Effects of viscous dissipation and fluid axial heat conduction on laminar heat transfer in ducts with constant wall temperature (Part II: Circular pipes)

by

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The problem of heat transfer for non-Newtonian fully developed laminar flow has been solved for circular ducts with constant wall temperature. The effects of viscous dissipation and fluid axial heat conduction were taken into account and a numerical scheme based on the finite difference method was applied to solve the governing elliptic type energy equation. The solution yielded the temperature distribution in the fluid flow for an infinite axial domain of $-\infty < z < \infty$ and the effects of Brinkman number and Peclet number on developing temperature distribution and Nusselt number at the wall are discussed.

1. Introduction

This is an extension of the previous work[1] which considered the heat transfer in parallel-plates ducts of constant wall temperatures. The problem of entrance region heat transfer with viscous dissipation and fluid axial heat conduction has been studied for circular pipes at constant wall temperature. The present work deals with the steady heat transfer for laminar flow of non-Newtonian fluids and the fluid is assumed to obey the power-law model.

In this paper, various relevant results and figures have been discussed mainly from the point of view on the validity of the numerical scheme, while the effects of viscous dissipation and fluid axial heat conduction on the heat transfer are also discussed. The results of the present study are presented in form of graphs and in order to ascertain the accuracy of the finite difference scheme, our results are compared with those reported in tabular forms by the previous researchers[2]–[7]. A literature survey revealed that there is abundant information on heat transfer analyses for ducts at uniform wall temperature.

Dang[8] has solved the present problem by applying two semi-infinite regions of $z \leq 0$ and $z \geq 0$ and the solutions for the two domains were matched at the origin $z = 0$. In his study the viscous dissipation was considered only in the region of $z \geq 0$. The fully developed Nusselt numbers for various non-Newtonian power-law fluids have been derived for the cases of non-zero Brinkman number. Hennecke[2] analyzed thermally developing flow of Newtonian fluids in a tube and presented Nusselt number for different Peclet values. In his study the viscous dissipation was neglected. Singh[3] reported Nusselt number and bulk temperature of a Newtonian fluid flowing in tubes for $Pe = 50$. In his study the viscous dissipation effect was also neglected. For the laminar heat transfer of a Bingham plastic in a circular pipe with uniform wall temperature, Min[9] has obtained a correlation formula between Nusselt number and Peclet number in thermally developed region and investigated the effects of viscous dissipation and fluid axial heat conduction in the thermally developing region. Olek[4] has studied developing heat transfer to laminar non-Newtonian fluids in circular and parallel-plates ducts including axial heat conduction, for the cases of (1) one wall insulated and the other with heat convection and (2) constant temperature at the walls. For the negligible fluid axial heat conduction case, the results of Nusselt number and bulk temperature were tabulated for the flow index, $n = 1/3$, 1 and 3, and compared with those of other researchers. He also

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compared his results of Nusselt numbers for $Pe = 50$ and $n = 1$ with those of by Singh[3]. Zanchini[10] has studied the effect of viscous dissipation on the asymptotic behavior of laminar forced convection in circular tubes under the assumption of the negligible fluid axial heat conduction. He reported the analytic expression of the asymptotic temperature profile for Newtonian fluids. Laminar heat transfer of Newtonian fluids in tube by considering the viscous dissipation effect was studied by Basu and Roy[11] for constant wall temperature case and constant wall heat flux case. They showed for the constant wall temperature case that the asymptotic Nusselt number attains the value 9.6 if Brinkman is other than zero. Viscous dissipation effects on laminar heat transfer in cylindrical pipes with constant wall temperature have been studied also by Manglik and Prusa[6], Barletta and Zanchini[12], Lin[13] and others by neglecting the fluid axial heat conduction effects.

