

# Influence of Cable Loosening on Nonlinear Parametric Response of Inclined Cables

by

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The effect of cable loosening on the nonlinear parametric vibrations of inclined cables is discussed in this paper. In order to calculate loosening for inclined cables without a small-sag limitation, it is necessary to first derive equations of motion for an inclined cable. Using these equations and the finite difference method, the effect of cable loosening on the nonlinear parametric responses of inclined cables under periodic support excitation is evaluated. A new technique that takes into account flexural rigidity and damping is proposed as a solution to solve the problem of divergence. The regions that generate compressive forces in inclined cables are also shown.

**Key words:** cable loosening, nonlinear vibration, inclined cable

## 1. Introduction

In conventional nonlinear vibration analysis of cables, the equations of motion are formulated based on the assumptions that the cable is a continuum that resists only axial forces, and that it is based on the same laws that apply to truss members. In other words, the cables are assumed to be able to resist both tensile and compressive axial forces [1, 2, 3]. However, the assumption is invalid when the sum of the initial and deflection-induced additional horizontal tensions produces compression, since the actual cables have no resistance to compressive forces. This situation may be easily observed in the nonlinear vibration of cables related to wind-rain vibration [4, 5, 6], wind uplift, strong earthquakes, etc. Therefore, it is necessary to evaluate the effect of cable loosening in handling the nonlinear vibrations of cables. An analysis that considered cable loosening was carried out on the stay cables of a cable-stayed bridge subjected to strong ground motions [6]. The nonlinear loosening effect has also been found in the hangers on the main cables of cable suspension bridges [7, 8, 9]. Cable loosening has been pointed out in the low-tension single cables such as power and signal transmission cables, underwater cables and mooring lines in offshore applications [10–15]. Nonlinear dynamic responses and vibrations of low-tension cables evaluated by introduced the small flexural rigidity of the cable to remove the

singularity when cable tension becomes zero [12, 13, 14]. Low-tension cable dynamics were solved by the finite difference method using the modified box scheme [15].

There appear to be few published research papers that take into account both loosening and the mass of the taut cables with small sags which include extension of cables. Therefore, the authors proposed a new technique for evaluating cable loosening and also examined the effect of loosening on the nonlinear vibrations of horizontal cables with small sags [16, 17]. The nonlinear equations of motion of a cable formulated as a continuum were made discrete by using the explicit formula of the finite difference method and assuming that the cable has no compressive resistance. The problem of divergence was solved by a proposed new technique that takes into account flexural rigidity and damping which cables possess as physical properties. Finally, the authors discussed the effect of cable loosening on the responses and the regions that generate compressive forces from nonlinear forced vibrations and parametric vibrations, focusing on horizontal cables with small sags. Research into nonlinear forced response [16] showed that loosening can easily occur in cable with a sag-to-span ratio corresponding to the region in which the mode transitions from the lower mode to the higher mode under periodic vertical loading. Research into nonlinear parametric response [17] revealed that cables with a specific

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sag-to-span ratio easily become loosened. In the range that is generated by cable loosening, the principal unstable region is larger than the second unstable region.

This paper discusses the effect of loosening on the nonlinear parametric vibrations of inclined cables. When a cable is inclined, the additional properties are different from those of a horizontal cable [2]. The crossover of natural frequencies of a symmetric mode toward the natural frequencies of an antisymmetric mode never occurs and the corresponding modes are neither symmetric nor antisymmetric. Therefore, in order to calculate loosening for inclined cables without a small-sag limitation, it is necessary to first derive equations of motion for an inclined cable. Using these equations and the proposed method, it is possible to evaluate the effect of loosening on the nonlinear parametric responses of inclined cables under periodic support excitation. In order to evaluate both the common and different properties of the inclined cables and horizontal cables expressed in Ref. [17], this paper looks at cables with small sags. The influence of the inclination angle on cable loosening is evaluated, and the regions that generate compressive forces in inclined cables are shown.

