Monetary and Fiscal Policy in a Liquidity Trap: The Japanese Experience 1999-2004

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Abstract

We characterize monetary and fiscal policy rules to implement optimal responses to a substantial decline in the natural rate of interest, and compare them with policy decisions made by the Japanese central bank and government in 1999–2004. First, we find that the Bank of Japan's policy commitment to continuing monetary easing until some prespecified conditions are satisfied lacks history dependence, a key feature of the optimal monetary policy rule. Second, the term structure of the interest rate gap (the spread between the actual real interest rate and its natural rate counterpart) was not downward sloping, indicating that the Bank of Japan's commitment failed to have sufficient influence on the market's expectations about the future course of monetary policy. Third, we find that the primary surplus in 1999–2002 was higher than predicted by the historical regularity, implying that the Japanese government deviated from the

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Ricardian rule toward fiscal tightening. These findings suggest that inappropriate conduct of monetary and fiscal policy during this period delayed the timing to escape from the liquidity trap.

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1 Introduction

Recent developments in the Japanese economy are characterized by the concurrence of two rare phenomena: deflation and zero nominal interest rates. The year-on-year CPI inflation rate has been below zero for about six years since the second quarter of 1998 (see Figure 1). On the other hand, the uncollateralized overnight call rate has been practically zero since the Bank of Japan (BOJ) policy board made a decision on February 12, 1999 to lower it to be "as low as possible" (see Figure 2).

(Insert Figure 1 and Figure 2 here)

The concurrence of these two phenomena has revived the interest of researchers in what Keynes (1936) called a liquidity trap, and various studies have recently investigated this issue. These studies share the following two features. First, regarding diagnosis, they argue that the natural rate of interest, which is defined as the equilibrium real interest rate, is below zero in Japan, while the real overnight call rate is above zero because of deflationary expectations, and that such an interest rate gap leads to weak aggregate demand. This diagnosis was first made by Krugman (1998) and shared by Woodford (1999), Reifschneider and Williams (2000), Jung et al. (2003), and Eggertsson and Woodford (2003a, b) among others.[1]

Second, based on this diagnosis, these studies write out a prescription that the BOJ should make a commitment to an expansionary monetary policy in the future. Woodford (1999) and Reifschneider and Williams (2000) argue that, even when the current overnight interest rate is close to zero, the long-term nominal interest rate

could be well above zero if future overnight rates are expected to be above zero.[2] In this situation, a central bank could lower the long-term nominal interest rate by committing itself to an expansionary monetary policy in the future, thereby stimulating current aggregate demand. As emphasized by Woodford (1999), Jung et al. (2003), and Eggertsson and Woodford (2003a, b), an important feature of this prescription is monetary policy inertia: a zero interest rate policy should be continued for a while, even after the natural rate of interest returns to a positive level. By making such a commitment, a central bank is able to achieve lower long-term nominal interest rates, higher expected inflation, and a weaker domestic currency in the adverse periods when the natural rate of interest significantly deviates from a normal level. This is as if a central bank "borrows" future monetary easing in the periods when current monetary easing is exhausted.

(Insert Table 1 here)

This idea of borrowing future easing has been discussed not only in the academic arena, but also in the policy-making process.[3] Just after the introduction of a "zero interest rate policy" in February 1999, there was a perception in the money markets that such an irregular policy would not be continued for long. Reflecting this perception, implied forward interest rates for longer than six months started to rise in early March. This was clearly against the BOJ's expectation that the zero overnight call rate would spread to longer-term nominal interest rates. Forced to make the bank's policy intention clearer, Governor Masaru Hayami announced on April 13, 1999 that

the monetary policy board would keep the overnight interest rate at zero until "deflationary concerns are dispelled".[4] Some researchers and practitioners argue that this announcement has had the effect of lowering longer-term interest rates by altering the market's expectations about the future path of the overnight call rate (Taylor (2000)). Given such a similarity between the BOJ's policy intention and the prescriptions proposed by academic researchers, a natural question is whether or not the BOJ's policy commitment is close to the optimal one. The first objective of this paper is to measure the distance between the optimal monetary policy rule derived in the literature and the BOJ's policy in practice.

The second objective of this paper is to think about the role of fiscal policy in a liquidity trap. The typical textbook answer to the question of how to escape from a liquidity trap is to adopt an expansionary fiscal policy, given that monetary policy is ineffective in the sense of no more room for current interest rate reductions (Hicks (1967)). Interestingly, however, researchers since Krugman (1998) pay almost no attention to the role of fiscal policy. This difference comes from their assumption about the behavior of the government: the government adjusts its primary surplus so that the government intertemporal budget constraint is satisfied for *any* possible path of the price level. That is, fiscal policy is assumed to be "passive" in the sense of Leeper (1991) or "Ricardian" in the terminology of Woodford (1995). Given this assumption, the government budget constraint is automatically satisfied, so that researchers need not worry about the government's solvency condition in characterizing the optimal

monetary policy rule in a liquidity trap.[5] However, this does not necessarily imply that fiscal policy plays no role in the determination of equilibrium inflation. Rather, as pointed out by Iwamura and Watanabe (2002) and Eggertsson and Woodford (2003b), a path for the primary surplus is uniquely selected when one chooses a monetary policy path by solving a central bank's loss minimization problem. Put differently, even if a central bank faithfully follows the optimal monetary policy rule derived in the literature, the economy might fail to achieve the optimal outcome if the government's behavior deviates from the one compatible with the optimal monetary policy rule. Then one might ask whether or not the assumption of passive fiscal policy was actually satisfied during the period in which the Japanese economy was in a liquidity trap. Specifically, one might be interested in whether or not the Japanese government has adjusted the primary balance as implicitly assumed in the literature.

The rest of the paper is organized as follows. Section 2 characterizes optimal policies in a liquidity trap with a special emphasis on the optimal fiscal policy rule. Sections 3 and 4 compare the optimal commitment solution with the monetary and fiscal policy adopted in 1999–2004. Section 5 concludes the paper.

2 Optimal commitment policy in a liquidity trap

2.1 A simple model

Household's consumption decision Let us consider a representative household that seeks to maximize a discounted sum of utilities of the form

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t+g_t)\right],$$

where $u(\cdot)$ is an increasing and concave function with respect to c_t+g_t, and β represents the discount factor. Following Woodford (2001), we assume that the private consumption expenditures c_t and the government purchases g_t are perfectly substitutable, so that government purchases have exactly the same effect on the economy as transfers to households of funds sufficient to finance private consumption for exactly the same amount. This assumption, together with the assumption of lump-sum taxes, creates a simple environment in which the government behavior affects the equilibrium only through changes in the household's budget constraint. Also, we do not treat money balances and labor supply explicitly in the utility function in order to make the exposition simpler (see Woodford (2003) for detailed discussions on these issues).

The representative household is subject to a flow budget constraint of the form

$$P_t c_t+\sum_{j=1}^{\infty}E_t[Q_{t,t+j}]\left[B^h_{t,t+j}-B^h_{t-1,t+j}\right]\leq P_t d_t+B^h_{t-1,t}, \tag{2.1}$$

where P_t is the price level, d_t is the household's disposable income, and $Q_{t,t+j}$ is a (nominal) stochastic discount factor for pricing arbitrary financial claims that matures in period $t+j$.[6] We assume that the government issues zero-coupon nominal bonds,

each of which pays one yen when it matures, and denote the face value of bonds held by the representative household at the end of period t that will come due in period $t+j$ by $B^h_{t,t+j}$. Since the nominal market price in period t of a bond that matures in period $t+j$ is $E_t[Q_{t,t+j}]$ $(=E_t[1 \times Q_{t,t+j}])$, the second term on the right-hand side represents the amount of repayment for bonds that mature in period t. The representative household allocates the sum of disposable income and the repayment between consumption expenditures and the purchases of government bonds. The term $B^h_{t,t+j} - B^h_{t-1,t+j}$ represents the change from the previous period in the face value of bonds that mature in period $t+j$, namely, an amount of net purchase in period t. These new bonds are evaluated at the market price in period t. Note that nominal bond prices must satisfy

$$E_t[Q_{t,t+j}] = E_t\left[Q_{t,t+1}\ Q_{t+1,t+2} \times \cdots \times Q_{t+j-1,t+j}\right],$$

and that the one-period risk-free nominal interest rate in period $t+k$ $(k \geq 0)$, which is denoted by i_{t+k}, satisfies

$$\frac{1}{1+i_{t+k}} = E_{t+k}[Q_{t+k,t+k+1}].$$

Under the assumption that the central bank can control the one-period risk-free interest rate, these two equations imply that the market's expectations about the future course of monetary policy, represented by the path of i_{t+k}, affects nominal bond prices.

The sequence of flow budget constraints and the No-Ponzi-game condition implies an intertemporal budget constraint, and necessary and sufficient conditions for house-

hold maximization are then that the first-order condition

$$1 + i_t = \beta^{-1} \left\{ E_t \left[\frac{u'(c_{t+1} + g_{t+1})}{u'(c_t + g_t)} \frac{P_t}{P_{t+1}} \right] \right\}^{-1} \tag{2.2}$$

holds at all times, and that the household exhausts its intertemporal budget constraint. We assume that some part, denoted by ν_t, of the economy's output y_t is distributed to another type of household that does not make consumption decisions based on intertemporal utility maximization, so that the market-clearing condition can be written as $y_t = c_t + \nu_t + g_t$. Substituting this condition into (2.2) yields

$$1 + i_t = \beta^{-1} \left\{ E_t \left[\frac{u'(y_{t+1} - \nu_{t+1})}{u'(y_t - \nu_t)} \frac{P_t}{P_{t+1}} \right] \right\}^{-1}. \tag{2.3}$$

Substituting the same condition into the flow budget constraint (equation (2.1) with an exact equality) and the corresponding intertemporal budget constraint leads to

$$P_t s_t + \sum_{j=1}^{\infty} E_t[Q_{t,t+j}] \left[B_{t,t+j} - B_{t-1,t+j} \right] = B_{t-1,t} \tag{2.4}$$

$$\sum_{j=0}^{\infty} E_t \left[Q_{t,t+j} P_{t+j} s_{t+j} \right] = \sum_{j=0}^{\infty} E_t[Q_{t,t+j}] B_{t-1,t+j} \tag{2.5}$$

where s_t represents the real primary surplus, which is defined as tax revenues less government expenditures, and $B_{t,t+j}$ is the supply of government bonds.[7]

We log-linearize equations (2.3) and (2.4) around the baseline path of each variable, which is specified as follows. With respect to the maturity structure of government debt, we assume

$$\frac{B^*_{t-1,t+j}}{B^*_{t+j-1,t+j}} = \theta^j \leq 1 \qquad \text{for} \quad j = 1, 2, \cdots, \tag{2.6}$$

where θ is a parameter satisfying $0 \leq \theta \leq 1$. We use $*$ to indicate the baseline path of a variable. The term $B^*_{t-1,t+j}$ represents the face value of bonds at the end of period $t-1$ that mature in period $t+j$, and $B^*_{t+j-1,t+j}$ represents the face value of the same type of bonds just before redemption in period $t+j$. Equation (2.6) simply states that the government issues additional bonds, which mature in period $t+j$, at a rate θ in each period between t and $t+j-1$. Note that $\theta = 0$ corresponds to the case in which all bonds mature in one period, while $\theta = 1$ corresponds to the case in which all bonds are perpetual bonds. With respect to other variables, we assume

$$c^*_t = c^*;\ y^*_t = y^*;\ s^*_t = s^*;\ P^*_t = P^*;\ Q^*_{t,t+j} = \beta^j;\ \nu^*_t = 0.$$

Note that the inflation rate is assumed to be zero on the baseline path.

Log-linearizing (2.3) around the baseline path, we obtain

$$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma^{-1}\left[(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \hat{r}^n_t\right], \tag{2.7}$$

where a variable with a hat represents the proportional deviation of the variable from its value on the baseline path (for example, $\hat{z}_t$ is defined as $\hat{z}_t \equiv \ln z_t - \ln z^*_t$),[8] and σ is a positive parameter defined as $\sigma \equiv -u''(y^*)y^*/u'(y^*)$. The output gap x_t is defined as $x_t \equiv y_t - y^n_t$, where y^n_t represents the natural rate of output or potential output. The inflation rate π_t is defined as $\pi_t \equiv \ln P_t - \ln P_{t-1}$. Finally, the deviation of the natural rate of interest from its baseline path, $\hat{r}^n_t$, is defined as

$$\hat{r}^n_t \equiv \sigma E_t\left[(\hat{y}^n_{t+1} - \hat{y}^n_t) - (\hat{\nu}_{t+1} - \hat{\nu}_t)\right]. \tag{2.8}$$

According to the above definition of $\hat{r}^n_t$, variations in the natural rate of interest are

caused by short-term factors such as changes in ν_t, as well as long-term factors such as the growth rate of potential output. Log-linearizing (2.4) around the baseline path, we obtain[9]

$$(1-\beta\theta)\left[\hat{B}_t-\beta^{-1}\hat{B}_{t-1}\right]=-(1-\beta\theta)(1-\theta)(\beta\theta)^{-1}\hat{Q}_t-\beta^{-1}(1-\beta)\left[\hat{P}_t+\hat{s}_t\right] \quad (2.9)$$

where $\hat{B}_t$ and $\hat{Q}_t$ are defined as

$$\hat{B}_t\equiv\sum_{j=0}^{\infty}(\beta\theta)^j\hat{B}_{t,t+1+j};\ \hat{Q}_t\equiv\sum_{j=0}^{\infty}(\beta\theta)^jE_t[\hat{Q}_{t,t+j}].$$

$\hat{B}_t$ and $\hat{Q}_t$ can be interpreted as a nominal debt aggregate, and an index of nominal bond prices.

Equation (2.7) can be seen as an "IS equation" that states that the output gap in period t is determined by the expected value of the output gap in period $t+1$ and the gap between the short-term real interest rate and the natural rate of interest in period t. Equation (2.7) can be iterated forward to obtain

$$\hat{x}_t=-\sigma^{-1}\sum_{j=0}^{\infty}E_t\left[(\hat{i}_{t+j}-\hat{\pi}_{t+j+1})-\hat{r}^n_{t+j}\right]. \quad (2.10)$$

According to the expectations theory, the expression $\sum_{j=0}^{\infty}E_t\left[(\hat{i}_{t+j}-\hat{\pi}_{t+j+1})-\hat{r}^n_{t+j}\right]$ stands for the deviation of the long-term real interest rate from the corresponding natural rate of interest in period t, which implies that, given the path of the natural rate of interest, the output gap depends inversely on the long-term real interest rate.

New Keynesian Phillips curve In addition to the IS equation, we need an "AS equation" to describe the supply side of the economy. We adopt a framework of staggered price setting developed by Calvo (1983). It is assumed that in each period a

fraction $1-\alpha$ of goods suppliers get to set a new price, while the remaining α must continue to sell at their previously posted prices. The suppliers that get to set new prices are chosen randomly each period, with each having an equal probability of being chosen. Under these assumptions, we obtain an AS equation of the form[10]

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta E_t \hat{\pi}_{t+1}, \tag{2.11}$$

where κ is a positive parameter which is conversely related to the value of α. Equation (2.11) is the so-called New Keynesian Phillips curve, which differs from the traditional Phillips curve in that current inflation depends on the expected rate of future inflation, $E_t\hat{\pi}_{t+1}$, rather than the expected rate of current inflation, $E_{t-1}\hat{\pi}_t$.

Locally Ricardian fiscal policy We assume that the government determines the (nominal) primary surplus each period following a fiscal policy rule of the form

$$P_t s_t = \sum_{j=0}^{\infty} \left[E_t(Q_{t,t+j}) - E_{t-1}(Q_{t-1,t+j}) \right] B_{t-1,t+j}, \tag{2.12}$$

where the term $E_t(Q_{t,t+j}) - E_{t-1}(Q_{t-1,t+j})$ represents the realized nominal one-period holding return, including interest payments and capital gains/losses, for a bond that matures in period $t+j$. Equation (2.12) simply states that the government creates a primary surplus by an amount just enough to cover these payments on existing liabilities. In a deterministic environment, in which there is no uncertainty about the sequence of bond prices, the absence of arbitrage opportunities implies $i_{t-1} = (Q_{t,t+j} - Q_{t-1,t+j})/Q_{t-1,t+j}$, so that (2.12) reduces to

$$P_t s_t = i_{t-1} \left(\sum_{j=0}^{\infty} Q_{t-1,t+j} B_{t-1,t+j} \right), \tag{2.13}$$

where the term $\sum_{j=0}^{\infty} Q_{t-1,t+j} B_{t-1,t+j}$ represents the market value of the existing government liabilities at the end of period $t-1$, and the right hand side of equation (2.13) represents the interest payments on existing liabilities. Equation (2.13) is equivalent to a budget deficit (not primary deficit but conventional deficit) targeting rule, and in that sense, is very close to the spirit of the fiscal requirement of the Maastricht treaty or the Stability and Growth Pact in the European Monetary Union. Also, the fiscal policy rule of this form is used in empirical studies such as Bohn (1998), in order to describe the actual government's behavior.

