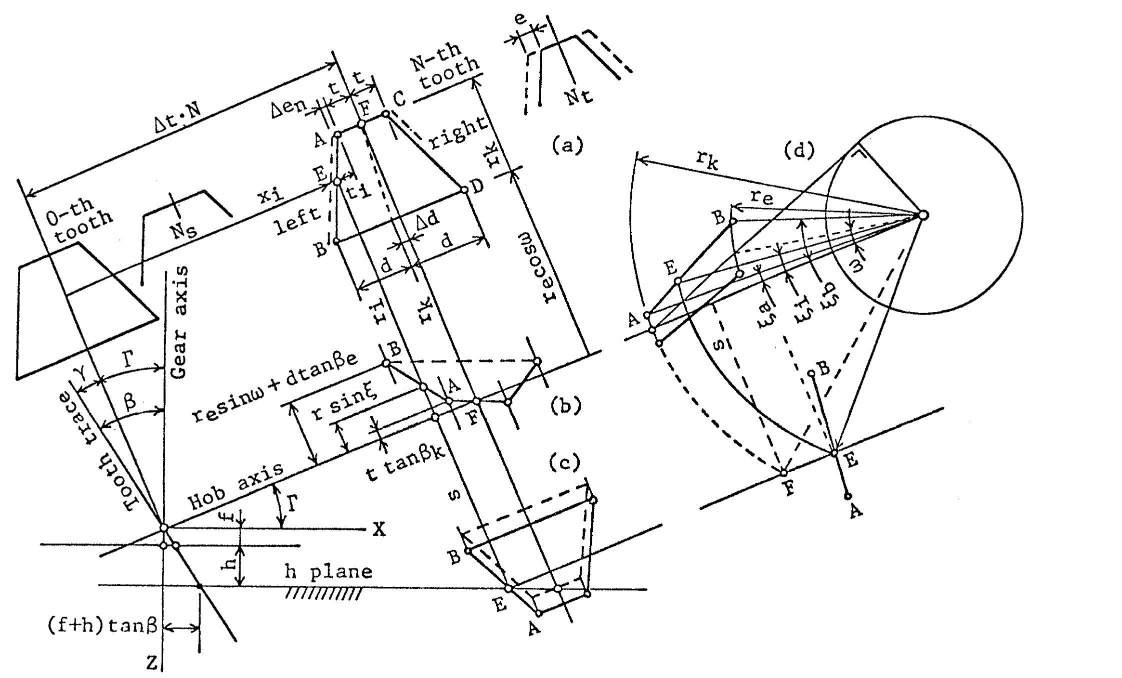


Fig.8 Front view of tooth profile, and its phase angle ξ

is cut off from the normal P_x with each hob tooth passing through the gear tooth space. When the hob has the rake angle δ and/or the helical gash angle $\bar{\beta}$, its tooth profile isn't bilaterally symmetrical and its side edge becomes a curved one. But in order to simplify this calculation, it is assumed that the side edge AB is straight as shown in Fig. 8(a). The tooth profile tilts by Δd because of a tooth profile deformation due to the rake angle. A revolution phase of the tooth top or the root is different by ξa or ξb from one of the top center F because of the helical gash of hob.

The tooth thicknesses diminish gradually from the N_sth tooth, and then the thickness of the N_tth tooth is decreased respectively by e at both flanks. Front view width t and d [Fig.8(a)] at the addendum and dedendum of the Nth tooth and the phase angles ξa, ξb [Fig.8(b)] of A, B positions can be calculated with the given figures.

$$\left.\begin{aligned} t &= m\{(0.25\pi - h_f\tan\alpha_c)/\\ &\quad \cos(\gamma+\bar{\beta}_k)\}\cos\bar{\beta}_k - \Delta e_n\\ d &= m\{(0.25\pi + h_k\tan\alpha_c)/\\ &\quad \cos(\gamma+\bar{\beta}_e)\}\cos\bar{\beta}_e - \Delta e_n\\ N &\leq N_s : \Delta e_n = 0\\ N &> N_s : \Delta e_n = \Delta e(N - N_s)\\ &\qquad \Delta e = e/(N_t - N_s)\\ \xi_a &= -\xi_c = \sin^{-1}(t\cdot\tan\bar{\beta}_k/r_k)\\ \xi_b &= -\xi_d = \sin^{-1}\{(r_e\sin\omega + d\tan\bar{\beta}_e)/r_e\} \end{aligned}\right\}\cdots(8)$$