Nomenclature

Br	: Brinkman number
c_p	: specific heat at constant pressure
D_h	: hydraulic diameter ($= 2R$)
f	: friction factor
k	: thermal conductivity
n	: flow index
m	: fluid consistency index
Nu	: Nusselt number
R	: radius of the tube
Pe	: Peclet number
r	: radial coordinate
r^*	: dimensionless radial coordinate
T	: temperature
u	: fully developed velocity profile
u_m	: fluid average velocity ($= \frac{2}{R^2}\int_0^R urdr$)
u^*	: dimensionless velocity ($=u/u_\mathrm{m}$)
z	: axial coordinate
z^*	: dimensionless axial coordinate

Greek Symbols

ρ	: density
τ	: shear stress
θ	: dimensionless temperature

Subscripts

b	: bulk
e	: entrance or inlet
fd	: fully developed
w	: wall

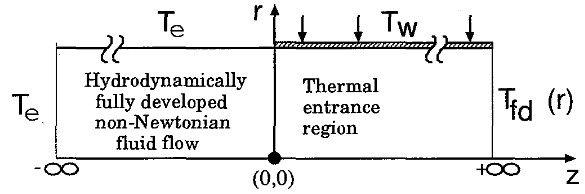

Fig.1 Geometrical configuration

2. Analysis

The geometry of the problem and the coordinate system for the analysis is shown in Fig.1. The assumptions and conditions used in the analysis are:

- The flow is steady, laminar and fully developed hydrodynamically.
- The fluid is non-Newtonian with constant physical properties. The shear stress may be discribed by the power-law model.
- The body forces are neglected.
- The entering fluid temperature, T_e, is constant at upstream infinity ($z \to -\infty$).
- There is a step change in the wall temperature at $z = 0$. For $z \leq 0$ the wall is kept at T_e. For $0 < z$, the wall is at a constant temperature T_w.

Fluid Flow

With the assumptions described above, the governing momentum equation with the non-slip condition is

$$\frac{1}{r}\frac{d}{dr}(r\tau) = -\frac{dP}{dz}. \tag{1}$$

$$\text{B.C.}: \begin{cases} \dfrac{du}{dr} = 0 \quad \text{at} \quad r = 0 \\ u = 0 \quad \text{at} \quad r = R. \end{cases} \tag{2}$$

The shear stress, τ, in Eq.(1) is

$$\tau = -m\left|\frac{du}{dr}\right|^{n-1}\frac{du}{dr} \tag{3}$$

By solving the momentum equation for the power-law fluids, the flow velocity is obtained as

$$u = \frac{n}{n+1}\left[\frac{1}{2m}\left(-\frac{dP}{dz}\right)\right]^{\frac{1}{n}}\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \tag{4}$$

Then the average velocity of the flow is

$$u_{\mathrm{m}} \equiv \frac{2}{R^2}\int_0^R u\ r\ dr$$

$$= \frac{n}{3n+1}\left[\frac{1}{2m}\left(-\frac{dP}{dz}\right)\right]^{\frac{1}{n}} R^{\frac{n+1}{n}} \tag{5}$$

Introducing the following dimensionless parameters

$$u^* = \frac{u}{u_{\mathrm{m}}}, \qquad r^* = \frac{r}{D_{\mathrm{h}}} \tag{6}$$

yields the exact solution for the velocity as

$$u^* = \frac{3n+1}{n+1}\left[1-(2r^*)^{\frac{n+1}{n}}\right] \tag{7}$$

Heat Transfer

The energy equation together with the assumptions above is written as

$$\rho c_{\mathrm{p}} u\frac{\partial T}{\partial z} = k\left[\frac{1}{r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right] - \tau\frac{du}{dr} \tag{8}$$

$$\text{in} \quad 0<r<R \quad \text{and} \quad -\infty<z<\infty$$

The boundary conditions are:

$$\begin{cases} \dfrac{\partial T}{\partial r}=0 & \text{at} \quad r=0 & \text{for} \quad -\infty<z<\infty \\ T=T_w & \text{at} \quad r=R & \text{for} \quad 0<z \\ T=T_e & \text{at} \quad r=R & \text{for} \quad z\le 0 \\ \lim\limits_{z\to-\infty} & T=T_e & \text{for} \quad 0<r<R \\ \lim\limits_{z\to+\infty} & T=T_{fd}(r) & \text{for} \quad 0<r<R. \end{cases} \tag{9}$$