Substituting (2.12) into the government's flow budget constraint (equation (2.4)), we observe that

$$\sum_{j=0}^{\infty} E_t[Q_{t,t+1+j}] B_{t,t+1+j} = \sum_{j=0}^{\infty} E_{t-1}[Q_{t-1,t+j}] B_{t-1,t+j}$$

holds each period. That is, the market value of the existing government liabilities does not change in each period as long as the government determines the primary surplus following (2.12). Using this property, we observe that

$$E_t \left[Q_{t,\tau+1} \sum_{j=0}^{\infty} Q_{\tau-1,\tau+j} B_{\tau-1,\tau+j} \right] = E_t \left[Q_{t,\tau+1} \right] \sum_{j=0}^{\infty} E_{t-1}[Q_{t-1,t+j}] B_{t-1,t+j}$$

holds for all $\tau > t$, which implies[11]

$$\lim_{\tau \to \infty} E_t \left[Q_{t,\tau+1} \sum_{j=0}^{\infty} Q_{\tau-1,\tau+j} B_{\tau-1,\tau+j} \right] = 0. \tag{2.14}$$

This equation states that the fiscal policy rule (2.12) guarantees the transversality condition for *any* path of the price level. Thus the government's transversality condition does not affect the price level in equilibrium as long as the government follows the rule

(2.12). Fiscal policy rules with this feature are called "passive" by Leeper (1991), and "locally Ricardian" by Woodford (1995).

Equations (2.7), (2.9), (2.11), and the log-linear version of (2.12)

$$\hat{s}_t + \hat{P}_t = (1-\beta\theta)\hat{B}_{t-1} + (1-\beta)^{-1}(1-\beta\theta)\left[\hat{Q}_t - \theta^{-1}\hat{Q}_{t-1}\right] \tag{2.15}$$

consist of four key equations of our model.[12] Given the natural rate of interest $\hat{r}_t^n$ as an exogenous variable and the short-term nominal interest rate $\hat{i}_t$ as a policy variable, which is determined as we see in the next subsection, these four equations determine the equilibrium paths of $\hat{x}$, $\hat{P}$ (or equivalently $\hat{\pi}$), $\hat{B}$, and $\hat{s}$.[13] It should be emphasized that fiscal variables ($\hat{s}_t$ and $\hat{B}_t$) do not appear in the IS and AS equations ((2.7) and (2.11)), so that, given the paths of $\hat{i}_t$ and $\hat{r}_t^n$, these two equations determine the paths of $\hat{x}_t$ and $\hat{\pi}_t$ (or equivalently $\hat{P}_t$), independently of the fiscal variables. In this sense, equations (2.7) and (2.11) constitute an independent block in the four equations system; namely, they first determine the paths of $\hat{x}_t$ and $\hat{\pi}_t$, and, given them, the other two equations determine the paths of the two fiscal variables ($\hat{s}_t$ and $\hat{B}_t$). This structure of the model is fully utilized when we characterize the optimal monetary policy rule in the next subsection.

2.2 Optimal monetary policy

Adverse shock to the economy Following Jung et al. (2003), we consider a situation in which the economy is hit by a large-scale negative demand shock; the central bank responds to it by lowering the short-term nominal interest rate to zero; but aggregate demand is still insufficient to close the output gap. More specifically,

we assume that a large negative shock to the natural rate of interest, denoted by ϵ_0^n, occurs in period 0, so that the natural rate of interest takes a large negative value in period 0 and subsequent periods. The deviation of the natural rate of interest from the baseline path is described by

$$\hat{r}_t^n \equiv \ln(1 + r_t^n) - \ln(1 + r_t^{n*}) = \rho^t \epsilon_0^n \quad \text{for} \quad t = 0, \cdots, \tag{2.16}$$

where r_t^{n*} is the baseline value of the natural rate of interest, which is assumed to be equal to $\beta^{-1}(1-\beta)$, and ρ is a parameter satisfying $0 \leq \rho < 1$.[14]

It is important to note that the natural rate of interest $\hat{r}_t^n$ appears only in the IS equation ((2.7)), and that fluctuations in the natural rate of interest could be completely offset if the central bank equalizes the short-term nominal interest rate to the natural rate of interest ($\hat{i}_t = \hat{r}_t^n$). In the usual situation, therefore, aggregate demand shocks can be completely offset by an appropriate monetary policy. However, this is not true if the natural rate of interest falls below zero and the non-negativity constraint of the short-term nominal interest rate, $i_t \geq 0$, or its log-linear version

$$\hat{i}_t + \beta^{-1}(1-\beta) \geq 0 \tag{2.17}$$

is binding.

Optimization under discretion The central bank chooses the path of the short-term nominal interest rates, starting from period 0, $\left\{\hat{i}_0, \hat{i}_1, \cdots\right\}$ to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \lambda \hat{x}_t^2\right],$$

subject to (2.7), (2.9), (2.11), (2.15), and (2.17). Since equations (2.7) and (2.11) consist of an independent block, and the fiscal variables ($\hat{s}_t$ and $\hat{B}_t$) do not appear in the loss function, the optimization problem can be solved in a step-by-step manner: we first minimize the loss function subject to (2.7), (2.11), and (2.17) and characterize the optimal paths for $\hat{i}_t$, $\hat{x}_t$, and $\hat{\pi}_t$; then we substitute them into (2.9) and (2.15) to obtain the optimal paths for $\hat{s}_t$ and $\hat{B}_t$.

Under the assumption of discretionary monetary policy, the central bank reoptimizes in each period. The optimization problem is represented by a Lagrangian of the form

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t + 2\phi_{1t}[\hat{x}_t - \hat{x}_{t+1} + \sigma^{-1}(\hat{i}_t - \hat{\pi}_{t+1} - \hat{r}_t^n)] + 2\phi_{2t}[\hat{\pi}_t - \kappa\hat{x}_t - \beta\hat{\pi}_{t+1}] \right\},$$

where ϕ_{1t} and ϕ_{2t} represent the Lagrange multipliers associated with the IS and AS equations. We differentiate the Lagrangian with respect to $\hat{\pi}_t$, $\hat{x}_t$, and $\hat{i}_t$ to obtain the first-order conditions

$$\hat{\pi}_t + \phi_{2t} = 0 \tag{2.18}$$

$$\lambda\hat{x}_t + \phi_{1t} - \kappa\phi_{2t} = 0 \tag{2.19}$$

$$\left[\hat{i}_t + \beta^{-1}(1-\beta)\right]\phi_{1t} = 0 \tag{2.20}$$

$$\hat{i}_t + \beta^{-1}(1-\beta) \geq 0 \tag{2.21}$$

$$\phi_{1t} \geq 0 \tag{2.22}$$

Equations (2.20), (2.21), and (2.22) are Kuhn–Tucker conditions regarding the nonnegativity constraint on the nominal interest rate. Observe that $\partial\mathcal{L}/\partial\hat{i}_t = 2\sigma^{-1}\beta^t\phi_{1t} \propto$

ϕ_{1t}. If the non-negativity constraint is not binding, $\partial\mathcal{L}/\partial\hat{i}_t$ is equal to zero, so that ϕ_{1t} is also zero. On the other hand, if the constraint is binding, $\partial\mathcal{L}/\partial\hat{i}_t$ is non-negative, and so is ϕ_{1t}.

Given the assumption that the natural rate of interest converges monotonically to its baseline value (see (2.16)), it is straightforward to guess that the non-negativity constraint is binding until some period, denoted by period T^d, but is not binding afterwards. By eliminating ϕ_{2t} from (2.18) and (2.19), we obtain

$$\phi_{1t} = -\kappa \left[\hat{\pi}_t + \kappa^{-1}\lambda\hat{x}_t\right].$$

Substituting $\phi_{1t} = 0$ into this equation leads to $\lambda\hat{x}_t + \kappa\hat{\pi}_t = 0$, which, together with the AS equation, imply $\hat{\pi}_t = 0$, $\hat{x}_t = 0$, and

$$\hat{i}_t = \hat{r}_t^n \tag{2.23}$$

for $t = T^d + 1, \cdots$. Thus the central bank sets the short-term nominal interest rate at zero during the periods in which the natural rate of interest is below zero, but, once the natural rate returns to a positive level, the central bank equalizes it with the level of the natural rate of interest. In this sense, the timing to terminate a zero interest rate policy is determined entirely by an exogenous factor, $\hat{r}_t^n$.

Optimization under commitment We now proceed to the commitment solution: the central bank makes a commitment about the current and future path of the short-term nominal interest rate, considering the consequences of the commitment on the

private sector's expectations. The first-order conditions become

$$\hat{\pi}_t - (\beta\sigma)^{-1}\phi_{1t-1} + \phi_{2t} - \phi_{2t-1} = 0 \tag{2.24}$$

$$\lambda\hat{x}_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0 \tag{2.25}$$

$$\left[\hat{i}_t + \beta^{-1}(1-\beta)\right]\phi_{1t} = 0 \tag{2.26}$$

$$\hat{i}_t + \beta^{-1}(1-\beta) \geq 0 \tag{2.27}$$

$$\phi_{1t} \geq 0 \tag{2.28}$$

which differ from those obtained earlier in that lagged Lagrange multipliers, ϕ_{1t-1} and ϕ_{2t-1}, appear in the first two equations. We eliminate ϕ_{2t} from equations (2.24) and (2.25) to obtain a second-order difference equation with respect to ϕ_{1t}.

$$\phi_{1t} - [1 + \beta^{-1} + \kappa(\beta\sigma)^{-1}]\phi_{1t-1} + \beta^{-1}\phi_{1t-2} = -\kappa\left[\hat{\pi}_t + \kappa^{-1}\lambda(\hat{x}_t - \hat{x}_{t-1})\right]$$
$$\text{for} \quad t = 0, \cdots, T^c + 1, \tag{2.29}$$

where T^c is the final period of a zero interest rate policy, and initial conditions are given by $\phi_{1-1} = \phi_{1-2} = 0$. A unique solution to this difference equation is given by

$$\phi_{1t} = -\kappa A(L)\left[\hat{\pi}_t + \kappa^{-1}\lambda(\hat{x}_t - \hat{x}_{t-1})\right], \tag{2.30}$$

where L is a lag-operator and $A(L)$ is defined by

$$A(L) \equiv \frac{1}{\xi_1 - \xi_2}\left(\frac{\xi_1}{1-\xi_1 L} - \frac{\xi_2}{1-\xi_2 L}\right),$$

and ξ_1 and ξ_2 are the two real solutions to the characteristic equation associated with the difference equation (2.29), satisfying $\xi_1 > 1$ and $0 < \xi_2 < 1$.

Equation (2.29) has the following implications regarding the differences between the discretionary and commitment solutions. First, as pointed out by Woodford (1999) and Jung et al. (2003), a zero interest rate policy is continued longer in the case of commitment. To see this, we observe from equations (2.10), (2.11), and (2.30) that

$$\phi_{1t} = B(L)\left[(\hat{i}_t - \hat{\pi}_{t+1}) - \hat{r}_t^n\right],$$

where

$$B(L) \equiv \kappa\sigma^{-1}A(L)\left[\kappa(1-\beta L^{-1})^{-1}(1-L^{-1})^{-1} + \kappa^{-1}\lambda(1-L^{-1})^{-1}(1-L)\right].$$

Note that the real interest rate will never be below the natural rate of interest ($(\hat{i}_t - \hat{\pi}_{t+1}) - \hat{r}_t^n \geq 0$) in the case of discretion. Thus, if a zero interest rate policy is terminated in period T^d, ϕ_{1t} takes a positive value at $t = T^d + 1$, indicating that

$$0 \leq T^d \leq T^c < \infty.$$

The optimal commitment solution is characterized by monetary policy inertia, in the sense that a zero interest rate policy is continued for a while even after the natural rate of interest becomes positive. This is in sharp contrast with the case of discretion, in which a zero interest rate policy is terminated as soon as the natural rate of interest becomes positive.

Second, we compare fiscal adjustments between the discretionary and commitment solutions. By log-linearizing the government's intertemporal budget constraint ((2.5)), we obtain

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\hat{P}_t+\hat{s}_t\right] = (1-\beta\theta)(1-\beta)^{-1}\hat{B}_{-1} + (1-\beta)^{-1}\left[(1-\beta\theta)\sum_{t=0}^{\infty}(\beta\theta)^t E_0(\hat{Q}_{0,t}) - (1-\beta)\sum_{t=0}^{\infty}\beta^t E_0(\hat{Q}_{0,t})\right].$$

In either discretionary or commitment solutions, the short-term nominal interest rate is set at zero for some periods and then returns to a normal level, which means that $E_0(\hat{Q}_{0,t})$ takes positive values in period 0 and subsequent periods and then returns to zero. Given that $\theta \in [0,1]$, this implies that the second term on the right-hand side is non-positive, therefore the (nominal) primary surplus must be on or below its baseline path.[15] Furthermore, the degree of fiscal expansion depends on the maturity structure of government bonds; the shorter the maturity, the larger the fiscal expansion. When the maturity of bonds is very long, reductions in the short-term nominal interest rate in the current and future periods raise bond prices significantly, therefore fewer fiscal adjustments are needed.[16]

To compare the discretionary and commitment solutions in terms of *real* fiscal adjustments, we compute

$$\sum_{t=0}^{\infty}\beta^t E_0[\hat{s}_t^c-\hat{s}_t^d] = -\left\{\sum_{t=0}^{\infty}\beta^t E_0[\hat{P}_t^c-\hat{P}_t^d]\right\} + (1-\beta)^{-1}\left\{\left[(1-\beta\theta)\sum_{t=0}^{\infty}(\beta\theta)^t E_0(\hat{Q}_{0,t}^c) - (1-\beta)\sum_{t=0}^{\infty}\beta^t E_0(\hat{Q}_{0,t}^c)\right] - \left[(1-\beta\theta)\sum_{t=0}^{\infty}(\beta\theta)^t E_0(\hat{Q}_{0,t}^d) - (1-\beta)\sum_{t=0}^{\infty}\beta^t E_0(\hat{Q}_{0,t}^d)\right]\right\},$$

where the first term on the right-hand side is negative since $\hat{P}_t^c$ is greater than $\hat{P}_t^d$ in every period, and the second term is also negative because $T^d \leq T^c$ implies $E_0(\hat{Q}_{0,t}^c) \geq$

$E_0(\hat{Q}^d_{0,t})$ in every period. Thus we observe

$$\sum_{t=0}^{\infty} \beta^t E_0[\hat{s}^c_t] \leq \sum_{t=0}^{\infty} \beta^t E_0[\hat{s}^d_t]. \tag{2.31}$$

This indicates that the commitment solution cannot be achieved by monetary policy alone, and that a close coordination with fiscal policy is indispensable.[17] A more expansionary stance should be taken on the side of fiscal policy, as well as on the side of monetary policy.

2.3 Numerical examples

(Insert Figure 3 here)

In this subsection we numerically compute the optimal path of various variables.[18] Figure 3 shows the responses of eight variables to an adverse shock to the natural rate of interest in the case of discretion. The paths for the short-term nominal and real interest rates and the natural rate of interest represent the level of those variables (i_t, $i_t - \pi_{t+1}$, and r^n_t), while those of other variables are shown by the deviations from their baseline values. The natural rate of interest, which is shown at the bottom left, stays below zero for the first four periods until period 3, and becomes positive in period 4, then gradually goes back to a baseline level. In response to this shock, the short-term nominal interest rate is set at zero for the first four periods, but becomes positive as soon as the natural rate of interest turns positive in period 4. Given the shock to the natural rate of interest and the monetary policy response to it, the short-term *real* interest rate rises and the spread between $i_t - \pi_{t+1}$ and r^n_t is widened, as shown in the

bottom-left panel. Consequently, inflation and the output gap stay below the baseline for the first four periods during which a zero interest rate policy is adopted, and return to zero as soon as that policy is terminated.

