Paper No. 248—24

The Effect of Radiation on Film Boiling Heat Transfer*
(2nd Report, Horizontal Cylinder and Sphere Perpendicular to Upward Vertical Flow)

By Tohru SHIGECHI**, Takehiro ITO***
and Kaneyasu NISHIKAWA****

The effect of radiation on forced convective film boiling heat transfer from a horizontal cylinder and sphere to a subcooled liquid is analyzed by means of the integral method of boundary-layer. Numerical solutions are determined for water under the atmospheric pressure. The effect of radiation on the location of separation and heat transfer is discussed using a new parameter for radiative contribution, and the method proposed by Bromley et al. to estimate the radiation effect on total heat transfer for a horizontal cylinder is also examined.

Key Words : Phase Change, Film Boiling, Radiation, Forced Convection, Subcooled Liquid, Horizontal Cylinder, Sphere

1. Introduction

In the previous report[1], the effect of radiation on forced convective film boiling heat transfer from a vertical plane to a subcooled liquid (water under the atmospheric pressure) was analyzed by the integral method of boundary-layer. The effect of radiation was discussed using the known parameters used in the conventional analyses of film boiling and a new parameter for radiative contribution, and the method proposed by Bromley for saturated pool film boiling was examined.

In this report, the previous analysis[1] for the vertical plane is extended to those for a horizontal cylinder and sphere perpendicular to upward vertical flow. Numerical solutions are determined for water under the standard atmospheric pressure and the effect of radiation is discussed in the same manner as in the first report. The method proposed by Bromley et al.[2] for saturated forced convective film boiling from a horizontal cylinder is also examined.

2. Nomenclature

a : absorptivity of liquid
c_p : specific heat at constant pressure
D : diameter of cylinder or sphere
F_r : Froude number, Eq.(15)
g : acceleration due to gravity
h : total heat transfer coefficient
h_r : radiation heat transfer coefficient
K : density ratio, Eq.(16)
l : latent heat of vaporization
M : dimensionless parameter for radiative contribution, Eq.(20)
P_r : Prandtl number
q_r : local heat flux by radiation
q_φ : local heat flux
q_D : average heat flux
R : $\rho\mu$ ratio, Eq.(17)
r : radius of cylinder or sphere
S_c : dimensionless subcooling, Eq.(19)
S_p : dimensionless superheating, Eq.(18)
T : absolute temperature
T_{sat} : saturation temperature
T_w : temperature of heating surface
T_∞ : bulk temperature of liquid
$T_w - T_{sat}$: degree of superheating
$T_{sat} - T_\infty$: degree of subcooling
u : tangential component of dimensionless velocity
U_∞ : approaching velocity of liquid
y : dimensionless co-ordinate normal to the heating surface
z : dimensionless co-ordinate normal to the vapor-liquid interface
δ : dimensionless thickness of vapor film
Δ : dimensionless thickness of liquid boundary-layer
ε : emissivity of heating surface
φ : angular position measured from the forward stagnation point
φ_s : separation point
λ : thermal conductivity
μ : viscosity
ν : kinematic viscosity
ρ : density
σ : Stefan-Boltzmann constant

Subscripts
1 : with physical dimension
L : liquid
V : vapor
δ : vapor-liquid interface
0 : no radiative contribution

* Received 9th March, 1984.
** Associate Professor, Faculty of Engineering, Nagasaki University (1-14 Bunkyomachi, Nagasaki, Japan)
*** Professor, Faculty of Engineering, Kyushu University (6-10-1 Hakozaki, Higashiku, Fukuoka, Japan)
**** Principal, Kurume Technical College (1232 Komorinomachi, Kurume, Japan)

3. Analysis

3.1 Horizontal cylinder

The film boiling heat transfer from a horizontal cylinder with uniform surface temperature T_w suspended in an upward stream of liquid (Fig.1) is analyzed. The gravitational force g acts in a direction entirely opposite to the approaching velocity of liquid U_∞. It is assumed that the potential flow prevails outside the liquid boundary-layer. The liquid is uniformly subcooled by $(T_{sat}-T_\infty)$ below the saturation temperature T_{sat} corresponding to the system pressure. The fundamental equations are formulated using the same assumptions as those employed in the previous report[1]. They are to be solved by means of the integral method of boundary-layer.