The locus of the side edge AB on the h-plane becomes a curve (l in Fig.9), because the h-plane rotates with the gear blank. It is difficult to express mathematically the intersection of the locus l and the normal Px. Accordingly, side edge profile AB is divided into j equal-length portions (e.g. AB is divided into equal portions in Fig.9). A polygonal line whose corners are loci of equally divided points passing through the h-plane is approximated to the true line l. In case that the E-point of the Nth tooth (Fig.8) is numbered i among divisions 0,1, 2,····, the distance x_i from Y-axis of E-point, the radius r_i and the phase angle ξ_i are calculated as follows,

$$\left.\begin{aligned} x_i &= \Delta tN \mp (t_i + t)\\ t_i &= (d - t \pm \Delta d)i/j\\ \xi_i &= (\xi_b - \xi_a)i/j + \xi_a\\ r_i &= (r_k\cos\xi_a - r_j)/\cos\xi\\ r_j &= (r_k\cos\xi_a - r_e\cos\xi_b) \end{aligned}\right\}\cdots\cdots\cdots\cdots(9)$$

θh is a revolution angle of E point when the Nth tooth revolves from a position where the gear blank is cut in most deeply by the Nth tooth up to the h-plane. The co-ordinates XY of E point can be expressed as in Eq.(10) with Fig.8 and 9.

$$\left.\begin{aligned} \theta_h &= \tau + \xi_i\\ \tau &= \sin^{-1}[\{x_i\tan\Gamma + (f+h)/\cos\Gamma\}/r_i]\\ X &= x_i/\cos\Gamma + (f+h)\tan\Gamma\\ Y &= a - r_i\cos\tau \end{aligned}\right\}\cdots\cdots\cdots\cdots(10)$$

Y-axis in Fig.9 is a common normal of gear and hob axes. When the 0th tooth cuts into the gear blank most deeply, the center lines of both hob tooth profile and gear tooth space must lie on Y-axis, which passes through 0 point on the base circle. When the Nth tooth cuts into the gear blank most deeply, the gear tooth space exists in a revolution position n represented by $\Delta\theta\cdot N\cdot k$. This tooth space turns at an angular position $m = (f+h)\tan\beta/R_o$ on the h-plane because of the tooth space helix. Additionally it comes to the angular position of $(\Delta\theta\cdot N - \theta_h)k$ when the E point passes through the h plane due to the hob revolution θ_h. Point p is a locus penetrated by E point on h plane and its angular position Θ from Y-axis and its tilt angle ψ between the normal Px and X-axis are expressed as follows. Where ϕ : half clearance angle at any radius R, k : revolution ratio n_g/n_h of gear and hob,