The four panels on the right-hand side of Figure 3 show the fiscal aspects of the model. The price level falls during the first four periods and continues to stay at a level below the baseline, while the bond price rises in period 0 and subsequent periods reflecting the market expectation of monetary easing in the current and future periods. This leads to a rise in the *real* value of the existing public debt, which puts the government under pressure to increase the real primary surplus, while lower interest payments due to the zero interest rate policy create room for the government to reduce the real primary surplus. Combining these two conflicting effects, the real primary surplus is below the baseline for the first eight periods until period 7, but slightly above the baseline path thereafter.

(Insert Figure 4 and Figure 5 here)

Figure 4 shows the responses of the same set of variables for the case of commitment. An important difference from the discretionary solution is that a zero interest rate policy is continued longer. Reflecting this, the cumulative sum of the deviation of the short-term *real* interest rate from the natural rate of interest becomes significantly smaller in comparison with the case of discretion, leading to a decline in the *real long*-term interest rate. This alleviates deflationary pressures on the inflation rate and the output gap. Turning to the fiscal aspects of the model, monetary policy inertia (i.e.,

prolonging a zero interest rate policy) keeps the price level higher than the baseline path, which is in sharp contrast with the case of discretion. As a result, the real primary surplus stays below the baseline path even after the zero interest rate policy is terminated. The differences between the commitment and discretionary solutions (the commitment solution minus the discretionary solution) are shown in Figure 5.

(Insert Table 2 here)

Table 2 shows the amounts of fiscal adjustments needed to achieve the optimal outcomes under discretion and commitment. Nominal adjustments ($\sum_{t=0}^{\infty}\beta^j[\hat{P}_t+\hat{s}_t]$) are negative in both solutions, indicating that fiscal expansion is needed to achieve the optimal outcomes. Note that the amount of fiscal adjustments is larger in the commitment solution in which a zero interest rate policy is continued longer. Also, note that the amount of fiscal adjustment depends on the maturity structure of government debt: the amount of fiscal adjustment is larger when the maturity is shorter. Turning to the real adjustments ($\sum_{t=0}^{\infty}\beta^j\hat{s}_t$), they are positive in the discretionary solution while negative in the commitment solution. This reflects a difference between the two solutions in terms of the path of the price level. In the case of the discretionary solution, the price level is lower than on the baseline (Figure 3), so that a larger primary surplus is needed to finance larger *real* redemption. On the other hand, the price level is higher than on the baseline in the commitment solution (Figure 4), thus a smaller surplus is sufficient to finance smaller *real* redemption. The difference between the two solutions again depends on the maturity structure of government debt: the real amount of fiscal

adjustment becomes larger when θ is smaller.[19]

3 Monetary policy in 1999–2004

3.1 Term structure of interest rate gaps

As emphasized by Woodford (1999), Jung et al. (2003), and Eggertsson and Woodford (2003a, b), history dependence is one of the most important features of the commitment solution. To see how history dependent monetary policy affects the output gap and inflation, we rewrite the IS and AS equations ((2.7) and (2.11)) as

$$\hat{x}_t = -\sigma^{-1}(1-L^{-1})^{-1}\left[(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \hat{r}_t^n\right];$$
$$\hat{\pi}_t = -\sigma^{-1}\kappa(1-\beta L^{-1})^{-1}(1-L^{-1})^{-1}\left[(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \hat{r}_t^n\right].$$

An important thing to note is that these two variables are determined soley by the current and future values of the interest rate gap (i.e., the spread between the actual real interest rate and its natural rate couterpart, $[\hat{i}_t - E_t\hat{\pi}_{t+1}] - \hat{r}_t^n$), and, in that sense, the interest rate gap is the key variable through which monetary policy affects the real side of the economy.[20] Given this structure, the central bank's commitment to continuing a zero interest rate policy even after the natural rate of interest becomes positive makes the private sector expect that the interest rate gap, $(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \hat{r}_t^n$, will shrink and become negative in the future periods, thereby weakening the deflationary pressure on the current output gap and inflation.

More specifically, as shown in Figure 3, the short-term real interest rate is never below the natural rate in the discretionary solution, thus the term structure of interest

rate gaps defined by

$$E_t \sum_{k=0}^{K} \left[(i_{t+k} - \pi_{t+k+1}) - r^n_{t+k} \right], \tag{3.1}$$

monotonically increases with K. In contrast, as shown in Figure 4, the short-term real interest rate stays below the natural rate in periods 3 to 6 in the case of the commitment solution, and therefore the gap defined by (3.1) *declines* during these periods. This is a direct reflection of monetary policy inertia, and a key feature to discriminate between the two solutions. These observations suggest a simple way to test whether the BOJ's actual policy is close to the optimal one: we estimate the term structure of interest rate gaps to see whether or not the gap declines with K towards the end of recession.

(Insert Figure 6 here)

We start by estimating the natural rate of interest using the methodology developed by Laubach and Williams (2003).[21] Equation (2.8) may be rewritten as

$$r^n_t = \sigma g^p_t + z_t, \tag{3.2}$$

where the potential growth rate g^p_t is defined as $g^p_t \equiv E_t(y^n_{t+1} - y^n_t)$, and the other stationary component z_t is defined as $z_t \equiv -\sigma E_t(\nu_{t+1} - \nu_t)$. Following Laubach and Williams (2003), we assume that g^p_t is a random walk process, while z_t follows an AR process. Using these two assumptions (together with other assumptions adopted in Laubach and Williams (2003)), we estimate the natural rate of interest for the period from 1982:1Q to 2003:4Q, which is presented in the upper panel of Figure 6. Note that

the natural rate of interest shown here represents the annualized overnight rate. Figure 6 shows that the natural rate of interest was seven percent in 1990, and then gradually declined until it reached almost zero in 1995. Furthermore, it declined below zero in 1998:1Q-1999:2Q, 2000:3Q-4Q, and 2001:2Q-2002:1Q, indicating that Krugman's (1998) prescription for the Japanese economy is not rejected by the data. The middle and bottom panels of Figure 6 decompose fluctuations in the natural rate of interest into the two components: the random walk component (σg_t^p) and the stationary component (z_t). The middle panel shows that the potential growth rate was barely above zero in the 1990s, but fell below zero for the three quarters starting from 2001:3Q. Negative values for the natural rate of interest are due to very low potential growth rates, as well as adverse temporary shocks that had occurred several times after the mid 1990s.

(Insert Figure 7 here)

Figure 7 compares the natural rate of interest with the overnight real interest rate, $i_t - E_t\pi_{t+1}$. We use the uncollateralized overnight call rate for i_t, and the actual inflation rate in period t as a proxy for the expected overnight inflation rate. Figure 7 shows that the real call rate is significantly lower than the natural rate of interest in the latter half of the 1980s, which is consistent with the results from the existing studies that the BOJ's policy was too expansionary, thereby contributing to the asset price inflation during this period. It also shows that the opposite (i.e., the real call rate is higher than the corresponding natural rate) happened in the period from 1998 to 2002. The nominal call rate had already been lowered to the zero lower bound during

this period, but deflationary expectations kept the real call rate above zero, thereby creating positive *overnight* interest rate gaps in these years.

Given that the time-series estimates for the natural rate of interest are to hand, we next construct a time-series for the expected values of the natural rate of interest $E_t \sum_{k=0}^{K} r^n_{t+k}$, as well as a time-series for the expected rate of inflation. We construct the first by utilizing the fact that the natural rate of interest consists of a random walk component and a stationary component.[22] As for the expected rate of inflation, we use the five-year forecasts published by a private research institute, the Japan Center for Economic Research (JCER) in December of each year.[23] By using these two time-series, we can compare the natural rate of interest and the real interest rate for various time horizons (namely, K in equation (3.1)).

(Insert Figure 8 here)

The results of these calculations are presented in Figure 8, which shows the term structure of interest rate gaps at the end of each year starting from 1998.[24] First, the term structure at the end of 1998, just before the introduction of the zero interest rate policy, was upward sloping although the one-year gap was very close to zero. The upward-sloping curve mainly comes from the term structure of nominal interest rates.[25] These two findings suggest that market participants expected that the BOJ would not adopt expansionary monetary policy sufficient to offset an expected decline in the natural rate of interest. Second, the term structure curve at the end of 1999 shifted downward from its position in 1998, and the gaps became negative for the

time-horizon up to two years. This suggests that the BOJ's new regime introduced in early 1999 had successfully affected the market's expectations. More importantly, however, we see no indication of a downward sloping curve, suggesting that the BOJ's commitment was not powerful enough to generate an expectation that the short-term real interest rate would decline below the level of the natural rate counterpart. Third, the term structure curve at the end of 2001 was also upward sloping: to make matters worse, it shifted up substantially from its positions in the preceding years, indicating that quantitative monetary easing combined with a renewed commitment in March 2001 was not strong enough to offset a pessimistic expectation about the future path of the natural rate of interest.[26]

3.2 Inflation targeting to implement the commitment solution

Eggertsson and Woodford (2003a) propose a version of price-level targeting to implement the optimal commitment solution characterized by Jung et al. (2003). However, as mentioned by Eggertsson and Woodford (2003a), price-level targeting is not the only way to implement it, but a version of inflation targeting can also implement the commitment solution. The BOJ's commitment relates the timing to terminate a zero interest rate policy (or quantitative easing policy) to the rate of inflation, so that it should be closer to inflation targeting rather than price-level targeting. In this subsection, we characterize a version of inflation targeting that achieves the commitment solution and compare it with the BOJ's policy commitment.

History dependent inflation targeting We start by defining an output-gap adjusted inflation measure $\tilde{\pi}_t$ as

$$\tilde{\pi}_t \equiv \hat{\pi}_t + \kappa^{-1}\lambda(\hat{x}_t - \hat{x}_{t-1}),$$

and then denote a target for this adjusted inflation by π_t^{Tar}. We also denote the target shortfall by Δ_t^{π} ($\Delta_t^{\pi} \equiv \pi_t^{Tar} - \tilde{\pi}_t$). Given these definitions, we substitute $\phi_{1t} = \kappa\Delta_t^{\pi}$ into equation (2.29) to obtain

$$\pi_t^{Tar} = [1 + \beta^{-1} + \kappa(\beta\sigma)^{-1}]\Delta_{t-1}^{\pi} - \beta^{-1}\Delta_{t-2}^{\pi}. \tag{3.3}$$

Now let us consider the following targeting rule. The inflation target for period 0 is set at zero ($\pi_0^{Tar} = 0$), and the targets for the subsequent periods are determined by equation (3.3). The central bank chooses the level of the overnight interest rate in each period, so that it can achieve the predetermined target level for the adjusted inflation rate. If the central bank successfully shoots the target in each period starting from period 0, then Δ_t^{π} is always zero, therefore the target in each period never deviates from zero. However, if the natural rate of interest falls below zero, the central bank cannot achieve the target even if it lowers the overnight interest rate to zero. Then, Δ_t^{π} takes a positive value, and consequently the predetermined target for the next period becomes higher than zero. Given that the natural rate of interest evolves over time following equation (2.16), the central bank fails to achieve the targets in period 0 and subsequent periods even though it lowers the overnight interest rate to zero. Therefore the central bank must continue a zero interest rate policy until it achieves the target

in some period, which is denoted by $T+1$. Since Δ^{π}_{T+1} equals to zero by definition, ϕ_{1T+1} must equal to zero as well, therefore $T = T^c$ must hold. Put differently, the central bank is able to implement the commitment solution by adopting a version of inflation targeting in which the target inflation rate is updated in each period following (3.3).[27] It is important to note that this inflation targeting has a feature of history dependence since the current target inflation rate depends on the values of the natural rate of interest and the performance of monetary policy in the past.

(Insert Figure 9 here)

The upper panel of Figure 9 shows the evolution of the target inflation rate that is needed to implement the commitment solution presented in Figure 4. The values for the adjusted inflation rate are below its target levels in the first six periods, but the target shortfall in each period gradually decreases until it finally reaches zero in period 6, when the central bank terminates the zero interest rate policy.

A comparison with the BOJ rule The regime of history dependent inflation targeting defined above has some similarities with the BOJ's commitment of continuing a zero interest rate policy (or quantitative easing policy) until some conditions regarding the inflation rate are met,[28] but these two rules differ in some important respects. To show this, we first express the BOJ's target criterion as

$$\hat{\pi}_t = \bar{\pi}^{Tar}.$$

The BOJ chooses overnight call rate in each period so as to achieve this target criterion if is possible; however, if it is not possible due to the zero interest rate bound, it simply sets the call rate at zero.

This BOJ rule differs from the regime of history dependent inflation targeting in the following respects. First, the output gap, x, is completely ignored in the BOJ's targeting criterion, while it plays an important role in the targeting criterion of the history dependent inflation targeting unless λ equals to zero. Put differently, under the BOJ rule, fluctuations in the output gap do not affect the timing to terminate a zero interest rate policy (or quantitative easing policy). Second, the target inflation rate is *never* revised under the BOJ rule, while equation (3.3) requires the central bank to revise the target for the next period depending on whether it successfully shoot the target in the current period.[29] In fact, despite the occurrence of a series of unanticipated adverse events including the failures of major banks, the target inflation rate has never been revised since the introduction of a zero interest rate policy in February 1999: some of the BOJ board members repeatedly showed an adherence to the commitment made in the past and no intention at all to revise its target level of inflation.[30] As seen in equation (3.3), the target inflation rate should have been upwardly revised in response to these additional shocks to the natural rate of interest. The lack of history dependent responses to unanticipated additional shocks implies the suboptimality of the BOJ rule.

To make a quantitative evaluation on the difference between the two rules, we

construct a time-series of π_t^{Tar} using the actual data. Specifically, we assume that the target level for the adjusted inflation rate was zero just before the introduction of a zero interest rate policy, and then compute π_t^{Tar} by substituting the actual values for the inflation rate and the output gap into equation (3.3). The basic idea of this exercise is as follows. If the BOJ rule is very close to the optimal one, then we should observe that the computed target rate is always close to $\bar{\pi}^{Tar}$, say, two percent per year. On the other hand, if the deviation of the BOJ rule from the optimal one is not negligible, then the exercise of computing target inflation using (3.3) would be a wrong one, which could yield unrealistically large numbers for the target rate of inflation.[31] The result presented in the upper panel of Figure 10 clearly shows that the computed target in each period is significantly higher than zero, suggesting that the deviation of the BOJ rule from the optimal one was not small.

(Insert Figure 10 here)

The lower panel of Figure 10 conducts the same exercise but now we take into account supply shocks to make the discussion closer to the reality. If deflation since the late 1990s is at least partly due to supply shocks (or equivalently, changes in relative prices), the target level of inflation that the BOJ seeks to achieve should be lowered to some extent.[32] To incorporate this type of argument into our model, we divide the items contained in the CPI into two subgroups, "goods" and "services", and denote the inflation rate in each sector by $\hat{\pi}_{1t}$ and $\hat{\pi}_{2t}$. The inflation rate in each sector is not necessarily identical, thus the relative price between the two sectors could change over

time. This is the situation in which Aoki (2001) and Benigno (2004) discuss the optimal monetary policy under the assumption of sticky prices. Benigno (2004) searches for a desirable index of the inflation rate that a central bank should target, and finds that it is not the traditional CPI inflation rate (namely, the simple average of the two inflation rates) but

$$\gamma\hat{\pi}_{1t} + (1-\gamma)\hat{\pi}_{2t},$$

where the weight γ is defined by

$$\gamma \equiv \frac{n\alpha_1(1-\alpha_1)^{-1}(1-\alpha_1\beta)^{-1}}{n\alpha_1(1-\alpha_1)^{-1}(1-\alpha_1\beta)^{-1} + (1-n)\alpha_2(1-\alpha_2)^{-1}(1-\alpha_2\beta)^{-1}}.$$

Here α_i represents the probability of no price adjustments being allowed (α_i takes a larger value for more sticky prices). Note that if the core inflation rate defined above equals to zero, the traditional CPI inflation rate ($n\hat{\pi}_{1t} + (1-n)\hat{\pi}_{2t}$, where n represents the CPI weight for the goods sector) equates to $(n-\gamma)[\hat{\pi}_{1t} - \hat{\pi}_{2t}]$.[33] Given that the central bank responds to relative price changes as recommended by Benigno (2004), this implies that (3.3) changes to the following rule[34]

$$\pi_t^{Tar} = (n-\gamma)\left[\hat{\pi}_{1t} - \hat{\pi}_{2t}\right] + [1 + \beta^{-1} + \kappa(\beta\sigma)^{-1}]\Delta_{t-1}^{\pi} - \beta^{-1}\Delta_{t-2}^{\pi}. \tag{3.4}$$

The lower panel of Figure 10 presents the implied target inflation rate π_t^{Tar} computed using equation (3.4).[35] The implied target inflation rate is now much closer to zero as compared with the upper panel, but it still requires high inflation of more than two percent per quarter. This implies that a quantitative difference between history

dependent inflation targeting and the BOJ rule is not trivial even if we take supply shocks into consideration.