The fundamental equations governing the vapor film thickness δ_1 and the liquid boundary-layer thickness Δ_1 are given by

$$-\lambda_V\frac{\partial T_V}{\partial y_1}\bigg|_{\delta_1}+q_r = l\rho_V\frac{1}{r}\frac{d}{d\varphi}\int_0^{\delta_1}u_{V_1}dy_1-\lambda_L\frac{\partial T_L}{\partial z_1}\bigg|_0 \quad \cdots(1)$$

$$c_{pL}\rho_L\frac{1}{r}\frac{d}{d\varphi}\int_0^{\Delta_1}u_{L_1}(T_L-T_\infty)dz_1 + c_{pL}(T_{sat}-T_\infty)\rho_V\frac{1}{r}\frac{d}{d\varphi}\int_0^{\delta_1}u_{V_1}dy_1 = -\lambda_L\frac{\partial T_L}{\partial z_1}\bigg|_0 \quad \cdots(2)$$

Equation (1) describes energy balance at the vapor-liquid interface, while equation (2) is an integrated form of the energy equation for the liquid boundary-layer. The second term on the left-hand side of Eq.(1) is heat flux due to radiation q_r. Here q_r is defined by

$$q_r\equiv h_r(T_w-T_{sat}) \quad \cdots(3)$$

$$h_r\equiv\{\sigma/(1/\varepsilon+1/a-1)\}\times\{(T_w^4-T_{sat}^4)/(T_w-T_{sat})\} \quad \cdots(4)$$

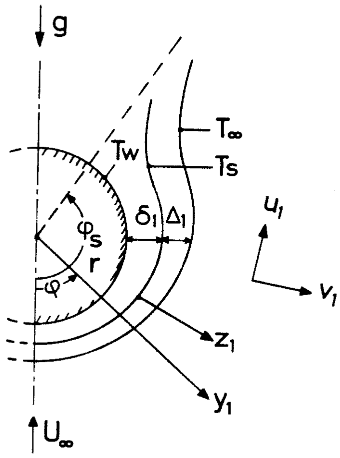


Fig.1 Physical model and co-ordinate system (horizontal cylinder and sphere)

First of all, we must prescribe distribution forms of velocity and temperature to determine δ_1 and Δ_1 in Eqs.(1) and (2).

3.1.1 Distributions of velocities u_{V_1} and temperatures T_V in vapor film

Taking into account the pressure gradient transmitted through liquid boundary-layer from the free stream (potential flow) and dropping the inertia term, we can reduce the momentum equation to $(\rho_L-\rho_V\approx\rho_L)$,

$$\mu_V\frac{\partial^2 u_{V_1}}{\partial y_1^2}+g\rho_L\sin\varphi+2\rho_L\frac{U_\infty^2}{r}\sin 2\varphi=0 \quad \cdots(5)$$

Integrating the above equation under the conditions $u_{V_1}=0$ at $y_1=0$ and $u_{V_1}=u_{\delta_1}$ at $y_1=\delta_1$, we obtain

$$u_{V_1}=u_{\delta_1}(y_1/\delta_1)+\delta_1^2(g\rho_L/2\mu_V)\{1+4(U_\infty^2/gr)\cos\varphi\}\sin\varphi\times\{(y_1/\delta_1)-(y_1/\delta_1)^2\} \quad \cdots(6)$$

while integrating the energy equation with neglected convection term

$$\frac{\partial^2 T_V}{\partial y_1^2}=0 \quad \cdots(7)$$

under the conditions $T_V=T_w$ (constant) at $y_1=0$ and $T_V=T_{sat}$ at $y_1=\delta_1$, we arrive at