## 2. Equations of Motion for an Inclined Cable

An inclined cable with an inclination angle  $\theta$ , a uniform cross section and uniform weigh per unit length hanging between two points, as shown in Figure 1, is analyzed. In this paper, the longitudinal and transverse directions ( $x^*$ ,  $z^*$ ) of the corresponding string are called as the local coordinate system.

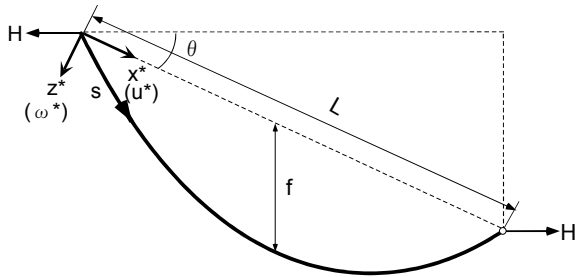


Figure 1 Geometry of an inclined cable

In the local coordinate system ( $x^*$ ,  $z^*$ ) of an inclined cable, the following equations are obtained by removing the self-weight term in the equations of motion.

$$m \frac{\partial^2 u^*}{\partial t^2} - \frac{\partial}{\partial s} \left( \Delta T \frac{dx^*}{ds} + (T + \Delta T) \frac{\partial u^*}{\partial s} \right) = p_{x^*}(x^*, t) \quad (1)$$

$$m \frac{\partial^2 w^*}{\partial t^2} - \frac{\partial}{\partial s} \left( \Delta T \frac{dz^*}{ds} + (T + \Delta T) \frac{\partial w^*}{\partial s} \right) = p_{z^*}(x^*, t) \quad (2)$$

where  $\Delta T$  is the additional tension generated,  $T = H / (\cos \theta \cdot dx^* / ds - \sin \theta \cdot dz^* / ds)$  is the initial tension obtained from the initial horizontal tension  $H$ ,  $u^*$  is the longitudinal displacement in the  $x^*$  direction,  $w^*$  is the transverse displacement in the  $z^*$  direction,  $p_{x^*}(x^*, t)$  and  $p_{z^*}(x^*, t)$  are the loads in the  $x^*$  and  $z^*$  directions,  $s$  is the coordinate along the cable,  $m$  is the mass per unit length of the cable and  $t$  is the time.

The additional tension  $\Delta T$  can be obtained from

$$\Delta T = EA \left\{ \frac{dx^*}{ds} \frac{\partial u^*}{\partial s} + \frac{dz^*}{ds} \frac{\partial w^*}{\partial s} + \frac{1}{2} \left( \frac{\partial u^*}{\partial s} \right)^2 + \frac{1}{2} \left( \frac{\partial w^*}{\partial s} \right)^2 \right\} \quad (3)$$

where  $E$  is the Young's modulus and  $A$  is the cross-sectional area of the cable.

By considering the effects of flexural rigidity and damping, the calculation is able to stably handle cable loosening, as proposed in the previous papers [16] and [17]. Equations (1) and (2) are rewritten as

$$m \frac{\partial^2 u^*}{\partial t^2} + c \frac{\partial u^*}{\partial t} + EI \frac{\partial^4 u^*}{\partial s^4} - \frac{\partial}{\partial s} \left( \Delta T \frac{dx^*}{ds} + (T + \Delta T) \frac{\partial u^*}{\partial s} \right) = p_{x^*}(x^*, t) \quad (4)$$

$$m \frac{\partial^2 w^*}{\partial t^2} + c \frac{\partial w^*}{\partial t} + EI \frac{\partial^4 w^*}{\partial s^4} - \frac{\partial}{\partial s} \left( \Delta T \frac{dz^*}{ds} + (T + \Delta T) \frac{\partial w^*}{\partial s} \right) = p_{z^*}(x^*, t) \quad (5)$$

where  $c$  is the damping coefficient and  $I$  is the geometrical moment of inertia.