4 Fiscal policy in 1999–2004

4.1 Did the Japanese government follow a Ricardian rule in 1999–2004?

It is assumed in section 2 that fiscal policy is passive (or locally Ricardian) in the sense that the government adjusts the primary surplus so that the government's solvency condition is satisfied for *any* path of the price level. In this subsection, we look at the behavior of the Japanese government to see whether or not this assumption has been satisfied since early 1999, when the BOJ introduced a new policy regime.

Evidences from the time-series data A positive linkage between the primary surplus and the real value of public debt is one of the most important implications of Ricardian fiscal policy.[36] Everything else equal, a fall in the price level leads to an increase in the real value of public debt, and then the Ricardian government responds to it by increasing the primary surplus.

(Insert Figure 11 here)

The upper panel of Figure 11 shows the gross public debt (relative to the nominal GDP) on the horizontal axis against the primary surplus (relative to the nominal GDP) on the vertical axis, for 1970–2003. This Figure shows that both variables tend to deteriorate simultaneously in the 1990s, indicating a negative correlation between them.

However, such a correlation may be spurious for the following reasons. First, cyclical fluctuations in economic activities lead to changes in the primary surplus, mainly through changes in tax revenues. Since we are mainly interested in the government's discretionary responses to various shocks, we need to remove the changes in primary surplus due to such an automatic stabilizer. Second, as emphasized by Barro (1986) and Bohn (1998), the government's tax-smoothing behavior could create a negative correlation between the two variables. For example, think about the consequence of a temporary increase in public expenditure. It is possible to increase taxes simultaneously in accordance with it, but changing marginal tax rates over time increases the excess burden of taxation. Therefore, an optimizing government minimizes the costs of taxation by smoothing marginal tax rates over time. This implies that a temporary increase in public expenditures would lead to a decrease in the primary surplus and an increase in the public debt.

(Insert Table 3 here)

Following Barro (1986) and Bohn (1998), we remove these two factors by estimating a regression of the form

$$SURPLUS_t = a_0 + a_1 GVAR_t + a_2 YVAR_t + a_3 DEBT_{t-1} + \upsilon_t, \tag{4.1}$$

where $SURPLUS_t$ is the primary surplus, $DEBT_{t-1}$ is the amount of the public debt at the end of the previous period, $GVAR_t$ is the level of temporary government spending, measured by the deviation of the government spending from its trend, and $YVAR_t$ is

the output gap, measured by the deviation of the GDP from its trend (all relative to GDP).[37] The columns [1] and [2] of Table 3 present the ordinary least squares estimates of this equation for the sample period 1970–2003: the column [1] uses the *gross* public debt while the column [2] uses the *net* public debt.[38] The coefficients on $GVAR$ and $YVAR$ are in the correct sign and statistically significant in both specifications, while the coefficient of our interest, a_3, is almost equal to zero in both specifications, rejecting the Ricardian fiscal policy rule. To see why it is rejected, the lower panel of Figure 11 plots the two variables again, but now the primary surplus is adjusted by subtracting the business cycle component as well as the temporary government spending component ($SURPLUS_t - (a_0 + a_1 GVAR_t + a_2 YVAR_t)$). As seen in the figure, there is indeed a positive correlation between the two variables during the period 1970–1993: the adjusted primary surplus tends to increase by about 0.10 percentage points for one percentage point increase in the public debt, which is close to the corresponding U.S. figures reported in Barro (1986) and Bohn (1998). On the other hand, during the period 1994–2003, we observe a slightly negative correlation between the two variables even after controlling for the business cycle factor and temporary government spending. The lack of a positive relationship in the latter period may be due to low nominal interest rates during the post-bubble period, particularly during the period of the zero interest rate policy and quantitative easing (see equation (2.13)).

To control for fluctuations in nominal interest rates in addition to the business cycle and temporary government spending, we now estimate a regression of the form

$$SURPLUS_t = b_0 + b_1 GVAR_t + b_2 YVAR_t + b_3 INTEREST_t + \upsilon_t, \tag{4.2}$$

where $INTEREST_t$ represents the government's debt interest payments, which corresponds to the expression on the right hand side of (2.13). Note that (4.2) can be a good approximation to (4.1) as long as the interest rate is constant over time, but not so during the period in which the interest rate exhibits a significant fluctuation as it did in the latter half of the 1990s. The estimate of this equation for the same sample period (1970–2003) is reported in the columns [3] (in which *gross* debt interest payments is used) and [4] (in which *net* debt interest payments is used). The coefficients on $GVAR$ and $YVAR$ are almost the same as before, but the coefficient on the debt interest payments is now positive and significantly greater than unity, implying that the Ricardian rule cannot be rejected. These sets of regression results indicate that the Japanese government adjusted the primary surplus in response to changes in the public debt, but only through changes in the debt interest payments.[39]

(Insert Figure 12 here)

Given that the Japanese government behavior was, *on average*, consistent with the Ricardian rule during 1970-2003, Figure 12 looks more closely at the difference between the actual and fitted values for the primary surplus, which can be interpreted as a measure for the deviation from the Ricardian rule.[40] There are three phases

in which the residual takes significant positive values: 1970-74, 1987-92, and 1999-2002. It is not surprising to observe positive residuals in 1987-92, a period of famous episode of fiscal reconstruction during which the Japanese government intensively cut expenditures to achieve a target of "no net issuance of government bonds".[41] But it might be somewhat surprising to observe positive residuals in 1999-2002, during which the Japanese economy had been in the midst of deflation. This result supports the view that the Japanese government started fiscal tightening just after the Obuchi Administration ended in April 2000.[42] It also suggests that policy coordination between the government and the BOJ did not work well during this period, in the sense that the government deviated from the Ricardian rule toward fiscal tightening while the BOJ adopted a zero interest rate policy and quantitative easing.

Evidences from the private sector's forecasts By taking innovations of the log-linear version of equation (2.13), we obtain

$$\begin{aligned}(E_t - E_{t-q})\hat{s}_t &= (1-\beta)^{-1}(E_t - E_{t-q})\hat{i}_{t-1} \\ &\quad +(E_t - E_{t-q})\left\{(1-\beta\theta)[\hat{B}_{t-1} + (\beta\theta)^{-1}\hat{Q}_{t-1}] - \hat{P}_t\right\},\end{aligned}$$

which simply states that the forecast errors in the primary surplus should be positively correlated with those in the real public debt as well as those in the nominal interest rate. This suggests that looking at the correlation between the forecast errors for those variables is another way to test the assumptions of Ricardian fiscal policy. Suppose that the private sector did not expect a change in the monetary policy regime from discretion

to commitment,[43] and that, at the end of 1998, just before the introduction of a new monetary policy regime, they expected the discretionary solution would continue to be realized in the coming years. Given the analysis in section 2, this implies that the private sector should be surprised not only by a change in monetary policy, but also by a shift in fiscal policy toward more expansionary (or less tightening) in 1999 and subsequent years, because the price level should be unexpectedly higher and thus the real debt burden should be unexpectedly lower.

(Insert Table 4 here)

Table 4 compares the forecasts about fiscal policy variables published in December 1998 by the JCER[44] with the corresponding actual values. The fiscal surplus, which is measured by the net saving of the general government (relative to the nominal GDP), was expected to deteriorate over time, starting from -0.085 in FY1999 to -0.117 in FY2003. But this expectation turns out to be too pessimistic: the corresponding actual values were -0.077 in FY1999 and -0.081 in FY2003. These forecast errors seem to be consistent with the theoretical prediction obtained in section 2. However, what is going on behind them is quite different from the theoretical prediction. First, the rate of deflation was higher than expected: very mild deflation in terms of the GDP deflator was expected (0.3 percent per year in 1998–2003), while the actual rate of deflation turned out to be much higher (1.8 percent per year during the same period). Second, in spite of the unexpectedly high rate of deflation, the public debt, measured by the gross debt (relative to the nominal GDP) of the general government at the beginning

of each fiscal year, was lower than expected. For example, the figure for FY2003 was expected to be 1.790 but turned out to be 1.619, mainly due to a slower accumulation of nominal government debt. Third, and most importantly, the combination of an overprediction of the public debt (i.e., an unexpectedly low government debt) and an underprediction of the fiscal surplus (i.e., an unexpectedly small fiscal deficit) is inconsistent with the assumption of Ricardian fiscal policy. Together with the fact that the nominal interest rate was lower than expected,[45] this suggests the possibility that the Japanese government deviated from the Ricardian fiscal policy rule toward tightening.

(Insert Table 5 here)

To investigate further the unanticipated improvement in fiscal deficits, Table 5 shows how forecasts for the amount of public investment were updated over time.[46] The amount of public investment tends to be decided on a discretionary basis; therefore the government's fiscal policy intention should be more clearly seen in its changes. Table 5 shows that downward revisions were consistently made for the years of FY1999, 2000 and 2001, while no substantial revisions were made for FY2002 and 2003. This suggests that an unanticipated shift in fiscal policy stance toward contraction took place around the year 2000.

4.2 Optimal monetary policy under the assumption of non-Ricardian fiscal policy

The above evidence suggests that the Japanese government has been deviating from Ricardian fiscal policy since the latter half of the 1990s. Given that evidence, the next question we would like to address is whether the deviation from Ricardian policy has some implications for optimal monetary policy commitment. As shown by Iwamura and Watanabe (2002) in a model with perfectly flexible prices, the optimal commitment solution differs depending on whether the government follows a Ricardian or a non-Ricardian policy. This is because the government solvency condition implies an equilibrium relation between current and expected future inflation under the assumption of non-Ricardian fiscal policy, so that the central bank must choose between deflation now or deflation later, a tradeoff analogous to the "unpleasant monetarist arithmetic" of Sargent and Wallace (1981). It is important to note that, in this situation, Krugman's (1998) prescription of making a commitment to a higher price level in the future would not work well, as emphasized by Iwamura and Watanabe (2002).

To see how the optimal monetary policy commitment would change, let us conduct the same exercise as we did in section 2.2, but now under the assumption of non-Ricardian fiscal policy. Since the government solvency condition (equation (2.31)) is no longer automatically satisfied, we have to consider (2.31) as an additional constraint for the central bank's loss minimization problem. To simplify the discussion, we assume

that all bonds are perpetual bonds ($\theta = 1$), then (2.31) reduces to[47]

$$\sum_{t=0}^{\infty} \beta^t \hat{\pi}_t = -(1-\beta) \sum_{t=0}^{\infty} \beta^t \hat{s}_t. \tag{4.3}$$

The Lagrangian becomes

$$\begin{aligned} \mathcal{L} \;=\; \sum_{t=0}^{\infty} \beta^t \Big\{ & L_t + 2\phi_{1t}[\hat{x}_t - \hat{x}_{t+1} + \sigma^{-1}(\hat{i}_t - \hat{\pi}_{t+1} - \hat{r}_t^n)] \\ & + 2\phi_{2t}[\hat{\pi}_t - \kappa \hat{x}_t - \beta \hat{\pi}_{t+1}] + 2\mu \left[\hat{\pi}_t + (1-\beta)\hat{s}_t\right] \Big\}, \end{aligned}$$

where μ is a new Lagrange multiplier associated with the government's solvency condition (4.3). Denoting the optimal value of L_t by L_t^*, the Lagrange multiplier μ must satisfy[48]

$$\mu = \frac{1}{2} \frac{\partial \sum_{t=0}^{\infty} \beta^t L_t^*}{\partial \sum_{t=0}^{\infty} \beta^t \hat{s}_t} \geq 0.$$

The difference equation that characterizes the timing to terminate a zero interest rate policy (equation (2.29)) now becomes

$$\phi_{1t} - [1 + \beta^{-1} + \kappa(\beta\sigma)^{-1}]\phi_{1t-1} + \beta^{-1}\phi_{1t-2} = -\kappa[\hat{\pi}_t + \kappa^{-1}\lambda(\hat{x}_t - \hat{x}_{t-1})] - \kappa\mu,$$

and its unique solution is given by

$$\phi_{1t} = -\kappa A(L)[\hat{\pi}_t + \kappa^{-1}\lambda(\hat{x}_t - \hat{x}_{t-1})] - \kappa\mu A(1), \tag{4.4}$$

where the definition of $A(L)$ is the same as before, and $A(1)$ satisfies $A(1) = (1 - \xi_1)^{-1}(1-\xi_2)^{-1} < 0$. Then, it is straightforward to see that if a zero interest rate policy is terminated in the same period as in section 2 (namely, period T^c), ϕ_{1t} takes

a positive value at $t = T^c + 1$, indicating that a zero interest rate policy should be continued longer in the case of non-Ricardian fiscal policy. Put differently, the property of history dependence plays a more important role in the case when the government deviates from Ricardian fiscal policy.

5 Conclusion

Have the Japanese central bank and the government adopted appropriate policies to escape from the liquidity trap? To address this question, we first characterize optimal policy responses to a substantial decline in the natural rate of interest, and then discuss monetary and fiscal policy rules to implement them. Based on this analysis, we compare the optimal policy rules with the actual policy decisions made by the Japanese central bank and the government in 1999–2004.

Our main findings are as follows. First, we find that the optimal commitment solution can be implemented through history dependent inflation targeting in which the target inflation rate is revised depending on the past performance of monetary policy. We compare this optimal rule with the Bank of Japan's policy commitment of continuing monetary easing until some conditions regarding the inflation rate are satisfied, and find that the BOJ rule lacks history dependence in the sense that the BOJ had no intention of revising the target level of inflation in spite of the occurrence of various additional shocks to the Japanese economy. Second, the term structure of the interest rate gap (i.e., the spread between the actual real interest rate and its natural rate counterpart) was not downward sloping, suggesting that the BOJ's

commitment failed to have a sufficient influence on the market's expectations about the future course of monetary policy. Third, we find time-series evidence that the primary surplus in 1999-2002 was higher than predicted by the historical regularity. Also, by comparing the private sector's forecasts with the corresponding actual values, we find a combination of an unexpectedly low government debt and an unexpectedly small fiscal deficit. Such evidence on the government's behavior suggests that the Japanese government deviated from Ricardian fiscal policy toward fiscal tightening. The optimal commitment solution obtained under the assumption of non-Ricardian fiscal policy implies that, given such government's behavior, the central bank should continue a zero interest rate policy longer.

Notes

1 Rogoff (1998) casts doubt on the plausibility of this diagnosis by pointing out that the investment-GDP ratio is well over 20 percent in Japan. Benhabib et al. (2002) show the existence of a self-fulfilling deflationary equilibrium, in which deflation and zero interest rates simultaneously occur even if the natural rate of interest stays above zero. Christiano (2004) investigates the numerical conditions under which the natural rate of interest falls temporarily below zero, using a model with endogenous capital formation.

2 Note that this argument assumes that an adverse shock to the natural rate of interest is not permanent but temporary. Otherwise, future overnight rates are also constrained by the zero lower bound, so that there is no room for lowering the long-

term nominal interest rate. Svensson (2001) names this a *temporary* liquidity trap to emphasize the difference from the original definition by Keynes (1936) in which the long-term nominal interest rate faces the zero bound constraint.

3 For example, Governor Toshihiko Fukui stated on June 1, 2003 that the idea behind the current policy commitment is "to achieve an easing effect by the Bank's commitment to keep short-term rates at low levels well into the future. In this way, even if short-term rates come up against the lower bound, the Bank can still "borrow" from the effect of the future low rates" (Fukui (2003)).

4 The BOJ terminated this commitment in August 2000, and made a new commitment of maintaining quantitative easing policy until "the core CPI registers stably a zero percent or an increase year on year" in March 2001. See Table 1 for the chronology of the BOJ's monetary policy decisions in 1999–2004.