$$T_V=T_w-(T_w-T_{sat})(y_1/\delta_1) \quad \cdots(8)$$

3.1.2 Distributions of velocities u_{L_1} and temperatures T_L in liquid boundary-layer

Assuming quadratic functions of z_1 for u_{L_1} and T_L, and then determining the coefficients of these polynomials in compliance with the conditions $u_{L_1}=u_{\delta_1}$ at $z_1=0$ and $u_{L_1}=2U_\infty\sin\varphi$, $(\partial u_{L_1}/\partial z_1)_{\Delta_1}=0$ at $z_1=\Delta_1$ for u_{L_1}, and $T_L=T_{sat}$ at $z_1=0$ and $T_L=T_\infty$, $(\partial T_L/\partial z_1)_{\Delta_1}=0$ at $z_1=\Delta_1$ for T_L, we establish

$$u_{L_1}=2U_\infty\sin\varphi-(2U_\infty\sin\varphi-u_{\delta_1})\times\{1-(z_1/\Delta_1)^2\} \quad \cdots(9)$$

$$T_L=T_\infty-(T_\infty-T_{sat})\{1-(z_1/\Delta_1)^2\} \quad \cdots(10)$$

From the compatibility condition for shearing stress at the vapor-liquid interface,

$$y_1=\delta_1:\mu_V\frac{\partial u_{V_1}}{\partial y_1}\bigg|_{\delta_1}=\mu_L\frac{\partial u_{L_1}}{\partial z_1}\bigg|_0 \quad \cdots(11)$$

a functional relation among u_{δ_1}, δ_1 and Δ_1 is deduced

$$u_{\delta_1}=\frac{\{\delta_1\Delta_1(g\rho_L/2\mu_V)\{1+4(U_\infty^2/gr)\cos\varphi\} + 4(\mu_L/\mu_V)U_\infty\}\sin\varphi}{(\Delta_1/\delta_1)+2(\mu_L/\mu_V)} \quad \cdots(12)$$

Substituting u_{V_1}, T_V, u_{L_1} and T_L into Eqs.(1) and (2) and eliminating u_{s_1} by Eq.(12), and introducing dimensionless variables and parameters defined by Eqs.(13) through (20), we finally arrive at the following simultaneous ordinary differential equations, Eqs.(21) and (22)

$$\delta \equiv \delta_1 (g/\nu_V^2 r)^{1/4} \quad \cdots (13)$$

$$\Delta \equiv \Delta_1 (g/\nu_V^2 r)^{1/4} (\mu_V/\mu_L) \quad \cdots (14)$$

$$F_r \equiv U_\infty^2/gD \quad \cdots (15)$$

$$K \equiv \rho_L/\rho_V \quad \cdots (16)$$

$$R \equiv (\rho_V \mu_V/\rho_L \mu_L)^{1/2} \quad \cdots (17)$$

$$S_p \equiv c_{p_V}(T_w - T_{sat})/P_{r_V} l \quad \cdots (18)$$

$$S_c \equiv c_{pL}(T_{sat} - T_\infty)/P_{r_L} l \quad \cdots (19)$$

$$M \equiv (h_r/\lambda_V)(\nu_V^2 r/g)^{1/4} \quad \cdots (20)$$

$$\delta^2 \Delta(\delta+\Delta)\{K(1+8F_r\cos\varphi)\delta(\delta+\Delta) + 4(2F_r)^{1/2}\}\sin\varphi\frac{d\delta}{d\varphi} + \frac{1}{2}\delta^3\Delta\{K(1 + 8F_r\cos\varphi)\delta^2 - 4(2F_r)^{1/2}\}\sin\varphi\frac{d\Delta}{d\varphi}$$
$$= \{(1+M\delta)\Delta S_p - 2\delta S_c\}(2\delta+\Delta)^2 - \delta^3\Delta(2\delta+\Delta)\left\{\frac{1}{6}K(\cos\varphi + 8F_r\cos 2\varphi) \times \delta(\delta+2\Delta) + 2(2F_r)^{1/2}\cos\varphi\right\} \quad \cdots (21)$$