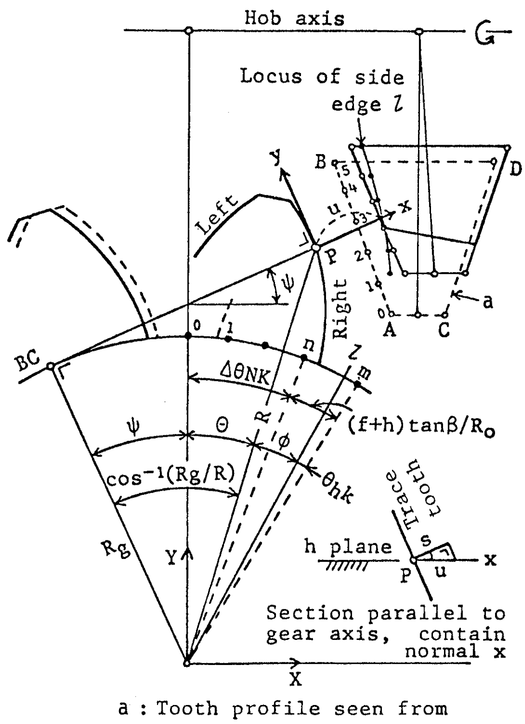
$$\left.\begin{aligned} \Theta &= (\Delta\theta N - \theta_h)K + (f+h)\tan\beta/R_o \mp \phi\\ k &= [1/\{1 - f\sin\beta/(\pi m Z)\}]Z_w/Z\\ \phi &= (0.5\pi - 2x_s\tan\alpha_s)/Z - \mathrm{inv}\alpha_s\\ &\quad + \mathrm{inv}\cos^{-1}(R_g/R)\\ \psi &= \cos^{-1}(R_g/R) - \Theta \end{aligned}\right\}\cdots\cdots\cdots\cdots(11)$$

(Upper compound symbol of $\mp$ is right tooth face, suffix s is transverse face of helix.)

X,Y co-ordinates of E point obtained by Eq.(10) are rearranged by the following x,y co-ordinates.

$$\left.\begin{aligned} x &= (X - R\sin\Theta)\cos\psi + (Y - R\cos\Theta)\sin\psi\\ y &= -(X - R\sin\Theta)\sin\psi + (Y - R\cos\Theta)\cos\psi \end{aligned}\right\}\cdots\cdots\cdots\cdots(12)$$

If $y > 0$ on $i = 0$, this calculation must be advanced to the next tooth, because this tooth does not cut the normal line. Under $y < 0$, the normal must be cut off by some portion of the side edge. In order to find the co-ordinates $x_t y_t$, $x_f y_f$ when the sign of y changes from negative to positive, the calculation from Eq.(9) to Eq.(12) must be repeated by increasing the value of i one by one. The normal is cut off by the side edge between these two points. In order to simplify this calculation, u on x co-ordinate



a : Tooth profile seen from back side of rake face

Fig.9 Positional relation between gear space and hob tooth of an intersection of x-axis and the straight line passing through these two points can be assumed in the position where the normal is cut. The u is represented by s (Fig.9) which is the u-component normal to the tooth surface.

$$\left.\begin{array}{l} u = x_t - y_t(x_t - x_f)/(y_t - y_f) \\ s = u\cos[\tan^{-1}(\tan\beta \cdot R/R_o)] \end{array}\right\} \cdots\cdots\cdots (13)$$

A flow-chart for finding the positions of cutting by the side tapered hob is shown in Fig.10. Any point on the gear tooth surface can be expressed by R and h. After the feed f is chosen, s_{min} can be found by the calculations changing the tooth number N. Thus, the tooth surface position s_b before one revolution of gear can be known. Finding the s_{min} under $f = 0$, a tooth surface position s_a generated in this hobbing and its tooth number N_a can be calculated. Next, finding

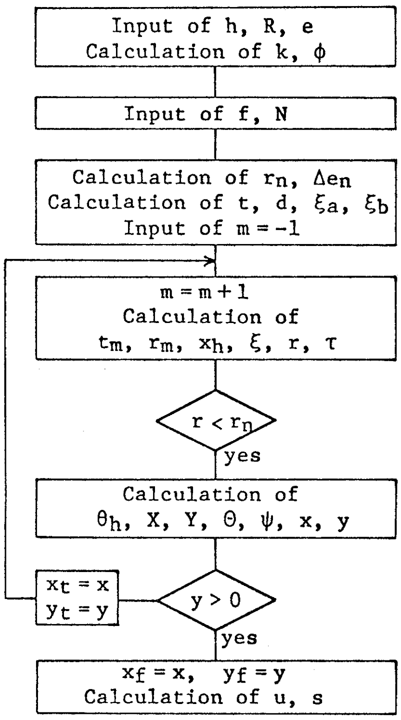


Fig.10 Calculation of cutting position s on normal line

$s_1, s_2, \cdots$ by calculating s in adding 1 to Na, then the cutting thicknesses of the side edge are expressed by $\Delta s_a = s_1 - s_a$, $\Delta s_1 = s_2 - s_1 \cdots$. If s is calculated just before s_b, the last tooth cuts off the thickness $s - s_b$. When the tooth surface is positioned at s_c by premachining, s_b is put as s_c, if s_b is larger than s_c.