5 With respect to this, Krugman (2000) states, "We assume ... that any implications of the [open market] operation for the government's budget constraint are taken care of via lump-sum taxes and transfers" (Krugman (2000), p. 225).

6 Under the assumption of complete financial markets, the existence and uniqueness of such an asset-pricing kernel follows from the absence of arbitrage opportunities.

7 Here we implicitly assume that the second type of household faces a flow budget constraint similar to (2.1), and that they exhaust their budget constraint.

8 The definition of $\hat{i}_t$ differs slightly from those of the other variables; namely, $\hat{i}_t \equiv \ln(1 + i_t) - \ln(1 + i_t^*)$.

9 The household's intertemporal budget constraint and the market clearing condition imply that $B^*_{t-1,t}/P^*_t = (1-\beta\theta)(1-\beta)^{-1}s^*_t$ holds on the baseline path. We use this to obtain (2.9).

10 See Woodford (2003) for more on the derivation.

11 Here we assume that the short-term nominal interest rate might be zero in the present and subsequent periods, but that it is strictly above zero in the sufficiently remote future, so that $\lim_{\tau\to\infty} E_t[Q_{t,\tau+1}] = 0$.

12 Note that equation (2.5), which is an equilibrium condition related to government solvency, is not a part of the key equations, since it is automatically satisfied as long as the government follows the rule (2.12).

13 Since $\hat{Q}_t = -\hat{i}_t - \sum_{j=1}^{\infty}(\beta\theta)^j E_t[\hat{i}_{t+1} + \hat{i}_{t+2} + \cdots + \hat{i}_{t+j-1}]$, the value of $\hat{Q}_t$ is determined by the path of the short-term nominal interest rate chosen by the central bank. Note that the expectations theory holds locally (i.e., as long as deviations of each variable from its baseline value are small enough).

14 Here we assume that, following Jung et al. (2003), the shock to the natural rate of interest is known in period 0 and that no new information arrives in the subsequent periods. Eggertsson and Woodford (2003a, b) extend the analysis by introducing stochastic disturbances of some special form. It is important to note that certainty equivalence does not hold in our optimization problem because of the non-negativity constraint on nominal interest rates, so that the difference between a deterministic and a stochastic environment is not trivial.

15 Note that, given the assumption that the economy is on the baseline before the natural rate of interest falls in period 0, $\hat{B}_{-1}$ in equation (2.31) must be zero.

16 For example, in the case of $\theta = 0$, in which all bonds are one-period bonds, reductions in the short-term nominal interest rate in the current and future periods have no influence on the current bond price, so that the first term in the squared bracket $((1-\beta\theta)\sum_{t=0}^{\infty}(\beta\theta)^t E_0(\hat{Q}_{0,t}))$ is zero, and the expression in the squared bracket takes a large negative value. On the other hand, if all bonds are perpetual bonds ($\theta = 1$), the expression in the squared bracket equals to zero.

17 See Iwamura and Watanabe (2002) for a similar argument in a setting of perfectly flexible prices.

18 The values for structural parameters are borrowed from Woodford (1999): $\lambda = 0.048/4^2$; $\beta = 0.99$; $\sigma = 0.157$; $\kappa = 0.024$. We assume that $\theta = 0.8$. The initial shock to the natural rate of interest, ϵ_0^n in equation (2.16), is equal to -0.10, which means a 40 percent decline in the annualized natural rate of interest. The persistence of the shock, which is represented by ρ in equation (2.16), is 0.5 per quarter. The parameter values are all adjusted so that the length of a period in our model is interpreted as a quarter.

19 Put differently, this implies that keeping the maturity of government debt longer during peacetime (i.e., on the baseline) is an effective way of insuring against the risk of falling into a liquidity trap. See Iwamura and Watanabe (2002) for more on this point.

20 Admittedly, this simple relationship between the interest rate gap and $\hat{x}_t$ or $\hat{\pi}_t$ depends on the structure of our model. However, Neiss and Nelson (2003) find a similar relationship, through simulation analysis, in a more complicated (and realistic) model with endogenous capital formation, habit persistence in consumption, and price setting of the Fuhrer-Moore type. Also, their empirical analysis using the UK data finds a reasonably strong negative relationship between the interest rate gap and the inflation rate.

21 Laubach and Williams (2003) use the Kalman filter method to estimate a system of equations consisting of the observation equations (i.e., the IS and AS equations) and the transition equations that describe the law of motion for the components of the natural rate of interest. The same methodology is applied to the Japanese data by Oda and Muranaga (2003). We would like to thank Thomas Laubach and John C. Williams for providing us with the program code used in their paper.

22 Specifically, z_t follows a AR (1) process, which is estimated as $z_t = 0.8304 * z_{t-1} + e_t$.

23 *The JCER Mid-term Economic Forecasts*, various issues.

24 The definition of the term structure of interest rate gaps is given in (3.1). Note that gaps are *not* annualized.

25 See Okina and Shiratsuka (2004) for the evolution of the term structure of nominal interest rates during the zero interest rate period.

26 The only example of a downward sloping curve we observe in Figure 8 is the

year 2002 (December 2002), in which the expected one-year real interest rate in each year was close to zero, but the corresponding natural rate was well above two percent, so that the interest rate gap declines by about two percent per year. This might be due to imprecise estimates of the natural rate of interest towards the end of the sample period.

27 Price-level targeting to implement the commitment solution can be derived in a similar way. We define an output-gap adjusted price-level index as $\tilde{P}_t \equiv \hat{P}_t + \kappa^{-1}\lambda\hat{x}_t$, and denote the target shortfall as $\Delta_t^P \equiv P_t^{Tar} - \tilde{P}_t$. Then, substituting $\phi_{1t} = \kappa\Delta_t^P$ into (2.29) leads to an equation describing the evolution of the target price level (equation (3.11) in Eggertsson and Woodford (2003b)). See the middle panel of Figure 9 for the path of P_t^{Tar} to implement the commitment solution. By a similar calculation, we can characterize an instrument rule to implement the commitment solution: $\hat{i}_t = \max\{0 - i_t^*, i_t^{Tar}\}$, where $i_t^{Tar} = \hat{r}_t^n + [1 + \beta\kappa\sigma(\kappa^2 + \lambda)^{-1}]E_t\hat{\pi}_{t+1} + \sigma E_t\hat{x}_{t+1} - \lambda\sigma(\kappa^2 + \lambda)^{-1}\hat{x}_{t-1} + [1 + \beta^{-1} + \kappa(\beta\sigma)^{-1}]\Delta_{t-1}^i - \beta^{-1}\Delta_{t-2}^i$, and $\Delta_t^i \equiv i_t^{Tar} - \hat{i}_t$. See the lower panel of Figure 9 for the path of i_t^{Tar} that implements the commitment solution.

28 For example, Governor Fukui emphasizes the importance of intentional policy delay by stating that the BOJ will continue to implement monetary easing "even after the economy has started to improve and inflationary expectations are emerging" (Fukui (2003)).

29 Most of the discussions about the BOJ's policy commitments have focused on whether $\bar{\pi}^{Tar}$ is high enough to escape from the liquidity trap. However, somewhat

surprisingly, little has been said about the absence of state-contingent responses to additional shocks.

30 However, this does not necessarily mean that the BOJ did not make any response to additional shocks. On the contrary, it responded to them by revising the target for the current account balances very frequently: it has been revised nine times during the last three years. However, as correctly pointed out by Eggertsson and Woodford (2003a), an additional provision of liquidity to the market without any implications about the future course of monetary policy has no effects on the economy as long as the demand for liquidity reaches a satiation level ("Irrelevance proposition").

31 For example, if one substitutes the values of π and x obtained in the discretionary solution (rather than those obtained in the commitment solution) into (3.3), then one would obtain extremely large numbers for the target rate of inflation.

32 With respect to an appropriate policy response to supply shocks, a BOJ policy board member stated, "It would be difficult for monetary policy to control the impact of supply shocks. If monetary policy were to try to control such impacts, it is likely that sustainable price stability would be impaired as production swings became larger and uncertainty regarding investment increased. Therefore, we should accept change in prices due to supply shocks to a certain extent" (Shinotsuka (2000)).

33 As pointed out by Benigno (2004), the traditional CPI inflation rate coincides with the core inflation rate if $\alpha_1 = \alpha_2$ or either of the two is equal to zero.

34 It should be noted that this rule is *not* derived by solving an optimization

problem. However, Kudo et al. (2005) explicitly solves a central bank's loss minimization problem in a two-sector economy with asymmetric sectoral shocks, and obtains an optimal monetary policy rule that is very close to (3.4) in the case when prices are perfectly flexible in one of the two sectors.

35 The values for α_1 and α_2 are taken from the estimates in Fuchi and Watanabe (2002): $\alpha_1 = 0.389$ and $\alpha_2 = 0.853$. Other parameter values are the same as before.

36 Woodford (1998) emphasizes that a positive linkage between these two variables is a necessary but not a sufficient condition for the Ricardian rule to hold, because a similar positive linkage could emerge even under the non-Ricardian fiscal policy rules, through a response of the price level to a change in the expected future primary surplus.

37 $GVAR_t$ and $YVAR_t$ are defined by $GVAR_t \equiv (G_t - G_t^*)/Y_t$ and $YVAR_t \equiv (1 - Y_t/Y_t^*)(G_t^*/Y_t)$, where G_t is the real government spending, Y_t is the real GDP, and G_t^* and Y_t^* represent the trend of each variable estimated by the HP filter. See Barro (1986) for more on the definition of these two variables.

38 The difference between the gross and net figures is not trivial in Japan: for example, the debt-GDP ratio in 2003 is 1.6 for the gross debt, while 0.7 for the net debt. Broda and Weinstein (2004) argue that the net figure should be used to evaluate the Japanese fiscal situation.

39 It should be noted that these results do not necessarily imply that the Japanese fiscal situation is not so bad. First, according to our definition of Ricardian rule (equation (2.12)), a government is required to generate primary surplus only to cover

debt interest payments in each period: it is not required to immediately repay the principal of debts. Given that interest rates are very close to zero, this requirement is not so difficult to fulfill even for a government with huge amount of public debts. Second, our Ricardian government is allowed to ignore "off-balance" debts, such as public pension expenditures that are expected to rise sharply in the near future. That is, a government is allowed to postpone fiscal reconstruction until off-balance items actually change to on-balance items. Our empirical results shown in Table 3 indicate that the Japanese government has a nice track record in the sense that it has not violated the Ricardian criterion at least so far; however, we do not have much to say about what will happen when the central bank turns to monetary tightening, or when public pension expenditures actually start to rise sometime in the future.

40 Here we use the estimates in the column [3] of Table 3; but we obtain the same result even when we use the specification [4] of Table 3.

41 See Ihori et al. (2001) for more on the fiscal reform during this period.

42 See, for example, Iio (2004). According to Iio (2004), the shift in fiscal policy stance toward tightening occurred during the Mori Administration (April 2000 to April 2001) and the Koizumi Administration (April 2001 to the present). Iio (2004) argues that a change in the electoral system from the middle-size district system to the single-member district & PR party lists parallel system has strengthened the influence of the prime minister relative to other political players, thereby creating a political environment for these administrations to start fiscal reconstruction. See, for example,

Persson and Tabellini (2000) for more on the relationship between electoral systems and fiscal policy making.

43 The BOJ had been conducting monetary policy in a discretionary manner before it started a zero interest rate policy (See, for example, Ueda (1993)). Also, Ueda (2000) emphasized the importance of the regime switch from discretion to commitment by stating that "the ZIRP [zero interest rate policy] was a unique experiment in the history of the BOJ not just because the level of the overnight rate was zero but because it involved some commitment about the future course of monetary policy".

44 *The JCER Mid-term Economic Forecast*, December 1998.

45 According to the JCER forecast in December 1998, the government bonds yield (10 years, benchmark) was expected to be 1.40, 1.72 and 1.94 percent in 2001, 2002, and 2003, much higher than the actual values.

46 The Nomura Research Institute, *The NRI Short-term Forecast*, various issues.

47 We continue to assume as before that the economy is on the baseline before period 0, so that $B_{-1} = 0$.

48 As we saw in section 2, the Ricardian government reduces $\sum_{t=0}^{\infty} \beta^t \hat{s}_t$ in response to a substantial decline in the natural rate of interest. The multiplier μ can be interpreted as a measurement of how much the government deviates from Ricardian policy.

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Table 1: Chronology of Monetary Policy Decisions in 1999–2004

Dates	Events
09/09/98	The BOJ reduces the target O/N rate to 0.25 from 0.50 percent
02/12/99	The BOJ introduces a zero interest rate policy (ZIRP)
04/13/99	Governor Hayami announces the BOJ will continue the ZIRP until "deflationary concerns are dispelled"
10/13/99	The BOJ expands the range of money market operations
08/11/00	The BOJ terminates the ZIRP. The target O/N rate is set at 0.25 percent
02/09/01	The BOJ introduces "Lombard-type" lending facility and reduces the official discount rate to 0.375 from 0.5 percent
02/28/01	The BOJ reduces the target O/N rate to 0.125 percent and the official discount rate to 0.25 percent
03/19/01	The BOJ announces to introduce "quantitative monetary easing policy" and continue it until "the core CPI records a year-on-year increase of zero percent or more on a stable basis." The target current account balance (CAB) is set at 5 trillion yen
08/14/01	The BOJ raises the target CAB to 6 trillion yen
09/18/01	The BOJ raises the target CAB to above 6 trillion yen
12/19/01	The BOJ raises the target CAB to 10–15 trillion yen
10/30/02	The BOJ raises the target CAB to 15–20 trillion yen
04/01/03	The BOJ raises the target CAB to 17–22 trillion yen
04/30/03	The BOJ raises the target CAB to 22–27 trillion yen
05/20/03	The BOJ raises the target CAB to 27–30 trillion yen
10/10/03	The BOJ raises the target CAB to 27–32 trillion yen. The BOJ announces more detailed description of its commitment regarding the timing to terminate "quantitative easing policy"
01/20/04	The BOJ raises the target CAB to 30–35 trillion yen

Table 2: Fiscal Adjustments in the Discretionary and Commitment Solutions

	$\theta = 0.1$	$\theta = 0.4$	$\theta = 0.8$	$\theta = 1.0$
Nominal adjustments $\sum_{t=0}^{\infty} \beta^t [\hat{P}_t + \hat{s}_t]$				
Commitment solution (A)	-7.174	-6.569	-3.937	0.001
Discretionary solution (B)	-5.345	-4.749	-2.581	0.035
(A)-(B)	-1.829	-1.820	-1.356	-0.034
Real adjustments $\sum_{t=0}^{\infty} \beta^t \hat{s}_t$				
Commitment solution (C)	-11.009	-10.404	-7.771	-3.832
Discretionary solution (D)	-0.465	0.132	2.300	4.916
(C)-(D)	-10.544	-10.535	-10.071	-8.748

Table 3: Estimates of Fiscal Policy Rules

	[1]	[2]	[3]	[4]
Constant	-0.021	-0.012	-0.079	-0.052
	(0.021)	(0.016)	(0.008)	(0.005)
$GVAR$	-1.904	-1.810	-1.640	-1.546
	(0.842)	(0.818)	(0.311)	(0.410)
$YVAR$	-1.549	-1.256	-2.334	-2.453
	(0.719)	(0.649)	(0.205)	(0.220)
Gross public debt	0.017			
	(0.029)			
Net public debt		0.012		
		(0.062)		
Gross debt interest payments			2.279	
			(0.260)	
Net debt interest payments				3.559
				(0.454)
R^2	0.330	0.322	0.815	0.746
σ	0.022	0.023	0.012	0.014
DW	0.237	0.243	0.515	0.363

Note: Dependent variable is the primary surplus (relative to GDP).

Figures in parenthesis represent standard errors.

Table 4: Private Sector's Forecast about Fiscal Policy

	Forecast	Actual
Net saving of the general government [Relative to the nominal GDP]		
FY1999	-0.085	-0.077
FY2000	-0.095	-0.066
FY2001	-0.105	-0.066
FY2002	-0.113	-0.081
FY2003	-0.117	-0.081
Gross debt of the general government at the beginning of each year [Relative to the nominal GDP]		
FY1999	1.200	1.218
FY2000	1.300	1.329
FY2001	1.510	1.430
FY2002	1.620	1.527
FY2003	1.790	1.619
GDP deflator [FY1998=100]		
FY1999	99.93	98.28
FY2000	99.44	96.39
FY2001	98.98	95.18
FY2002	98.59	93.01
FY2003	98.25	91.35

Note: Forecast was published in December 1998 by the Japan Center for Economic Research (JCER).