The bulk temperature and Nusselt number are defined as

$$T_{\mathrm{b}} \equiv \frac{\int_0^R uTrdr}{\int_0^R urdr} \tag{10}$$

$$Nu \equiv \frac{hD_{\mathrm{h}}}{k} \tag{11}$$

where

$$h = \frac{q_{\mathrm{w}}}{T_{\mathrm{w}}-T_{\mathrm{b}}}, \qquad q_w = k\left.\frac{\partial T}{\partial r}\right|_{r=R} \tag{12}$$

The following dimensionless quantities are introduced

$$z^* = z/(Pe{\cdot}D_{\mathrm{h}}) \tag{13}$$

$$Pe = \rho c_{\mathrm{p}} u_{\mathrm{m}} D_{\mathrm{h}}/k \tag{14}$$

$$\theta = \frac{T-T_e}{T_w-T_e} \tag{15}$$

$$Br = \frac{m\ u_m^{n+1}\ D_{\mathrm{h}}^{1-n}}{k(T_w-T_e)} \tag{16}$$

The substitution of the above quantities into the dimensional formulation gives

$$u^*\frac{\partial\theta}{\partial z^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial\theta}{\partial r^*}\right)+\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial z^{*2}}$$

$$+ Br\left(-\frac{du^*}{dr^*}\right)^{n+1} \tag{17}$$

$$\text{in} \quad 0<r^*<\frac{1}{2} \quad \text{and} \quad -\infty<z^*<\infty$$

$$\begin{cases} \dfrac{\partial\theta}{\partial r^*}=0 & \text{at} \quad r^*=0 & \text{for} \quad -\infty<z^*<\infty \\ \theta=1 & \text{at} \quad r^*=\dfrac{1}{2} & \text{for} \quad 0<z^* \\ \theta=0 & \text{at} \quad r^*=\dfrac{1}{2} & \text{for} \quad z^*\le 0 \\ \lim\limits_{z^*\to-\infty} & \theta=0 & \text{for} \quad 0<r^*<\dfrac{1}{2} \\ \lim\limits_{z^*\to+\infty} & \theta=\theta_{fd}(r^*) & \text{for} \quad 0<r^*<\dfrac{1}{2}. \end{cases} \tag{18}$$

The bulk temperature in the dimensionless form is calculated as

$$\theta_{\mathrm{b}} \equiv 8\int_0^{1/2} u^*\ \theta\ r^*\ dr^* \tag{19}$$

Nusselt number at the wall is

$$Nu = \frac{1}{(1-\theta_{\mathrm{b}})}\left.\frac{\partial\theta}{\partial r^*}\right|_{r^*=1/2} \tag{20}$$

In the fully developed region the dimensionless temperature is a function of r^* alone. Then the dimensionless temperature θ_{fd} corresponding to the boundary condition of constant wall temperature is the particular solution of the following equation.

$$\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d\theta_{fd}}{dr^*}\right) = -Br\left(-\frac{du^*}{dr^*}\right)^{n+1} \tag{21}$$

$$\begin{cases} \dfrac{d\theta_{fd}}{dr^*} = 0 & \text{at} \quad r^* = 0 \\ \theta_{fd} = 1 & \text{at} \quad r^* = \dfrac{1}{2} \end{cases} \tag{22}$$

In the fully developed region, the temperature gradient is

$$\frac{d\theta_{fd}}{dr} = -Br\, 2^{\frac{(n+1)^2}{n}} \left(\frac{3n+1}{n}\right)^n r^{*\frac{2n+1}{n}} \tag{23}$$

The solution for θ_{fd} is

$$\theta_{fd} = 1 + 2^{\frac{(n+1)^2}{n}} \left(\frac{n}{3n+1}\right)^{1-n} \times Br \left(2^{-\frac{3n+1}{n}} - r^{*\frac{3n+1}{n}}\right) \tag{24}$$

The bulk temperature in the fully developed region is

$$\theta_{b_{fd}} = 1 + Br \left(\frac{2}{n}\right)^{n-1} \frac{(4n+1)(3n+1)^{n-1}}{(5n+1)} \tag{25}$$

The asymptotic values of Nusselt number for the different fluid behaviours are calculated as

$$Nu_{fd} = \frac{2(3n+1)(5n+1)}{n(4n+1)} \tag{26}$$

for non-zero Brinkman numbers. For $Br = 0$, the value of Nu_{fd} cannot be determined as

$$\frac{d\theta_{fd}}{dr^*} = 0, \quad \theta_{fd} = 1 \quad \text{and} \quad \theta_{b_{fd}} = 1$$

from Eqs. (23) - (25).