$$\delta\Delta^3\{K(1+8F_r\cos\varphi)\delta(\delta+\Delta) + 4(2F_r)^{1/2}\}\sin\varphi\frac{d\delta}{d\varphi} + \frac{1}{2}\delta\Delta\{K(1+8F_r\cos\varphi)\delta^2\Delta(4\delta+\Delta) + \frac{8}{3}(2F_r)^{1/2}(10\delta^2 + 4\delta\Delta + \Delta^2)\}\sin\varphi\frac{d\Delta}{d\varphi}$$
$$= -\frac{1}{2}\delta^3\Delta^3(2\delta+\Delta)K(\cos\varphi + 8F_r\cos 2\varphi) - \frac{4}{3}\delta\Delta^3(2\delta+\Delta)(5\delta+\Delta)(2F_r)^{1/2}\cos\varphi + 5(R^2/P_{r_L})\{2(1+S_c P_{r_L})\delta - (1+M\delta)\Delta S_p P_{r_L}\}(2\delta+\Delta)^2 \quad \cdots (22)$$

Solving the equations (21) and (22) for a prescribed set of parameters, Froude number F_r, density ratio K, radiation parameter M (note that M is different from N defined for the vertical plate in the previous report [1]), liquid Prandtl number P_{r_L}, $\rho\mu$ ratio R, dimensionless subcooling S_c and dimensionless superheating S_p, local heat flux q_φ and average heat flux q_D can be calculated as follows.

$$q_\varphi = -\lambda_V \frac{\partial T_V}{\partial y_1}\bigg|_0 + h_r(T_w - T_{sat}) = (1/\delta + M)\lambda_V (g/\nu_V^2 r)^{1/4}(T_w - T_{sat}) \quad \cdots (23)$$

$$q_D = \frac{1}{\pi}\int_0^\pi q_\varphi d\varphi = \left\{\frac{1}{\pi}\int_0^\pi (1/\delta + M)d\varphi\right\} \times \lambda_V (g/\nu_V^2 r)^{1/4}(T_w - T_{sat}) \quad \cdots (24)$$

Since the simultaneous differential equations, Eqs.(21) and (22), have a singular point at $\varphi = 0$, the forward-integration scheme with the conditions of $(d\delta/d\varphi) = 0$ and $(d\Delta/d\varphi) = 0$ at $\varphi = 0$ does not work there. To overcome this difficulty, we expand δ and Δ such that $\delta = a_0 + a_2\varphi^2$ and $\Delta = b_0 + b_2\varphi^2$ in the neighborhood of $\varphi = 0$. The coefficients a_0, a_2, b_0 and b_2 are determined by substituting these expressions for δ and Δ into Eqs.(21) and (22). These polynomial solutions were employed up to φ of 0.1 or 0.2 rad. Starting therefrom, Eqs.(21) and (22) are integrated numerically by Runge-Kutta-Gill method up to the separation point of boundary-layer. When the boundary-layer separates, the mechanism of heat transfer in the entire downstream region of the separation point is assumed to be due to radiation only. For this situation, equation (24) reduces to

$$q_D = \frac{1}{\pi}\left\{\int_0^{\varphi_s}(1/\delta)d\varphi + \pi M\right\} \times \lambda_V (g/\nu_V^2 r)^{1/4}(T_w - T_{sat}) \quad \cdots (25)$$

where φ_s is the location of separation point.

3.2 Sphere

For spheres, a similar analysis to that for a horizontal cylinder was performed on a physical model like Fig.1. In the analysis, the geometrical symmetry, potential flow for general flow $(3/2)\ U_\infty \sin\varphi$ and so forth were taken into consideration. Dimensionless variables and parameteres for the sphere are the same as those defined by Eq.(13) through (20), if r and D are the radius and diameter of the sphere, respectively.

