# Distribution of Response Words from the Free Association for a Single Stimulating Word

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(Received March 15,2004)

#### **Abstract**

The main target of the association researches was the diagnosis for the mental condition. So, these researches were focused the response time from the stimulating something and the response words which were recalled by the individual answerer, and the association of the arbitrary mass group did not aim at the researches of association.

Author has been researching the association of mass group by seizing the relation between the stimulating word and response words as the probability process. And, author proposed quantities, which are the association entropy (H), the association distance (D) and the quantity of association (A) by using some ideas of information theory.

This paper describes on the distribution of response words at the free association from a single stimulus word to clear the behavior of association entropy and discusses on the influence of the distribution of response words for the association entropy.

**Key Words:** Association Entropy, Distribution of Response Words, Probability for Association

#### 1. Introduction

Association has been researching many problems as one of the world of psychology by many researchers. The main target of these researches was the diagnosis for the mental condition, which are a human complex, psychoanalysis and so on. So, these researches were focused the response time from the stimulating something and the response words which were recalled by the individual answerer, and the association of the arbitrary mass group did not aim at the researches of association.

Itoyama et al. have been researching the association of mass group, for example pupils in the arbitrary classroom, to evaluate the class work. Namely, we have been proposing the method, for which we evaluate the teaching and learning for the

changes of response words in pre-and post association test from the arbitrary stimulating word in the class work [1]-[5].

In case that we treat the association of mass group, we seized the relation between the stimulating word and response words as the probability process, we proposed some quantities, which are the association entropy (H), the association distance (D) and the quantity of association (A) by using some ideas of information theory [6].

In this paper, the author describes on the distribution of response words at the free association from a single stimulating word (SSE) to clear the behavior of association entropy. And the author discusses on the size of answerer's group which is not affected by new response words recalled by new answerer through discussing on the mentioned above.

## 2. Basic Quantities for Response of Association

## 2-1. Model of A Single Stimulating Word (SSE)

Fig.1 shows a schematic diagram on free association for a single stimulating word, which is an association process that is recalled some words (R) from a single stimulating word (SSE). Author defines this process the fee association of SSW.

Assuming that the Z kinds of response words,  $R_1$ ,  $R_2$ , ...,  $R_Z$ , are recalled from single stimulating word, SW, by the number of answerer, M, as shown in Fig.1. And, assuming that the number of response for each response word is  $n_1$ ,  $n_2$ , ...,  $n_Z$ , the number of total response, N, is given in Eq.(1),

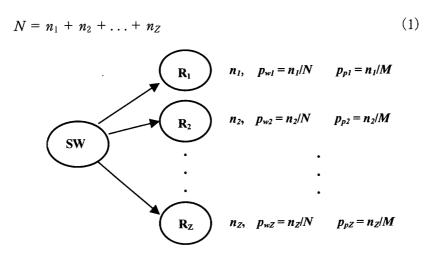


Fig. 1 A schematic diagram on single free association;

SW: stimulating word,

 $R_i$ : *i*th response word

N: total number of response word,

M: the number of answerer

Z: all kinds of response word

 $n_i$ : the number of response for *i*th response word

 $p_{wi}$ : the probability for word,

 $p_{pi}$ : the probability for person

Authors assumed that the ratio of  $n_i$  and N is probability, we represented in  $p_{wi}$ . Namely,  $p_{w1}$ ,  $p_{w2}$ , ...,  $p_{wZ}$  are given in Eq. (2).

$$\begin{array}{ccc}
p_{w1} &= n_1 / N \\
p_{w2} &= n_2 / N \\
\vdots \\
p_{wZ} &= n_Z / N
\end{array}$$
(2)

Author calls  $p_{wi}$  "the probability for word". Where

$$p_{w1} + p_{w2} + \ldots + p_{wZ} = 1 \tag{3}$$

Author can consider that the ratio of  $n_i$  and M is also a kind of probability, so author represented in  $p_{pi}$  for this probability. Namely,  $p_{p1}$ ,  $p_{p2}$ , ...,  $p_{pz}$  are given in Eq. (4).

In this case,

$$p_{b1} + p_{b2} + \ldots + p_{bZ} \ge 1 \tag{5}$$

From Eq.(5), many readers will have some doubts about that this quantity is treated as the probability. However, author can consider that the treatment as the probability is admitted under the discussion on the ratio of each stimulating and response word. So, author calls  $p_{pi}$  "the probability for person".