3. Results and Discussion

The temperature distribution of the non-Newtonian power-law fluids flowing in a circular pipe was calculated for an axial domain of $-\infty < z < \infty$, where at the origin ($z = 0$) there is a step jump in the wall temperature. The calculation has been carried out by using the finite difference method. The range of parameters considered are:

Brinkman number: -1, -0.5, -0.1, 0.0, 0.1, 0.5, 1

Peclet number: ∞, 100, 50, 20, 10, 5, 2

Flow index: 1, 1/3, 0.5, 1.5 and 3.

The computed results are shown graphically in the following figures and the main features are discussed. Typical developing temperature profiles are given in Fig. 2 to show the effects of viscous dissipation and fluid axial heat conduction. The two figures compare the case of negligible viscous dissipation and fluid axial heat conduction with the case of considerable viscous dissipation and fluid axial heat conduction. The curves in Fig. 2(a) illustrate the development of local temperature profile for the case of negligible viscous dissipation and fluid axial heat conduction or for $Pe \to \infty$ and $Br = 0$. The circles show the results by Olek[4]. Figure 2.(b) displays the temperature profile development for the case of considerable viscous dissipation and fluid axial heat conduction or for $Pe = 10$ and $Br = 0.1$. At $z^* = 0$, there is a step change in the wall temperature. The solid lines in Fig. 2(b) correspond to the axial locations listed in Fig. 2(a). The dashed lines are for the temperature profiles at $z^* \le 0$, where the wall temperature is kept equal to the entering fluid temperature. By comparing the temperature development in Figs. 2(a) and 2(b), it is seen that the fluid temperature increases due to fluid axial heat conduction and viscous dissipation before the fluid enters into the region of $z^* > 0$ or into the heated wall region.

In the following figures the heat transfer results are illustrated in terms of the conventional Nusselt number at the wall. Figure 3 presents the results in the thermally developing range for the three different fluids. It is worthwhile to compare the present results with those reported by Blackwell[5] and by Prusa and Manglik[6] for the limiting case of neglected viscous dissipation and fluid axial heat conduction for the power-law fluids. Even at small values of z^*, the agreement is excellent.

In Fig. 4, the Nusselt number is shown as a function of the axial coordinate with Peclet number as a parameter. These Nusselt curves are for the case of negligible viscous dissipation and for Newtonian fluids. The circles show results by Hennecke[2] and the triangles are for the results of Singh[3]. It is also seen that the agreement is good.

The effects of both Peclet number and Brinkman number on the Nusselt number are demonstrated in Figs. 5 - 7 for Newtonian, pseudoplastic and dilatant fluids. The solid lines stand for the case of negligible viscous dissipation. The dashed lines are for the heat transfer with considerable viscous dissipation. The case with $Br = 0$ and $Pe \to \infty$ is the limiting case of neglected viscous dissipation and axial heat conduction. It is worthwhile to compare the results for this particular case with those reported by Lawal and Mujumdar[7] whose predictions were considering the effect of viscous dissipation for Newtonian fluids. However, in their studies the domain of $-\infty < z < 0$ was not considered as the fluid axial heat conduction was assumed to be negligible. Therefore their results for non-zero Brinkman in