4. Results and Discussion

Numerical calculations were carried out for water under the standard atmospheric pressure (0.1013 MPa). The ranges of calculations for dimensionless parameters are as follows

$K = 1600$, $R = 0.0051$, $P_{r_L} = 1.76$

$S_p = 0.25,\ 0.5,\ 1.0$

$S_c = 0,\ 0.005,\ 0.01,\ 0.02,\ 0.04$

$M = 0.5,\ 1.0,\ 2.0,\ 4.0,\ 8.0$

In addition, the magnitude of radiation parameter M is shown in Table 1 for water at the atmospheric pressure.

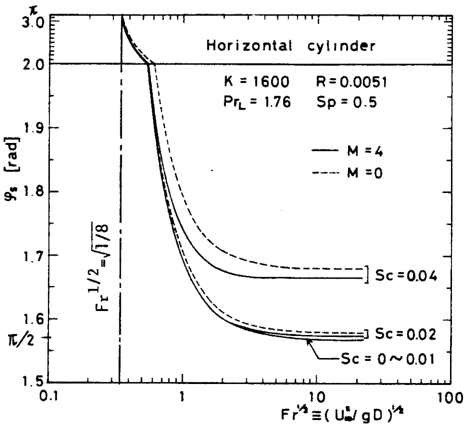
Sp	Tw-Tsat [K]	M(1/ε+1/a-1)		
		D [mm]		
		5	10	20
0.25	280	0.772	0.918	1.09
0.50	560	1.75	2.08	2.48
1.0	1120	5.87	6.98	8.30

Table 1 The magnitude of radiation parameter M for water at the atmospheric pressure

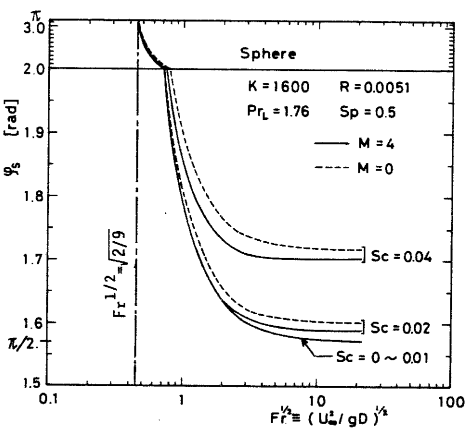
4.1 Separation point

Since the flow outside the liquid boundary-layer is assumed to be a potential one in the present analysis, the vapor flow shall separate (i) when $F_r>1/8$ for the horizontal cylinder and (ii) when $F_r>2/9$ for the sphere. The effects of approaching velocity U_∞ and the degree of subcooling $(T_{sat}-T_\infty)$ on the location of separation were already discussed in the previous reports[3][4]. So, the effect of radiation only will be examined.

The relation between the location of separation φ_s and Froude number F_r is shown in Figs.2(a) and (b) with dimensionless subcooling S_c as a parameter. In these figures, dashed curves indicate purely convective heat transfer without radiation ($M=0$). From these figures, the location of separation φ_s seems to be practically independent of radiation parameter M in the range of $S_c=0\sim0.01$ for the horizontal cylinder and sphere. For $S_c=0.02$ and 0.04, the effect of S_c on φ_s is appreciable and the point of separation φ_s moves toward the forward stagnation point when M increases. That is, when there exists a substantial radiative heat transfer, the separation occurs at smaller φ, unlike that for purely convective heat transfer without radiation. This is due to the increasing thickness of the vapor film. This effect is more remarkable with larger subcooling, i.e., for thinner vapor film. However, the effect is relatively small in the range of parameters in this report.