## 2-2. Association Entropy of Stimulating Word: H

Author will discuss on the entropy of stimulating word. For example, what meaning is that the entropy of stimulating word shows the change of 0.1 bit on the size of answerer? Why does the entropy of stimulating word change between pre-and post association test in the class work for the same stimulating word?

As shown in Fig. 1, assuming that the response words recalled from stimulating

word, SW, are  $R_1$ ,  $R_2$ , ...,  $R_Z$ , the association entropy of stimulating word, H, is given in Eq. (6).

$$H = -\sum_{i=1}^{Z} p_{wi} \log p_{wi} \tag{6}$$

From Eq. (6), we can obtain following equations;

$$H = -\left\{ \left( \frac{n_1}{N} \right) \log \left( \frac{n_1}{N} \right) + \left( \frac{n_2}{N} \right) \log \left( \frac{n_2}{N} \right) + \dots + \left( \frac{n_Z}{N} \right) \log \left( \frac{n_Z}{N} \right) \right\}$$

$$= -\left\{ \left( \frac{n_1}{N} \right) \left( \log n_1 - \log N \right) + \left( \frac{n_2}{N} \right) \left( \log n_2 - \log N \right) + \dots + \left( \frac{n_Z}{N} \right) \left( \log n_Z - \log N \right) \right\}$$

$$= \left( \frac{\log N}{N} \right) \left( n_1 + n_2 + \dots + n_Z \right) - \left( \frac{1}{N} \right) \left( n_1 \log n_1 + n_2 \log n_2 + \dots + n_Z \log n_Z \right)$$

$$(7)$$

Substituting Eq. (1) to Eq. (7),

$$H = \log N - \left(\frac{M}{N}\right) \left\{ \left(\frac{n_1}{M}\right) \log \left(\frac{n_1 M}{M}\right) + \left(\frac{n_2}{M}\right) \log \left(\frac{n_2 M}{M}\right) + \dots + \left(\frac{n_Z}{M}\right) \log \left(\frac{n_Z M}{M}\right) \right\}$$

$$= \log N - \left(\frac{M}{N}\right) \left(\frac{1}{M}\right) (n_1 + n_2 + \dots + n_Z) (\log M)$$

$$- \left(\frac{M}{N}\right) \left\{ \left(\frac{n_1}{M}\right) \log \left(\frac{n_1}{M}\right) + \left(\frac{n_2}{M}\right) \log \left(\frac{n_2}{M}\right) + \dots + \left(\frac{n_Z}{M}\right) \log \left(\frac{n_Z}{M}\right) \right\}$$

$$= \log N - \log M$$

$$- \left(\frac{M}{N}\right) \left\{ p_{p_1} \log p_{p_1} + p_{p_2} \log p_{p_2} + \dots + p_{p_Z} \log p_{p_Z} \right\} (8)$$

Therefore, we can obtain Eq. (9) as the association entropy of stimulus word.

$$H = -\left\{\log\left(\frac{M}{N}\right) + \left(\frac{M}{N}\right) \sum_{i=1}^{Z} p_{pi} \log p_{pi}\right\}$$
(9)

Equations (8) or (9) are the theoretical equation which expresses the relation between the entropy of stimulating word and sample size, which means the number of answerer.

In Eq.(9), the first term is related to the mean value of the number of response word and the second term depends on the kinds of response word and these distribution.

The association entropy expresses the situation of disorder of response words for

the stimulating word. In other words, the association entropy expresses whether the schema, which is the knowledge, concept, image and so on, which the answerer group has from the stimulating word is divergent or convergent. This viewpoint is the important thing for discussing the evaluation for the class work. Because the result of learning is changed like as the divergent or the convergent one by the instruction for learning and the learning material. It is very effective method that the association method is able to evaluate the instruction for learning and the learning material.

#### 2-3. On the Association Test

The association test was enforced for students in Nagasaki University under the condition of the free association from a single stimulating word and 30 seconds in association time. In this work, author employed "run", "believe", "white", "stinking", "friend" and "angry" as the stimulating word. Answerers wrote the response words on the test paper freely in the limited time.