The side taper is adopted on the side of the peripheral one, but the peripheral taper is not always necessary in case of a small tooth number gear or a long hob. Therefore setting h and R respectively at a little inside of Z_{max} and R_k and assuming $e = 0$, $(s_b - s_a)$ is calculated on the end tooth on which the cutting load concentrates. This $s_b - s_a$ is important to judge whether the taper is necessary or not. If the side taper is necessary, it is desirable to choose e when the cutting thickness $s_b - s_a$ obtained by increasing e-value gradually from $e = 0$ is an allowable value.

The cutting behavior of the left side edge without a side taper is represented in Table 4(a) where R in the example is set at 2227 mm. The cutting load of length 30 mm and thickness 0.295 mm concentrates on the -18th tooth of the left end. In calculating $s_b - s_a$ increasing the e-value, the -18th tooth does not cut at all at $e = 2.8$ mm. Table 4 (b) shows a cutting behavior calculated under $e = 2.0$ mm (this 2.0 mm is equal to about 70% of $e = 2.8$ mm), where the cutting load is distributed uniformly on each tooth. The concentration recognized on the -10th tooth can be removed by the corner cut method as well as by the peripheral taper.

2.4 Shift and other Remarks

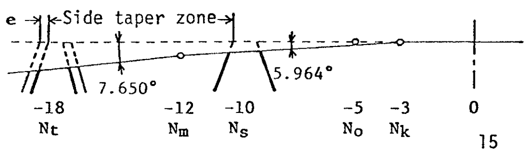
The most suitable shapes and their dimensions are determined by the hobbing condition (* marks in Table 1), so that it is not adequate to use the same hob in other hobbing conditions. The use under the other conditions can be made according to the change of the setting position (shift) and/or the feed. When the tapered hobs are used for hobbing a gear whose number of teeth and helix angle are larger than those under the designed condition, it is a well known practice to shift the hob to its taper side, but its shift length has been chosen by intuition.

In order to find a suitable shift length,

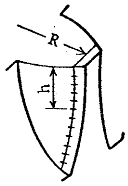
Table 4 Cutting state of side tapered hob

R = 2227 mm

h mm	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
0																				8
5																			32	
10																	1	23		
15																28	14			
20														5	39					
25													22	30						
30												48	6						-10	
35										15	45								↓	15
40									41	28									41	28
45									119				[μm]				8	60		
50									168							34	51			
55									196						59	16				
60								21	192		-18		26	66						
65							41	86	85		↓	51	39							
70					16	61	104	40	↑		78	48								
75	-18			36	80	109			-10		155	7								
80	↓	10	55	96	68						199									
85	29	72	105	25							232									
90	92	138	41								266									
95	153	152									295									



[-18th tooth doesn't cut at e = 2.8 mm]



(b) Side tapered hob (e = 2.0)

(a) No side taper hob (e = 0)