(a) horizontal cylinder



(b) sphere

Fig.2 The effect of radiation parameter M on separation point

4.2 Heat transfer

The effect of radiation on film boiling heat transfer will be discussed in terms of $(h-h_0)/h_r$ (h: total heat transfer coefficient, h_0: purely convective heat transfer coefficient without radiation) in the same manner as for the vertical plane in the previous report[1]. $(h-h_0)/h_r$ at the forward stagnation point ($\varphi=0$) is local value:

$$\{(h-h_0)/h_r\}_{\varphi=0}=(1/\delta_{\varphi=0}+M-1/\delta_{0\varphi=0})/M$$
$$=(1/a_0+M-1/a_{0\,M=0})/M \quad\cdots\cdots(26)$$

for the horizontal cylinder and sphere. The average values between $\varphi=0$ and $\varphi=\pi$ are for horizontal cylinder:

$$\{(h-h_0)/h_r\}_m=\frac{1}{\pi M}\left\{\int_0^{\varphi_s}\frac{1}{\delta}d\varphi+\pi M-\int_0^{\varphi_{s0}}\frac{1}{\delta_0}d\varphi\right\}\quad\cdots\cdots(27)$$

and for sphere:

$$\{(h-h_0)/h_r\}_m=\frac{1}{2M}\left\{\int_0^{\varphi_s}\frac{1}{\delta}\sin\varphi\,d\varphi+2M-\int_0^{\varphi_{s0}}\frac{1}{\delta_0}\sin\varphi d\varphi\right\}\quad\cdots\cdots(28)$$

where $a_{0\,M=0}$, δ_0 and φ_{s0} are the pure convection counterparts of a_0, δ and φ_s, respectively.

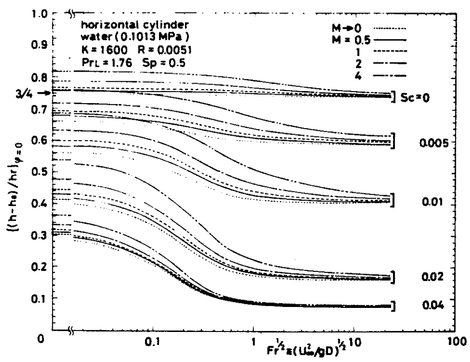
In addition, the asymptotic value of $(h-h_0)/h_r$ as $M\to0$, i.e., $\lim_{M\to0}\{(h-h_0)/h_r\}$ can be evaluated only for the case of local value by taking the limit on the right-hand side of Eq.(26).

The heat transfer results for the horizontal cylinder are shown in Figs.3 (a) and (b), where $\{(h-h_0)/h_r\}_{\varphi=0}$ or $\{(h-h_0)/h_r\}_m$ is taken as the ordinate and square root of $F_r(\equiv U_\infty^2/gD)$ as the abscissa. In these figures, the region of larger values of $F_r^{1/2}$ on the abscissa corresponds to the situation where the forced convection predominates, whereas the region of smaller values of $F_r^{1/2}$ does to that where the body force predominates (natural convection). The values of $(h-h_0)/h_r$ for $F_r^{1/2}=0$, i.e., for the case of pure natural convection, are indicated by horizontal line segments on the left-hand end in Fig.3. In Fig.3 (a), $M\to0$ indicates the values of above mentioned $\lim_{M\to0}\{(h-h_0)/h_r\}_{\varphi=0}$. It is seen that $(h-h_0)/h_r$ at the forward stagnation point becomes small for (i) large F_r, (ii) large S_c, and (iii) small M with the other parameters fixed constant. This means that the effect of radiation is small for thin vapor film. Furthermore, $\{(h-h_0)/h_r\}_{\varphi=0}$, i.e., the effect of radiation is small in the region of predominating forced convection when degree of subcooling S_c is large. Incidentally, the ordinate approaches a constant value in the limit of $M \to 0$ (see Fig.3) when $S_c=0$ and $F_r=0$. It is exactly 3/4.

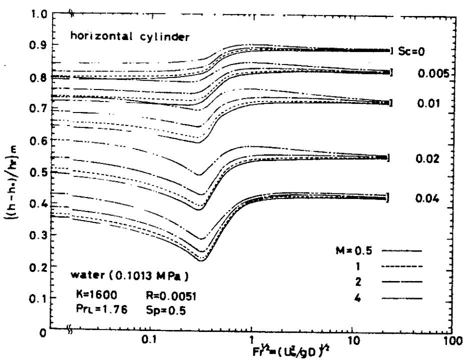
For the average value $\{(h-h_0)/h_r\}_m$, it is seen from Fig.3(b) that the curves do not monotonously decrease with F_r and that they behave abnormally near $F_r=1/8$ ($\sqrt{F_r}=0.35$). This is because the convective contribution was neglected in the region downstream of the separation point in calculating the average values of heat transfer coefficient. When the average values are compared with the local one (at the forward stagnation point), the former is always larger than the latter. Numerical values of the average heat transfer coefficients for $S_p=0.25$ and $S_p=1.0$ are shown in Figs.4 (a) and (b).