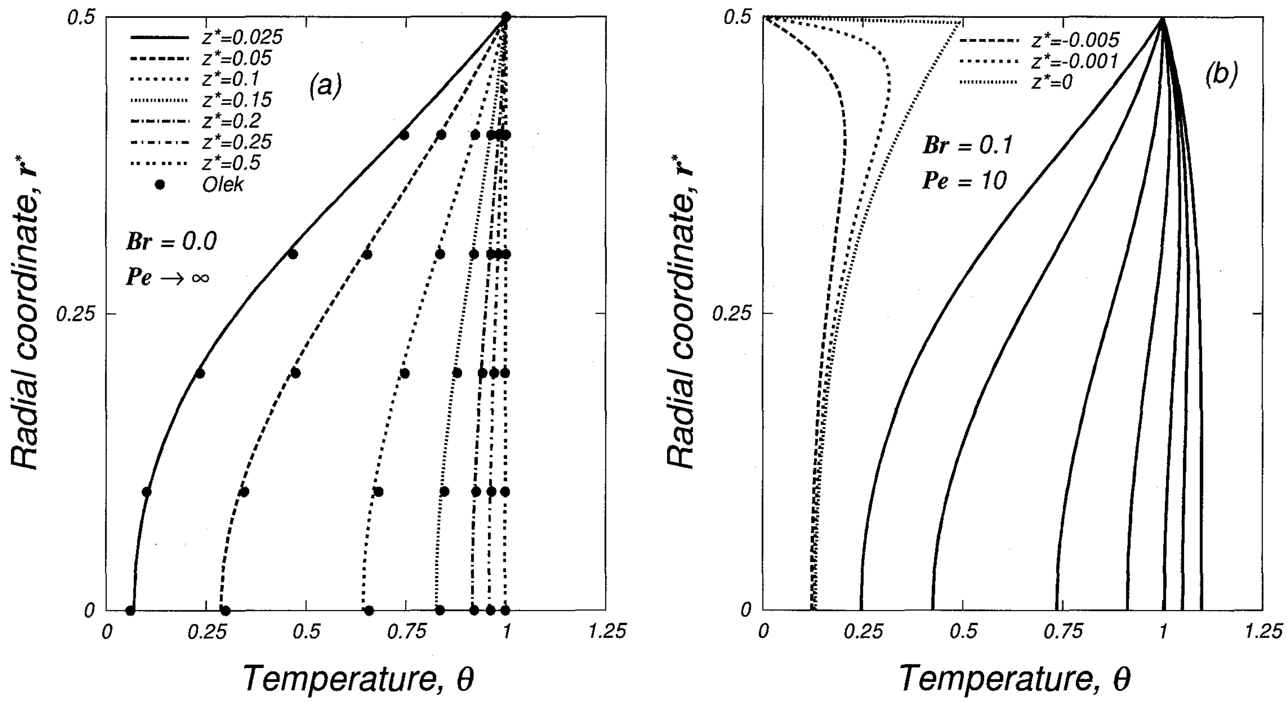

Fig. 2 Developing temperature profiles ($n = 1$)

(a) Negligible viscous dissipation and fluid axial heat conduction ($Br = 0$ and $Pe \to \infty$).

(b) Both viscous dissipation and fluid axial heat conduction are considerable ($Br = 0.1$ and $Pe = 10$).

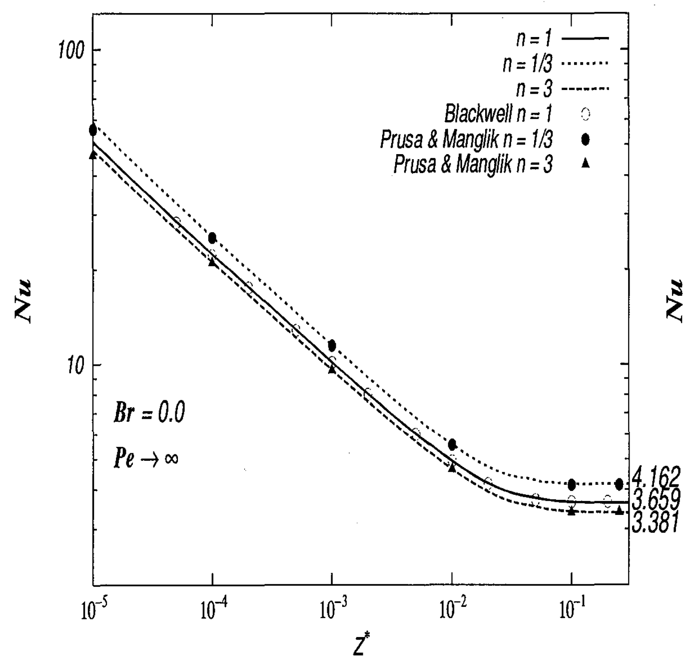

Fig. 3 Nusselt number for different fluids ($n = 1$, $1/3$ and 3)