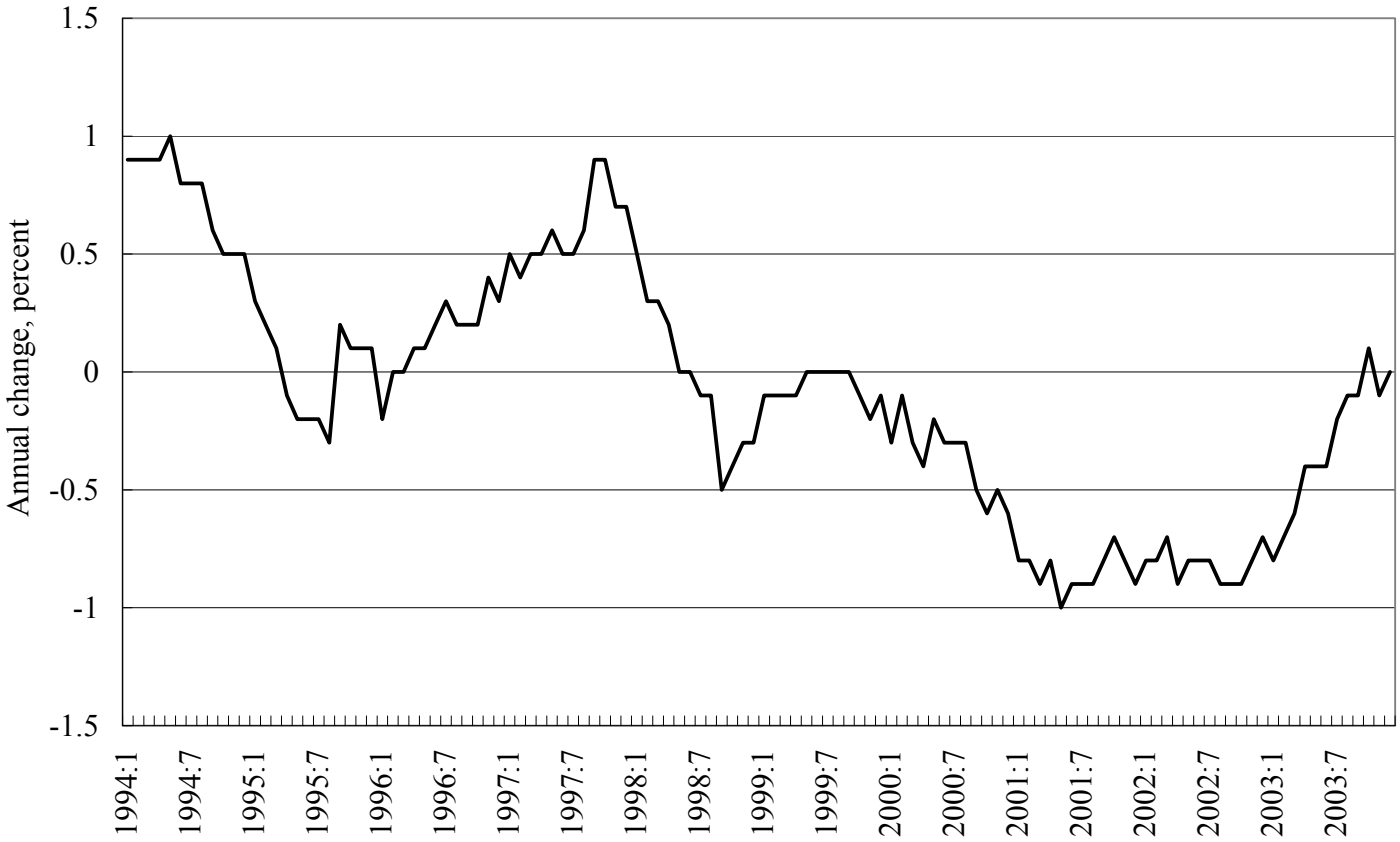
Table 5: Private Sector's Forecast about Public Investment

	The amount of public investment in				
	FY1999	FY2000	FY2001	FY2002	FY2003
Forecasted in					
1999.03	115.2	116.5			
1999.06	112.6	113.8			
1999.09	108.0	111.1			
1999.12	106.3	110.4	110.4		
2000.03	104.6	104.6	104.4		
2000.06	100.8	99.4	98.6		
2000.09		100.2	99.5		
2000.12		94.2	90.8	83.7	
2001.03		96.8	95.0	87.3	
2001.06		92.9	90.1	81.7	
2001.09			88.9	83.0	
2001.12			87.3	86.6	81.8
2002.03			87.9	87.5	83.3
2002.06			88.1	87.0	77.4
2002.09				89.0	79.8
2002.11				82.9	76.7
2003.02				83.4	79.3
2003.05				83.7	77.5
2003.09					77.2
2003.11					75.4
2004.02					74.1

Note: Figures represent forecasts made by the Nomura Research Institute. Index, FY1997=100.

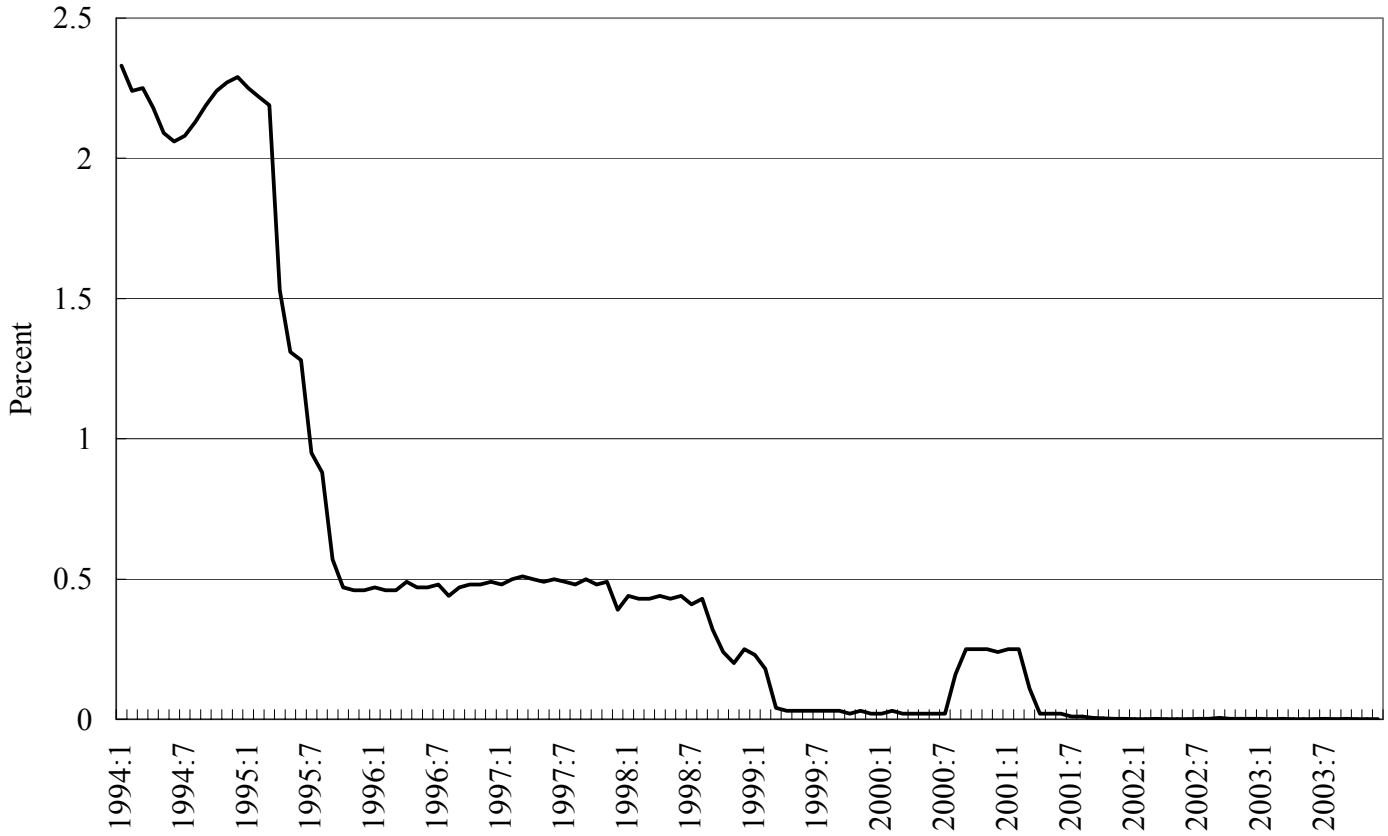
Source: *The NRI Short-term Forecast*, various issues

Figure 1: CPI Inflation



Source:"Consumer Price Index", The Ministry of Public Management, Home Affairs, Posts and Telecommunications.