The results for a sphere are similar to those for a horizontal cylinder. Numerical values of the average heat transfer coefficients are shown in Figs.5 (a), (b) and (c).

Bromley et al.[2] calculated the total heat transfer coefficient h for forced convective film boiling from a horizontal cylinder to a saturated liquid in the following manner. Heat transfer coefficients in the region between forward stagnation point ($\varphi=0$) and separation point ($\varphi=\varphi_s$) are given by Bromley's procedure for saturated pool film boiling [5]



(a) local value(at the stagnation point)



(b) average value

Fig.3 The effects of Froude number F_r, dimensionless subcooling S_c and radiation parameter M (horizontal cylinder)

$$h=h_0+(3/4)h_r \quad \cdots\cdots (29)$$

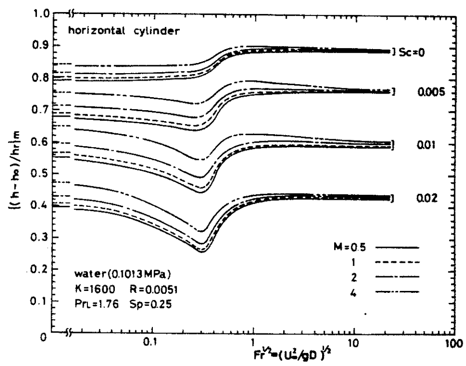
and in the region downstream of the separation point only by radiation. Thus, they proposed the following expression

$$h=h_0+(1-\varphi_s/4\pi)h_r \quad \cdots\cdots (30)$$

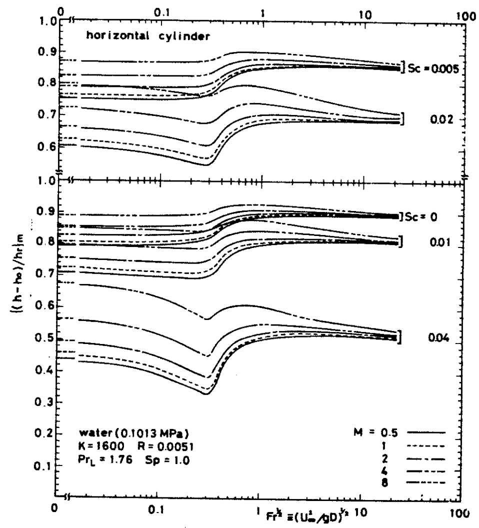
Since Eq.(30) was obtained for the average value, then, Eq.(30) is rewritten

$$\{(h-h_0)/h_r\}_m=1-\varphi_s/4\pi \quad \cdots\cdots (31)$$

Figure 6 shows the comparison between the values calculated by Eq.(31) and those obtained in the present analysis for the horizontal cylinder. Since Eq.(31) was derived using Eq.(29), it should be applied to the case of small M. Figure 6 is for $M=0.5$ and the locations of separation in Eq.(31) are those obtained in the present analysis for $S_c=0\sim0.04$.