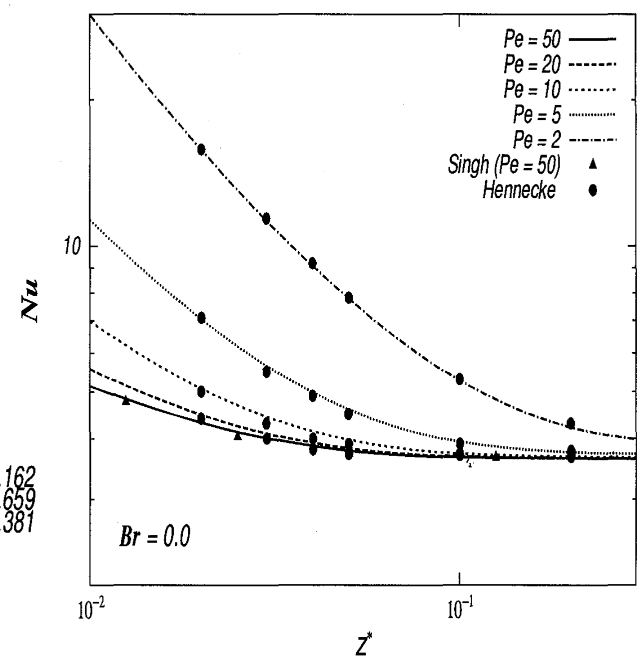

Fig. 4 Nusselt number for various Peclet number for the negligible viscous dissipation case ($n = 1$)