Figure 2: Uncollateralized Overnight Call Rates



Source: Bank of Japan

Figure 3: Optimal Responses under Discretion

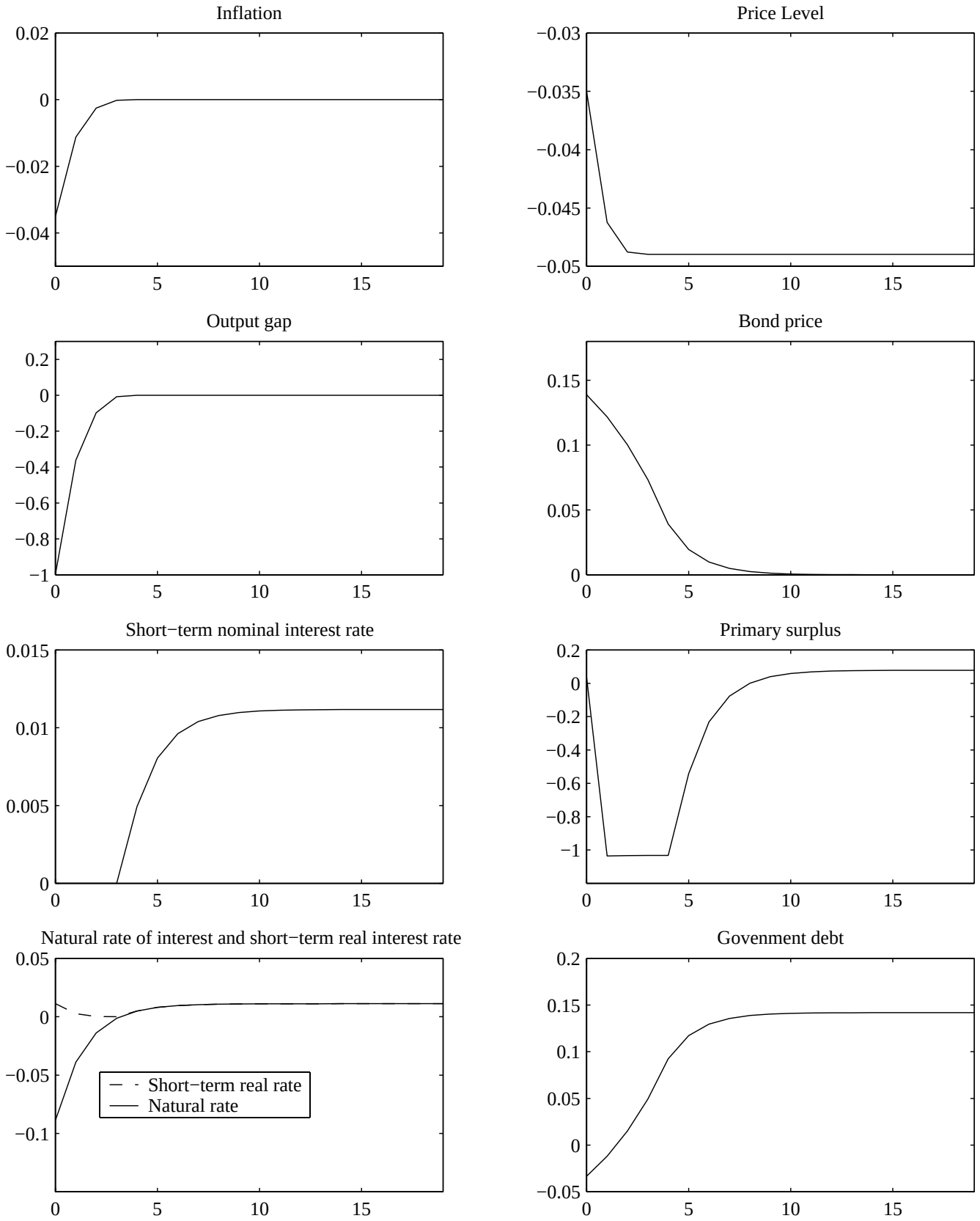


Figure 4: Optimal Responses under Commitment

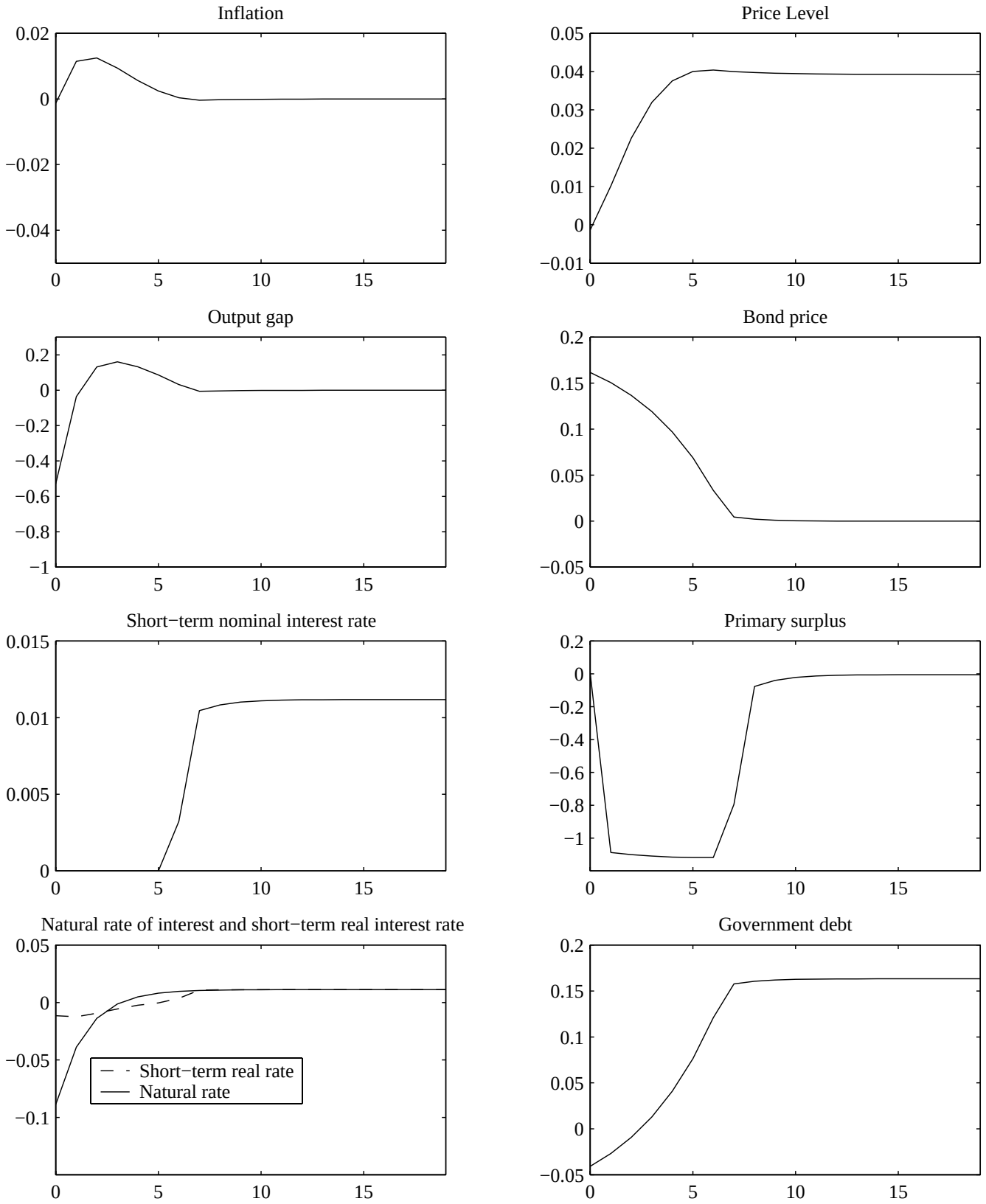


Figure 5: Differences between the Commitment and Discretionary Solutions

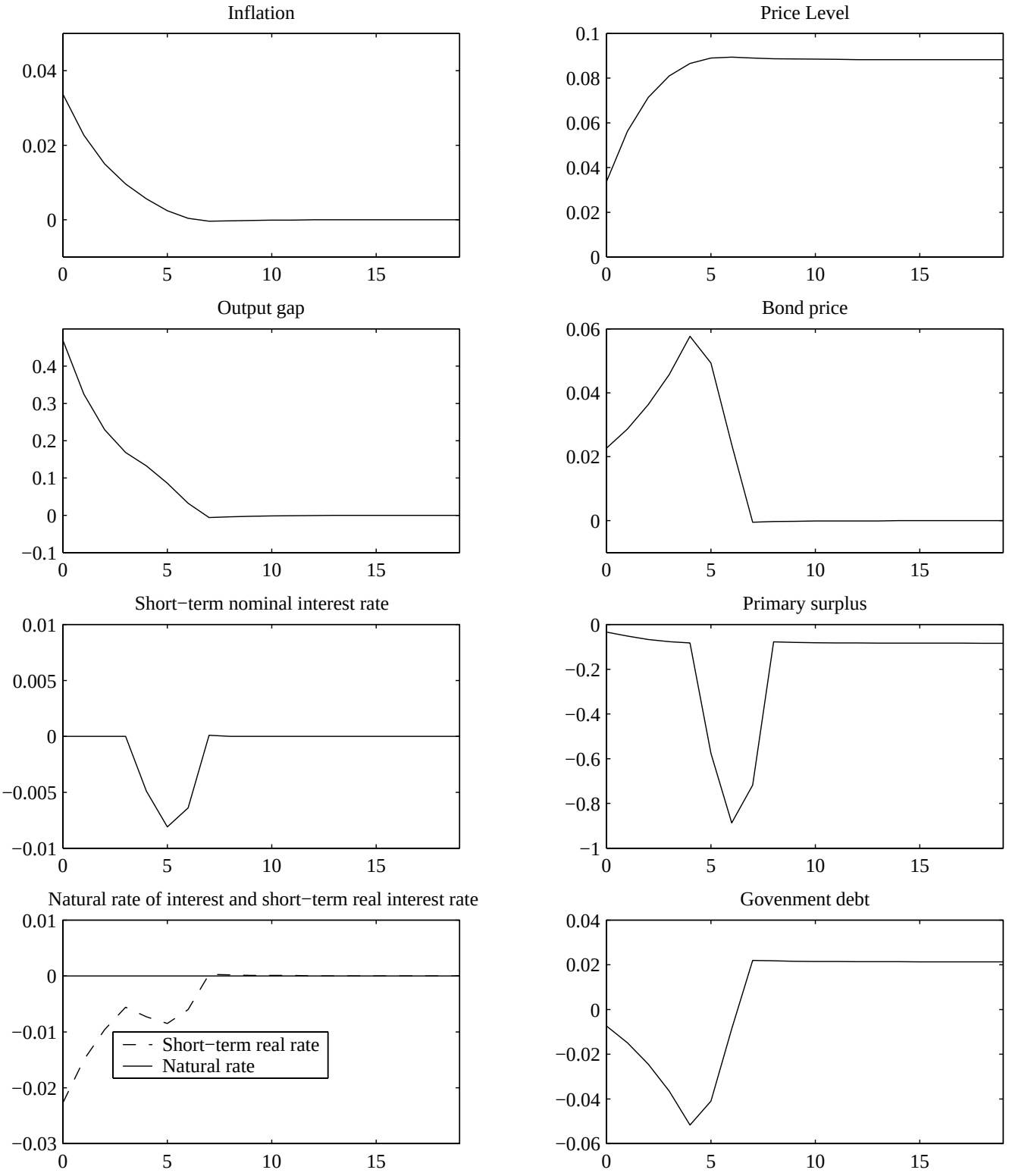
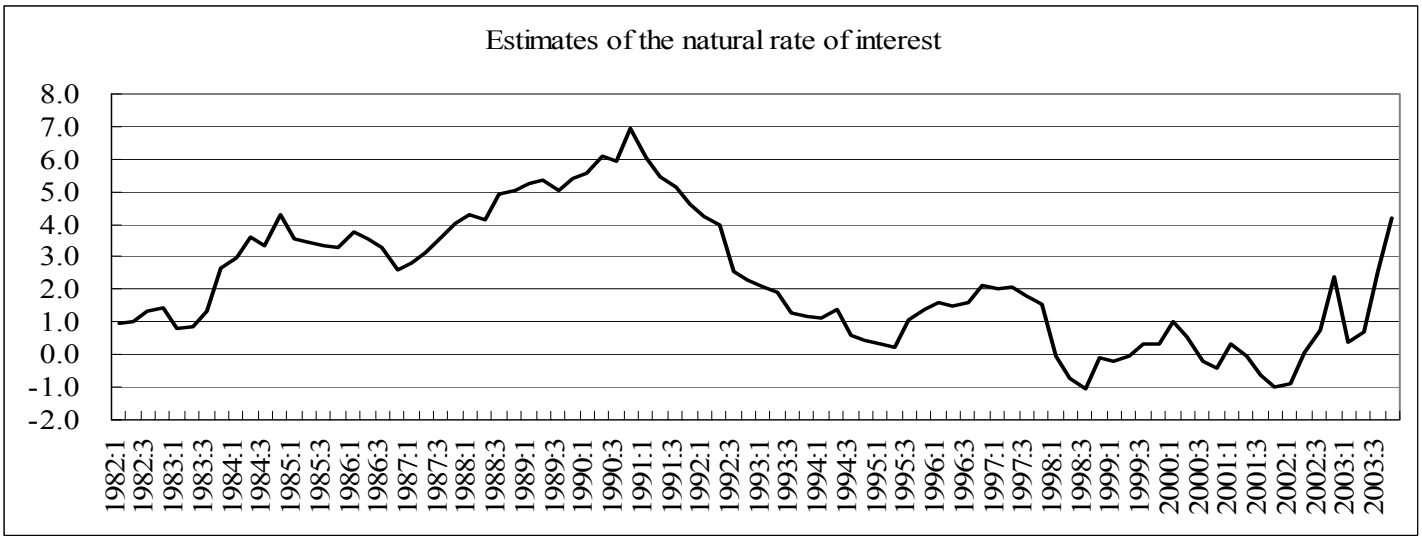
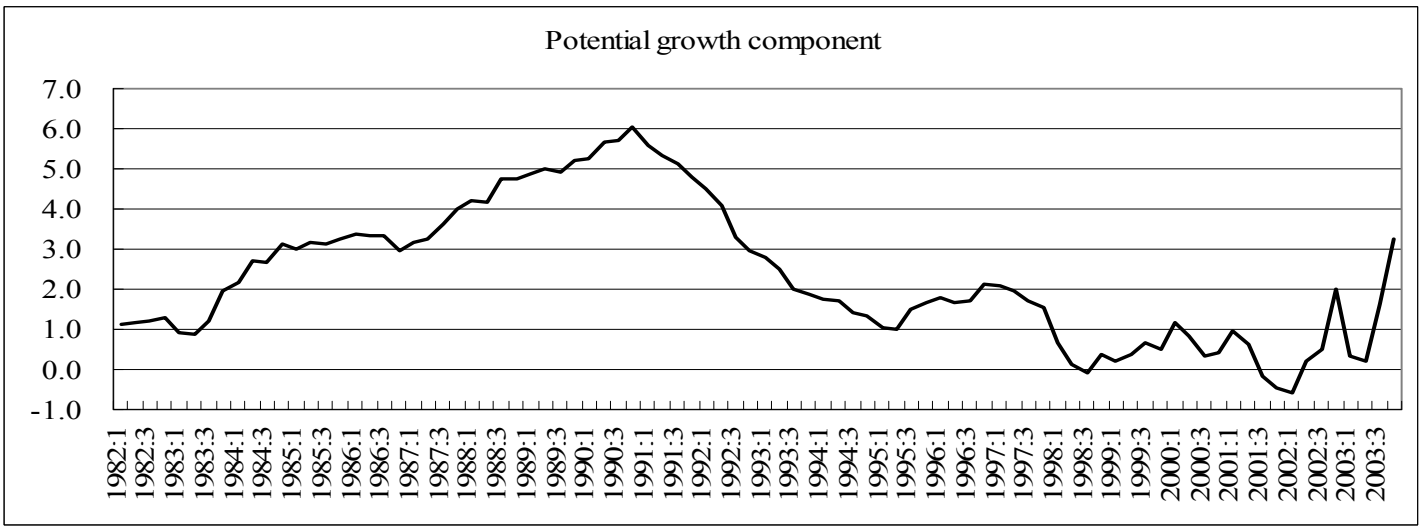