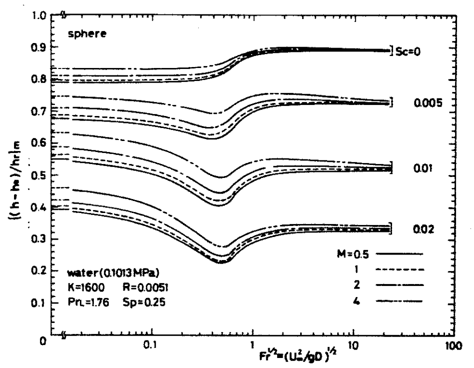


(a) Sp=0.25

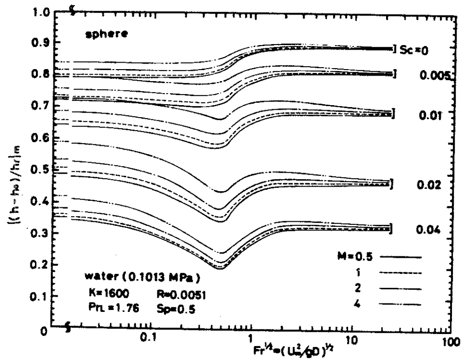


(b) Sp=1.0

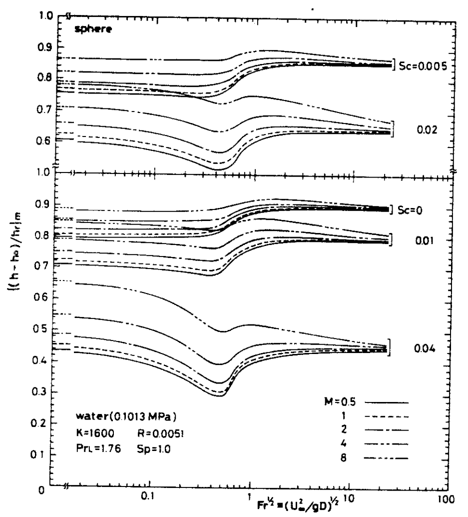
Fig.4 The effects of Froude number F_r, dimensionless subcooling S_c and radiation parameter M (horizontal cylinder, average value)



(a) Sp=0.25



(b) Sp=0.5



(c) Sp=1.0

Fig.5 The effects of Froude number F_r, dimensionless subcooling S_c and radiation parameter M (sphere, average value)

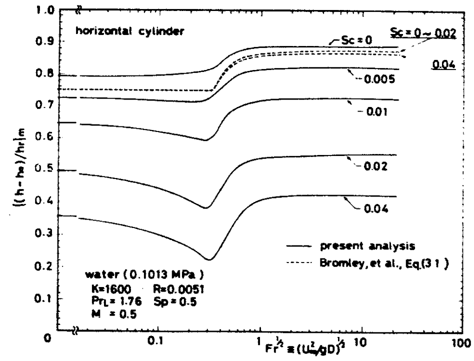


Fig.6 The comparison between present analysis and method of Bromley et al. (horizontal cylinder)

For $S_c=0$ the ordinate calculated by the method of Bromley et al.[(Eq.(31)] is smaller a little than that obtained in the present analysis. Both results show qualitatively a similar tendency. With an increasing S_c, the ordinate by the present analysis becomes smaller than those for $S_c=0$, whereas those by Bromley et al. are insensitive to S_c. This is caused by the fact that the location of separation φ_s in Eq.(31) varies only a little with S_c as seen from Fig.2 (a). This means that the method proposed by Bromley et al. overestimates the effect of radiation for subcooled liquids.

5. Conclusions

The effect of radiation on forced convective film boiling heat transfer from a horizontal cylinder and sphere to a subcooled liquid was analyzed and the validity of the method [Eq.(31)] proposed by Bromley et al. for saturated forced convective film boiling from a horizontal cylinder was tested.

The authors wish to thank Messrs. K. Kunihiro and T. Matsunaga who eagerly carried out this work as their graduation theses in Nagasaki University.

Referenecs

(1) Shigechi,T. et al., Bull.JSME, Vol.27, No.227(1984), p.939.
(2) Bromley,L.A. et al., Ind.Eng.Chem., Vol.45, No.12(1953), p.2639.
(3) Ito,T. et al., Bull.JSME, Vol.24, No.198(1981), p.2107.
(4) Shigechi,T. et al., Bull.JSME, Vol.26, No.214(1983), p.554.
(5) Bromley,L.A., Chem.Eng.Prog., Vol.46, No.5(1950), p.221.