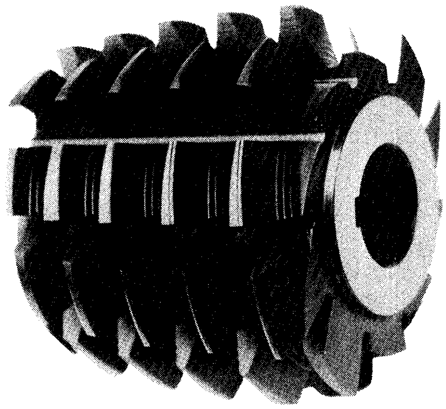
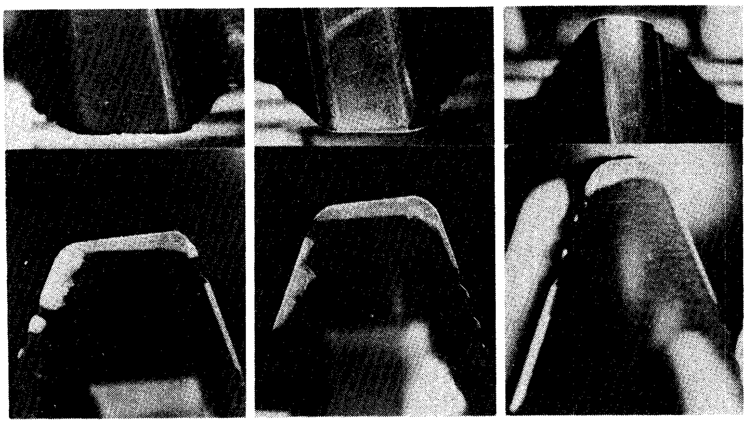


Fig.11 This taper hob is made on trial



-18th tooth -12th tooth -3rd tooth

Fig.12 Cutting allotment and wear on main teeth

the gear items in Table 1 are changed to the new gear ones and the cutting behavior is calculated under new tooth numbers N_o, N_k, N_m, N_t increased or decreased according to the shift direction, where the shift length of one tooth is Δt. Calculate the cutting thicknesses $R_b - R_a$ and $s_b - s_a$ of taper end tooth in shifting one after another tooth number until both $R_b - R_a$ and $s_b - s_a$ become appropriate for the cutting thicknesses of taper end tooth. When the optimum shift length is determined, the cutting behavior will be predictable in detail through Tables 3 and 4 calculated on the shift condition. Two or three spare teeth will be necessary outside the N_oth tooth corresponding to the shift.

The tapered hobs designed in A type can not be used under B type condition. The taper angle ζ of B type hob becomes larger than that of A type one under the same length. It is also necessary for accommodation to the feed change to prepare the hob longer by two or three teeth outside the end tooth. It is important to prevent the corner wear by positioning differently the corner teeth N_k, N_m, N_s of both peripheral taper and side one.

3. Practical hobbing with the trial hob

A hob whose optimum taper is designed in an example, where m 12, Z 320, β 30° reduction gear in a marine turbine is generated with the r_k 105, Z_w 2, L 180 tapered hob, is made on trial (Fig.11). The practical hobbing is done with TOSHIBA 7.5 m hobbing machine (HHA650/750A). This is a roughing hob and has the side edges waved alternately on the addendum or the dedendum. Hob material is equivalent to SKH55 and gear is SCM430(Hs 39). After 42 hours of hobbing under v = 12 m/min and 1.5 mm/rev, the wear of the main teeth is shown in Fig.12.

According to observation of the white areas rubbed by chip flowing on the rake face, the cutting loads concentrate slightly on the corner teeth, the shallow crater wears are recognized on the tooth top and the other teeth share the load moderately. Because the flank wear was within 0.4 mm width, no corner wear was recognized and the vibration was less than 2μm amplitude, the calculated taper was confirmed as a suitable one.

4. Conclusions

This paper, which shows how the cutting load is distributed uniformly on each cutting tooth by the peripheral taper and/or the side taper, can be summarized as follows;

(1) There are two kinds of the cutting types depending on the direction of feed f and setting angle Γ. One is suitable as the taper hob and the other is not suitable, because the taper becomes steep and the hob length changes with feed.

(2) A new design method of determining the suitable shape and dimensions of peripheral taper is shown on a design example.

(3) A side taper method of softening the cutting load concentration on the side edge is proposed and its cutting mechanism is made clear.

(4) Both the reasonableness and the availability of the theories above mentioned are proved through a practical hobbing with a trial hob.

Acknowledgement

We would like to thank AZUMI Co. and TOYO TURBINE Co. for preparing a hob and doing actual hobbing.

Reference

(1) Terashima, K., et al., Bulletin of the JSME (in English), Vol.23, No.180 (1980-6), p.983.