Figure 6: Estimates of the Natural Rate of Interest





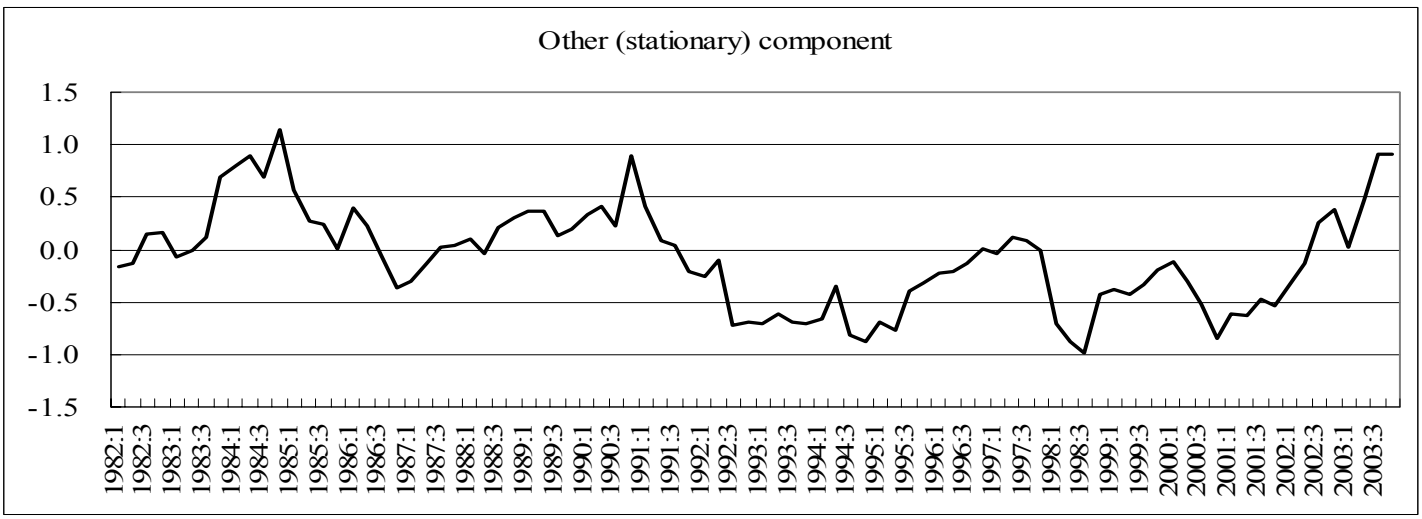


Figure 7: Overnight Interest Rate Gap

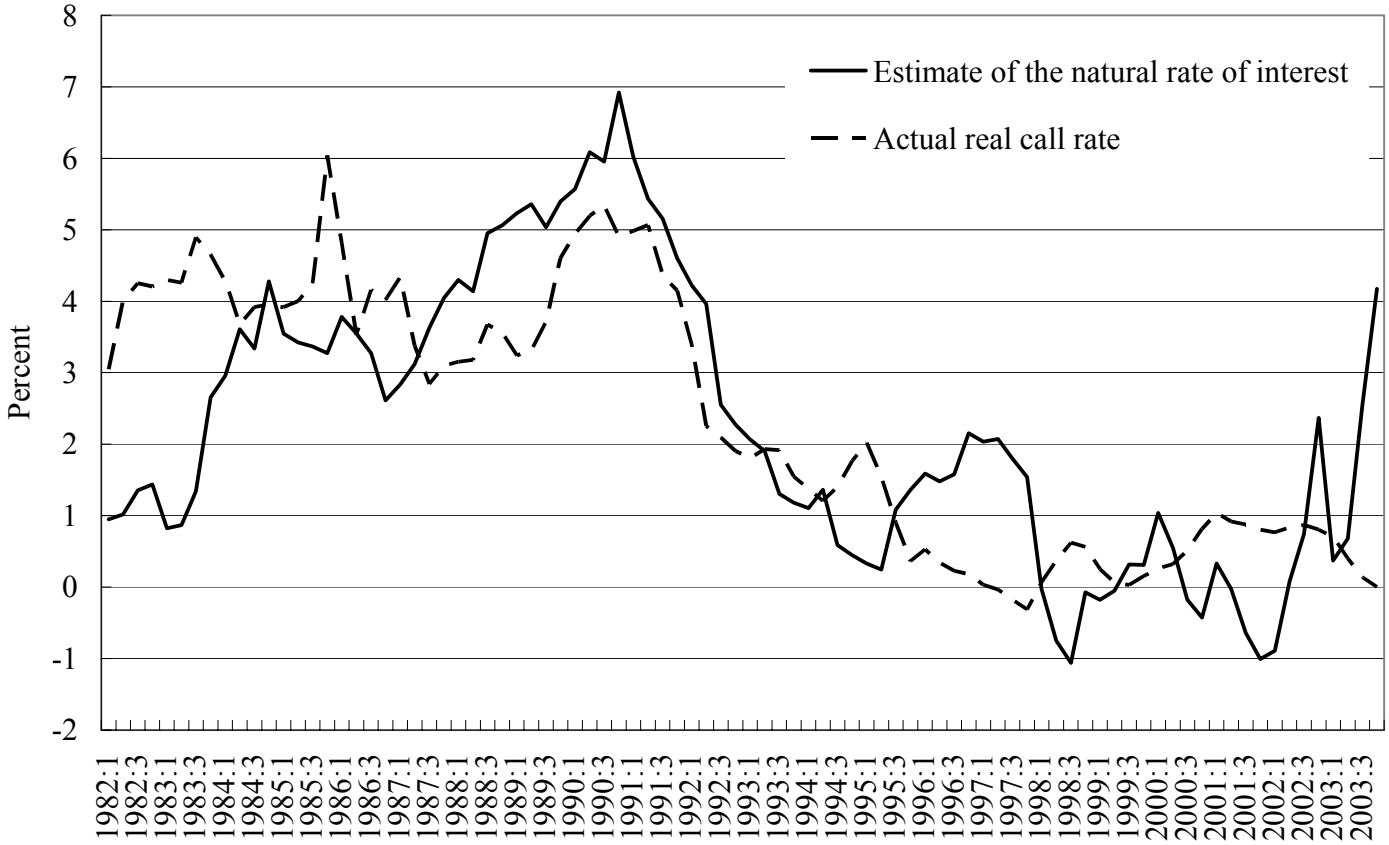


Figure 8: Term Structure of Interest Rate Gaps

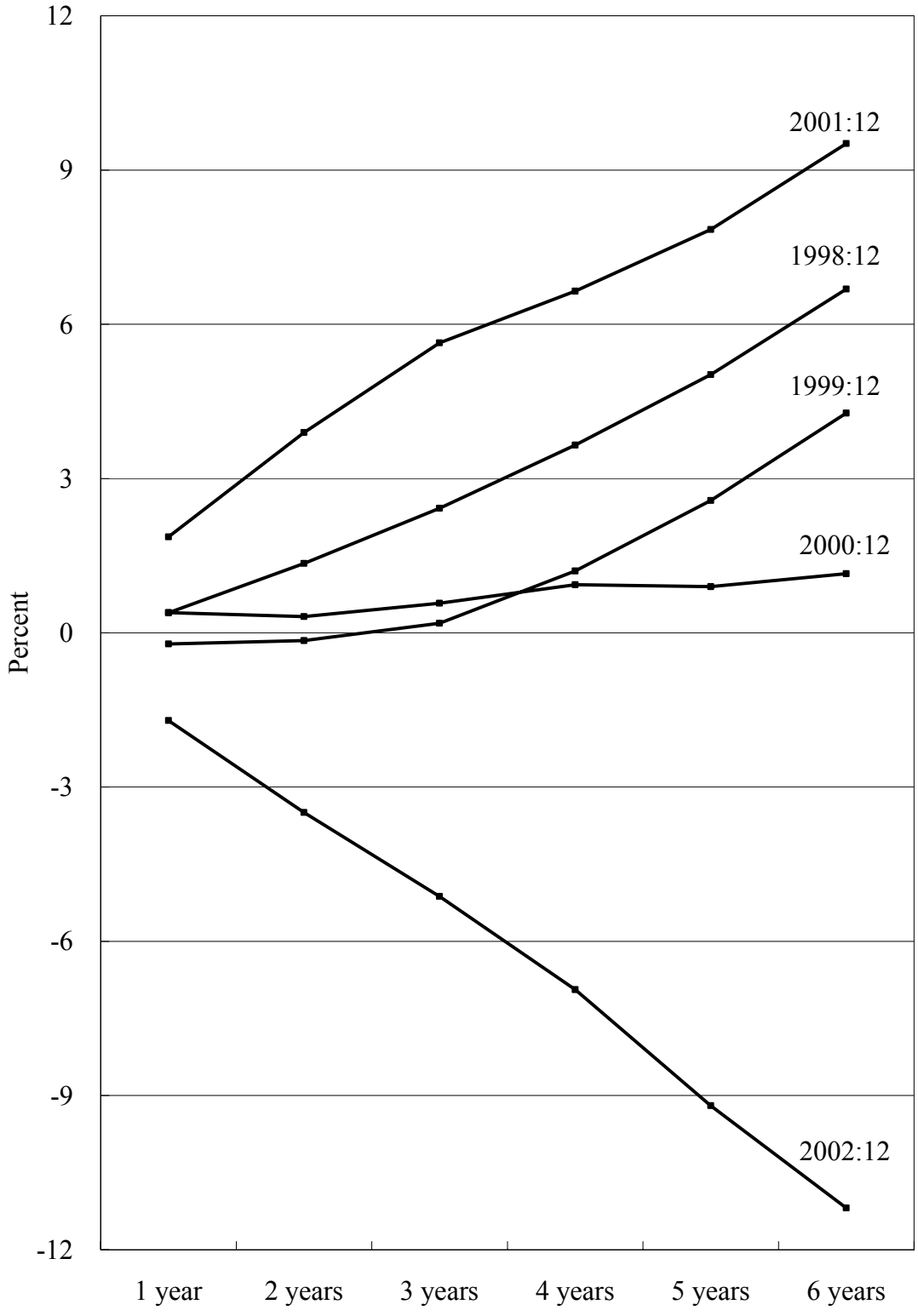


Figure 9: Monetary Policy Rules to Implement the Commitment Solution

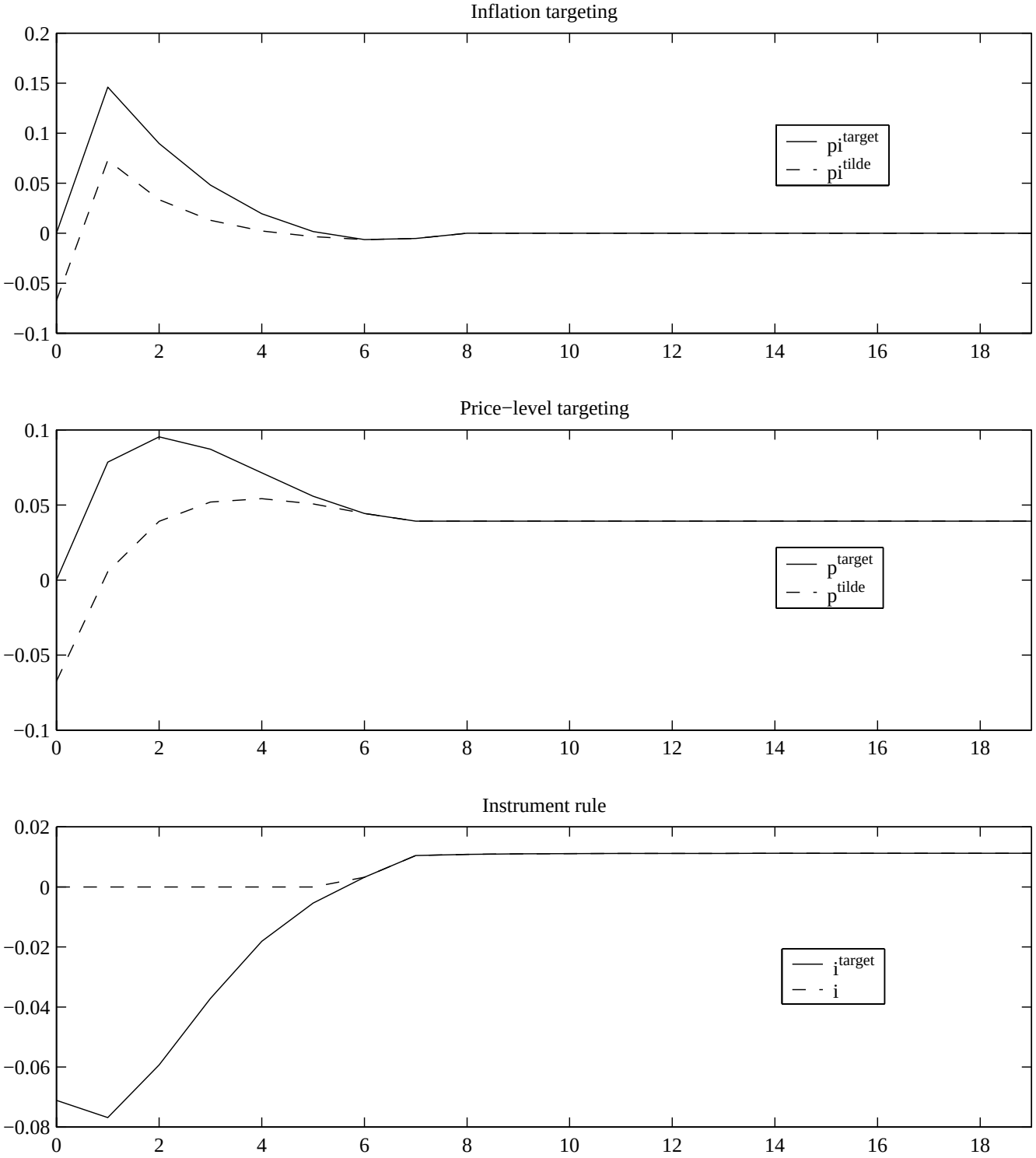


Figure 10: Is the BOJ Rule Close to the Optimal Targeting Rule?

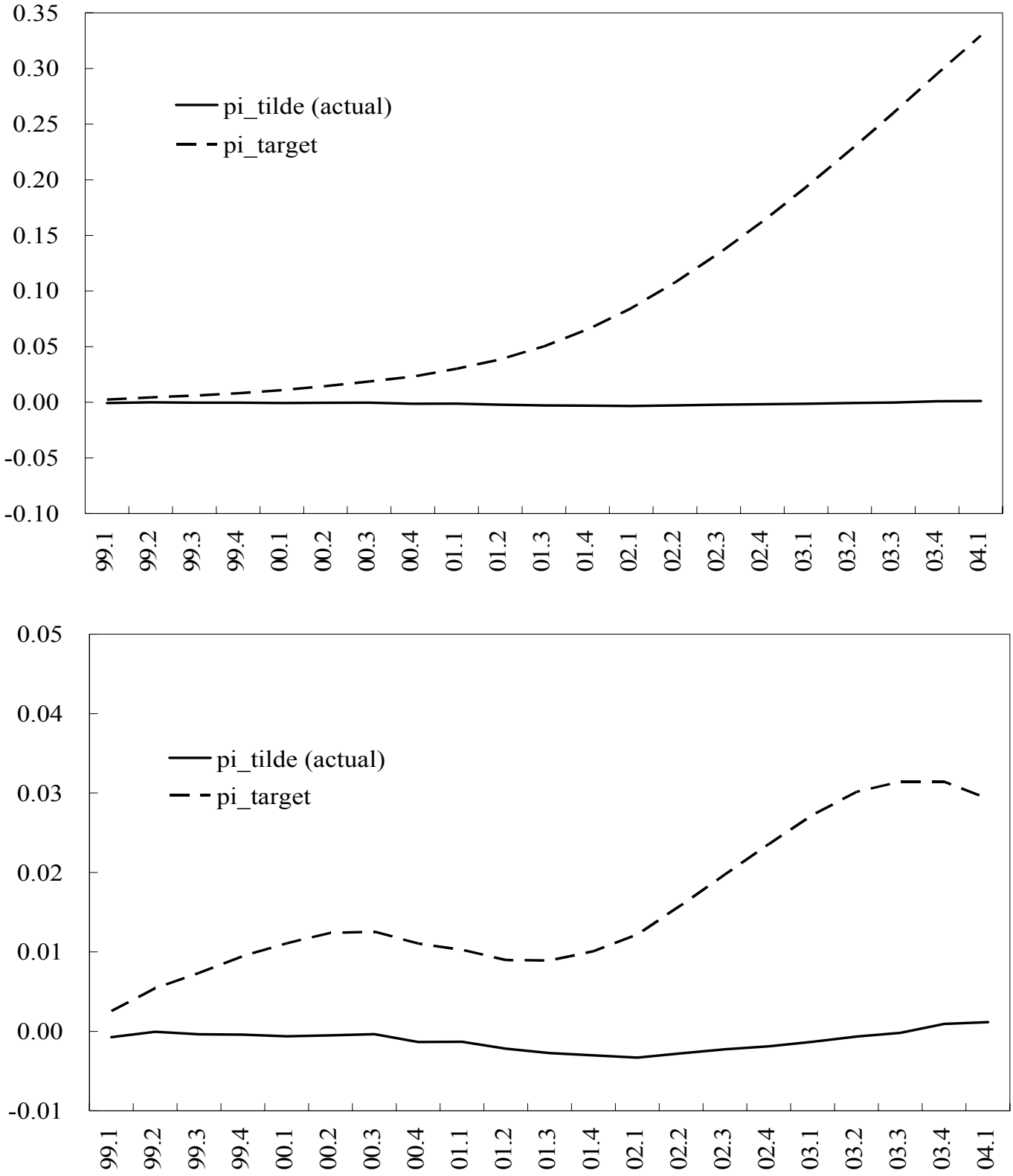
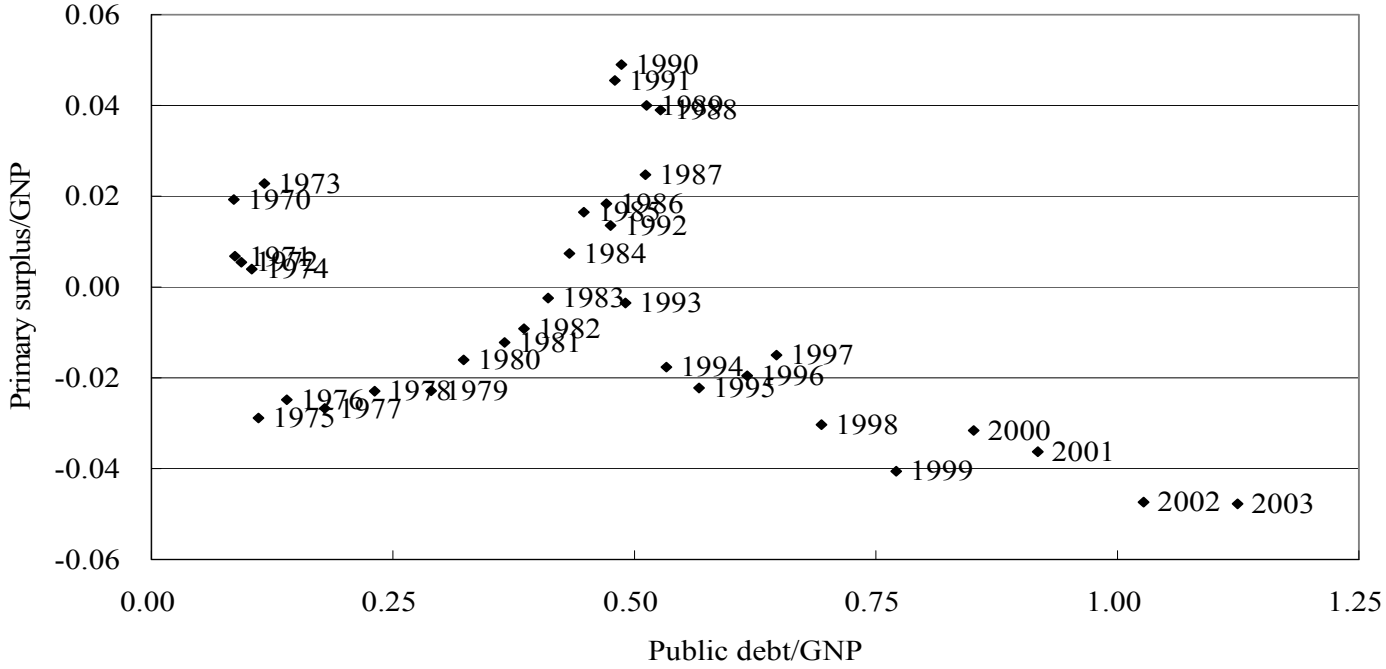


Figure 11: Primary Surplus versus Public Debt, 1970-2003

The simple correlation



The adjusted correlation

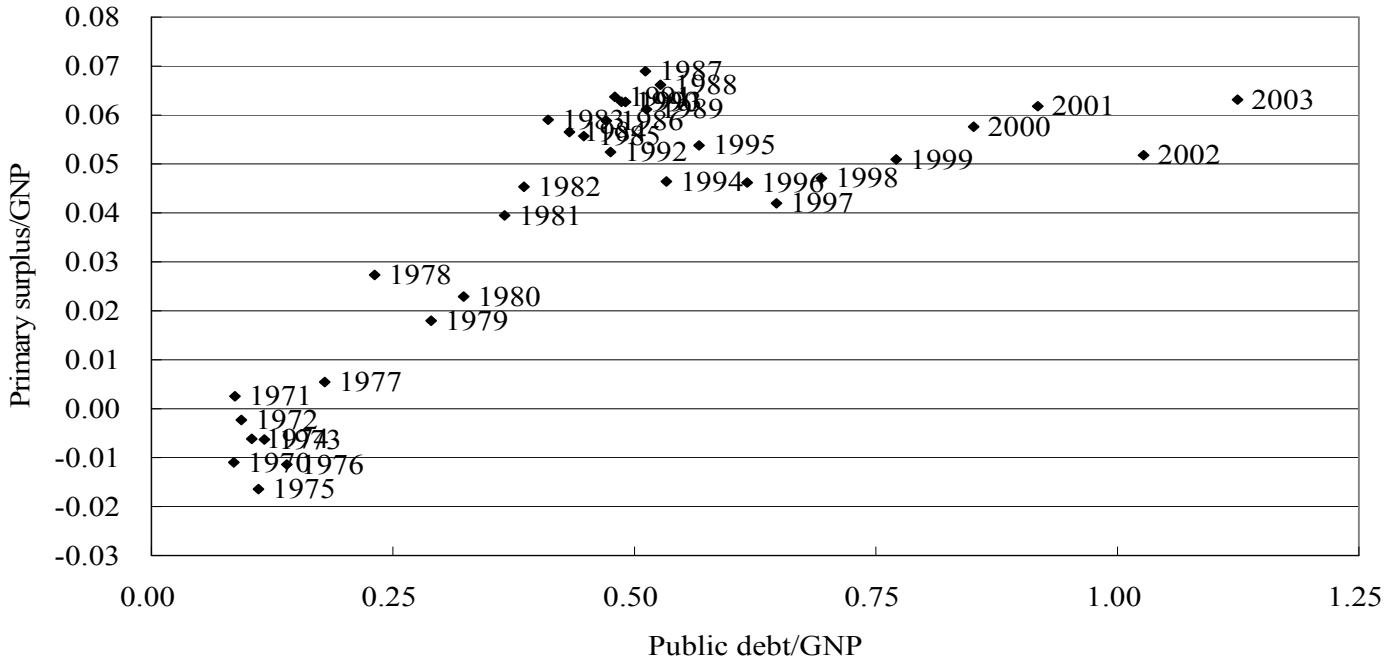


Figure 12: Deviations from the Ricardian Fiscal Policy Rule