#### 3. Results and Discussion

# 3-1. The Change of Number of Response Word (N) and Kinds of Response Word (Z)

Fig. 2 shows the relation between the number of answerer, M, and the number of response word, N, with changing sample size for some stimulating words. As shown in Fig. 2, this shows the linear relation. In Fig. 2, this gradient is the mean value of number of response word. And, this figure shows that the gradient does not depend on the sample size. The linear relation can be found for all stimulating word, but the gradient of the linear relation depends on the quality of the stimulating word largely.

Fig. 3 shows the relation between all kinds of response word (Z) and the number of answerer (M). As shown in Fig. 3, this relation between Z and M is not a linear relation. The increasing rate of Z decreases with increasing the number of answerer.

Fig. 4 shows the relation between the different response words per person (Z/M) and the number of answerer (M). As shown in Fig. 4, the values of (Z/M) decrease with increasing the number of answerer and these values are 1.5 to 2.8 (words/person) in 100 answerer at the sample size. In case that the sample size, which means the number of answerer, becomes large, it is very reasonable that the probability of which the response words of new one answerer overlap to the other response words is very high. The stimulating word which has high value of (Z/M) corresponds to high value of the association entropy.

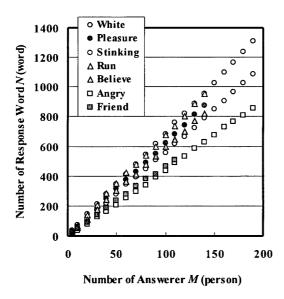


Fig. 2 The number of response word (N) vs. the number of answerer (M).

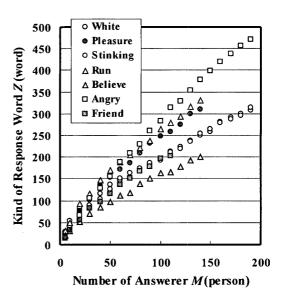


Fig.3 The relation between the number of answerer (M) and the all kind of response word (Z).

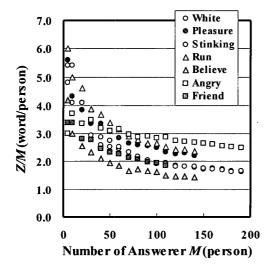


Fig.4 The different response words per person (Z/M) vs. the number of answerer (M).

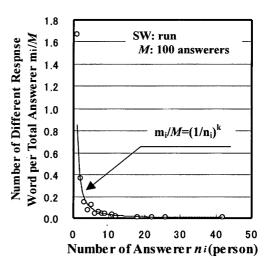


Fig. 5 Numbers of different response words per person (m/M) vs. the number of answerer  $(n_i)$ .

#### 3-2. Distribution of the Number of Response Word

Fig.5 shows the relation between numbers of different response words per answerer and number of answerer at the sample size of 100 answerers. The stimulating word is "run". In this Fig.5, the response words that only one answerer recalled from "run" appear about 170 words and the response words that two answerers

recalled appear about 40 words. In many cases, the response word that many answerer, for example 30 % or more answerers, will recall is one word. So the value of  $m_i$  will approach to zero in larger value of  $n_i$  and the approximate carve shows the type of  $(1/n_i)^k$ .

For discussing the distribution of the number of response word, we will propose the schematic diagram of free association from single stimulating word (SSE) as shown in Fig. 6. In case that we will treat the association entropy, we ought to study the distribution of  $m_i$ .

At first, author will discuss the distribution of  $m_1$ , which is the response word recalled by only one answerer. Because, the numbers of  $m_1$  are many and these have a great influence upon the association entropy. Fig. 7 shows the relation between numbers of  $m_1$  per answerer  $(m_1/M)$  and probability for person  $(p_{pi})$ . As shown in Fig. 7, values of  $(m_1/M)$  decrease linearly with decreasing probability for person  $(p_{pi})$  under the condition of value at 0.05 or less. This value corresponds to 20 answerers or less.