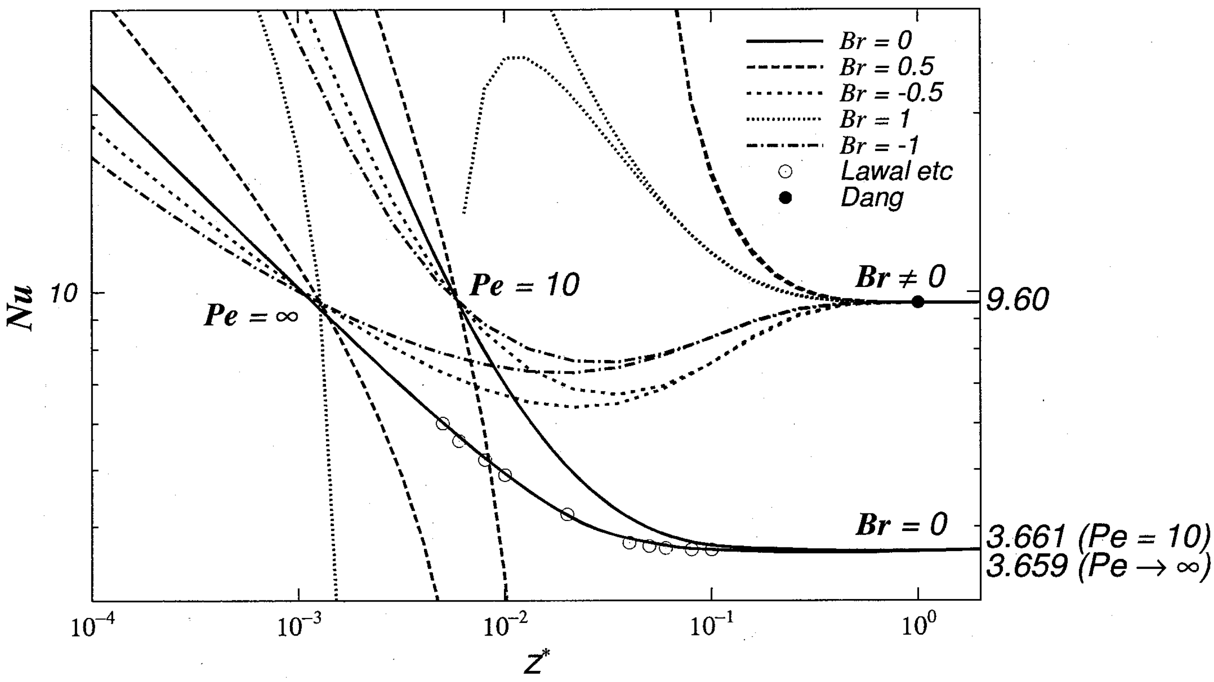

Fig. 5 Effects of Br and Pe on Nusselt number for $n = 1$

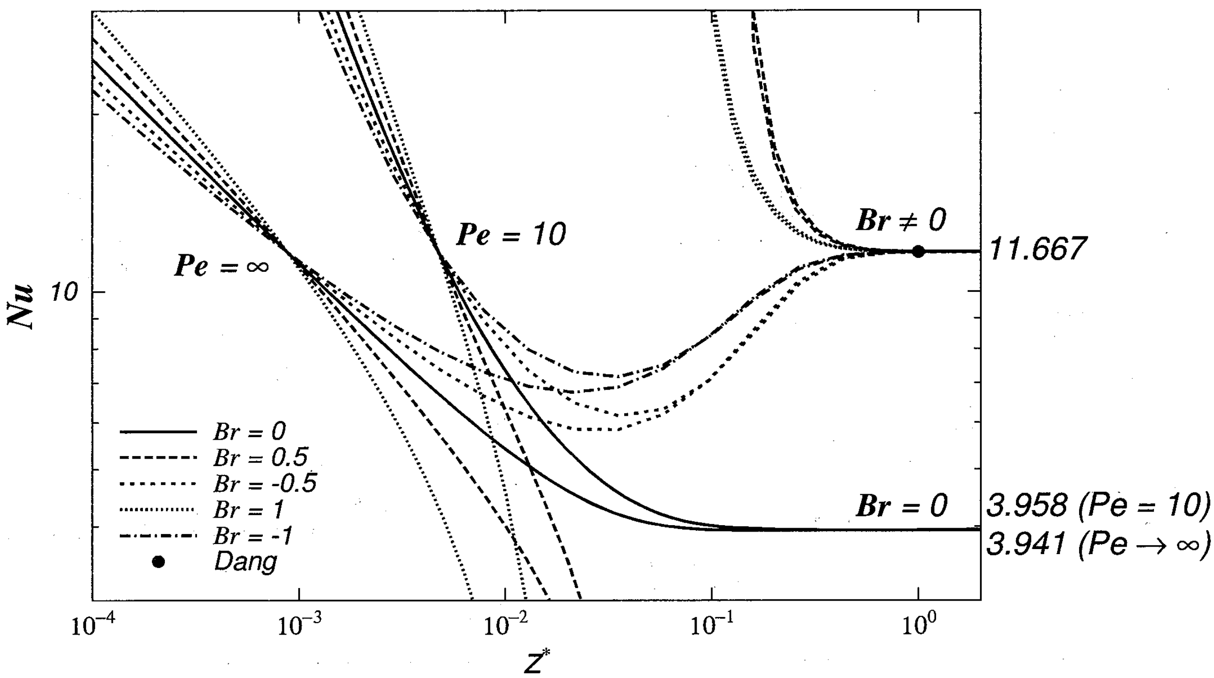

Fig. 6 Effects of Br and Pe on Nusselt number for $n = 0.5$

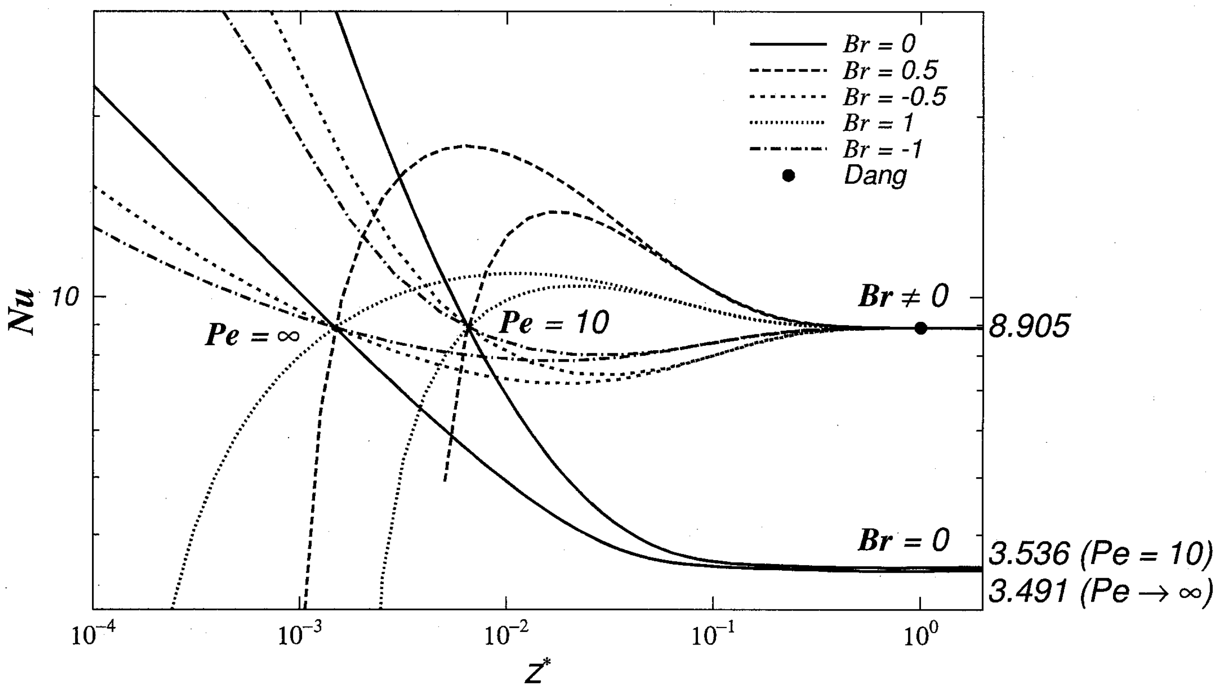

Fig. 7 Effects of Br and Pe on Nusselt number for n = 1.5

the thermally developing region are not shown in Fig. 5 which shows the Nusselt curves for Newtonian fluids. The results for Nusselt numbers in the fully developed range by Lawal and Mujumdar[7] , and the asymptotic Nusselt values by Dang[8] are in excellent agreement with our corresponding results. In this study, according to the Brinkman number definition, for minus Brinkman numbers the fluid is considered as being cooled and positive Brinkman numbers show that fluid is being heated from the wall. Thus Figs. 5 - 7 illustrate the cases of negligible viscous dissipation and also the cases of cooling and heating processes with considerable viscous dissipation. It is seen there is a singular value of the fully developed Nusselt number for various non-zero Br for a given fluid. Equation (26) ensures that for a particular fluid, the asymptotic Nu has a single value for any non-zero values of Br.

4. Conclusions

Thermally developing heat transfer of non-Newtonian power-law fluids in a circular tube under the boundary conditions of constant wall temperature has been analyzed taking into account of the effects of viscous dissipation and fluid axial heat conduction. In view of the mathematical formulation, the energy equation was an elliptic type problem and it was solved by considering two semi-infinite axial domains.

The results are presented graphically in dimensionless form. In order to verify the numerical scheme applied in this study, our results for special case studies are compared with data sets published in open literature.

An inspection of the temperature profile reveals that the fluid temperature increases at $z < 0$ due to fluid axial heat conduction and viscous dissipation before the fluid flow reaches the heated wall. The results indicate that, for a given fluid the asymptotic value of Nusselt number at the wall has a single value for different non-zero values of Brinkman number. For non-zero Brinkman numbers, the asymptotic Nusselt number does not depend on the Peclet number values. However, for zero Brinkman number, the asymptotic Nusselt number depends on the Peclet number value and with a decrease in Peclet number the asymptotic Nusselt number increases slightly.

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