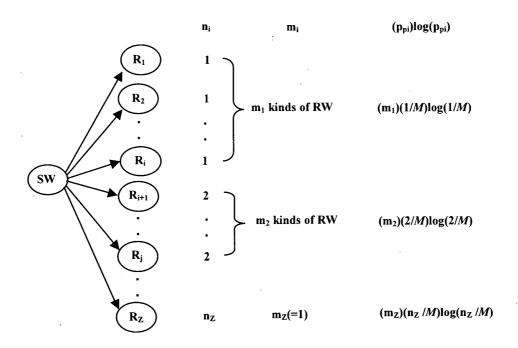


Fig. 6 A schematic diagram of Free Association for SSW and association entropy.

SW: stimulating word R<sub>i</sub>: response word

 $m_1$ : numbers of response words recalled by only one answerer

 $m_2$ : numbers of response words recalled by two answerers

 $m_Z$ : numbers of response words recalled by Z answerers

Fig. 8 shows the relation between numbers of  $m_1$  per answerer  $(m_1/M)$  and probability for word  $(p_{wi})$ . In also case, values of  $(m_1/M)$  decrease linearly with decreasing probability for word  $(p_{wi})$  under the condition of value at 0.007 or less. In Fig. 8, the point of which the value of  $(m_1/M)$  is 0.0075 is very important to change the gradient of  $(m_1/M)$ . This point is correspondent to the numbers of answerer at 20 persons. Namely, in case that sample size is over 20 answerers or more, this means that the influence of which each individual answerer has becomes smaller. In case that we will apply the association entropy for the smaller group at 20 persons or less, we must pay attention above mentioned point.

## 3-3. The Changes of Distribution of $m_1$ and $m_2$

The changes of distribution of  $m_1$ ,  $m_2$  and  $m_3$  are shown in Fig. 9. Fig. 9 shows the relation between values of  $m_1$ ,  $m_2$ ,  $m_3$  and the probability for person  $(p_{pi})$ . As shown in Fig. 9, the values of  $m_1$ ,  $m_2$  and  $m_3$  decrease with increasing the value of  $(p_{pi})$  as well as the trend of Fig. 5. Namely, the numbers of response words recalled by only one answerer  $(m_1)$  increase with increasing the numbers of answerer. The changes of  $m_2$  also shows as same trend as  $m_1$ . Because, the probability for person  $(p_{pi})$  is in inverse proportion to the numbers of answerer. The changes of which the numbers of response words are recalled by 3 answerers or more are not different so much. In case that the value of  $n_i$  is 3 or more, the value of  $m_i$  is estimated about 10 to 1.

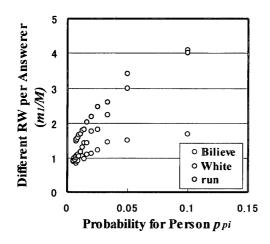


Fig. 7 Numbers of  $m_1$  per answerer (m / M) for each stimulating word vs. probability for person  $(p_{pi})$ .

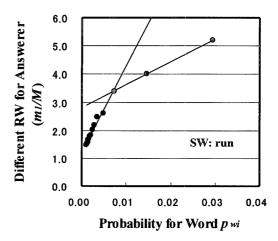


Fig. 8 Numbers of  $m_1$  per answerer (m/M) vs. probability for word  $(p_{wi})$ ; Stimulating Word is "run".

#### 4. Conclusion

- (1) The different response words per person (Z/M) decrease with increasing the number of answerer and these values are 1.5 to 2.8 (words / person) in 100 answerer at the sample size.
- (2) The stimulating word which has high value of (Z/M) corresponds to high value of the association entropy
- (3) The values of  $(m_1/M)$  decrease linearly with decreasing probability for person  $(p_{pi})$  under the condition of value at 0.05 or less. This value corresponds to 20 answerers or less.
- (4) The numbers of response words recalled by only one answerer  $(m_1)$  increase with increasing the numbers of answerer.

The changes of  $m_2$  also shows as same trend as  $m_1$ .

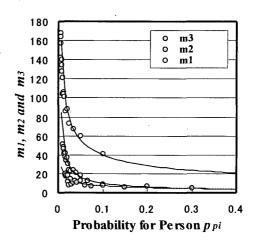


Fig. 9 Numbers of  $m_1$ ,  $m_2$  and  $m_3$  vs. probability for person  $(p_{pi})$ ; Stimulating Word is "white".

## Acknowledgement

This work is supported in part by a Grant-in-Aid for Scientific Research, which number is C-15530596, from the Ministry of Education, Culture, Sports, Science and Technology.

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