By making equations (3), (4) and (5) non-dimensional by means of  $H \sec \theta$ , the length  $L$  between supports and the first natural circular frequency  $\omega_0$  of the inclined taut string, the following non-dimensional equations are obtained.

$$\begin{aligned} & \frac{\partial^2 \bar{u}^*}{\partial \tau^2} + 2h\omega_1 \frac{\partial \bar{u}^*}{\partial \tau} + \frac{k^2 \delta}{\pi^2} \frac{\partial^4 \bar{u}^*}{\partial \bar{s}^4} \\ & - \frac{1}{\pi^2} \frac{d}{d\bar{s}} \left\{ \Delta \bar{T} \frac{d\bar{x}^*}{d\bar{s}} + \left( \frac{\cos \theta}{\frac{d\bar{x}^*}{d\bar{s}} \cos \theta - \frac{d\bar{z}^*}{d\bar{s}} \sin \theta} + \Delta \bar{T} \right) \frac{\partial \bar{u}^*}{\partial \bar{s}} \right\} \\ & = \frac{8\beta}{\pi^2} \cdot \frac{p_{x^*}(\bar{x}^*, \tau)}{mg} \end{aligned} \quad (6)$$

$$\begin{aligned}
& \frac{\partial^2 \bar{w}^*}{\partial \tau^2} + 2h\bar{\omega}_1 \frac{\partial \bar{w}^*}{\partial \tau} + \frac{k^2 \delta}{\pi^2} \frac{\partial^4 \bar{w}^*}{\partial s^4} \\
& - \frac{1}{\pi^2} \frac{d}{ds} \left\{ \Delta \bar{T} \frac{d\bar{z}^*}{ds} + \left( \frac{\cos \theta}{\frac{dx^*}{ds} \cos \theta - \frac{dz^*}{ds} \sin \theta} + \Delta \bar{T} \right) \frac{\partial \bar{w}^*}{\partial s} \right\} \\
& = \frac{8\beta}{\pi^2} \cdot \frac{p_{z^*}(\bar{x}^*, \tau)}{mg}, \quad (7)
\end{aligned}$$

$$\Delta \bar{T} = k^2 \left\{ \frac{d\bar{z}^*}{ds} \frac{\partial \bar{w}^*}{\partial s} + \frac{d\bar{x}^*}{ds} \frac{\partial \bar{u}^*}{\partial s} + \frac{1}{2} \left( \frac{\partial \bar{w}^*}{\partial s} \right)^2 + \frac{1}{2} \left( \frac{\partial \bar{u}^*}{\partial s} \right)^2 \right\} \quad (8)$$

where  $\Delta \bar{T} = \Delta T / H \sec \theta$  is the non-dimensional additional tension,  $\beta = mgL / 8H \sec \theta$ ,  $\bar{u}^* = u^* / L$  and  $\bar{w}^* = w^* / L$  are the non-dimensional displacements in the  $x^*$  and  $z^*$  directions,  $\tau = \omega_0 t$  is the non-dimensional time,  $\omega_0 = \pi / L \sqrt{H \sec \theta / m}$  is the first natural circular frequency of an inclined taut string,  $k^2 = EA / H \sec \theta$  is the ratio of the elongation stiffness to the longitudinal tension of the cable [2],  $\delta = EI / L^2 EA$  is the ratio of the flexural rigidity to the elongation stiffness [2],  $H$  is the initial horizontal tension of the inclined cable,  $\bar{\omega}_1 = \omega_1 / \omega_0$  is the first natural non-dimensional circular frequency of the inclined cable,  $\omega_1$  is the first natural circular frequency of the inclined cable,  $h$  is damping constant,  $g$  is the gravitational acceleration,  $\bar{x}^* = x^* / L$ ,  $\bar{z}^* = z^* / L$ , and  $\bar{s} = s / L$ .

If equations (6) and (7) are written by using the global co-ordinate system, the results coincide with the equations in Ref. 2 in the case of  $h = 0$  and  $\delta = 0$ .

### 3. Analytical Condition and Numerical Analysis Method

Figure 2 shows the small-sag ( $\beta \ll 1$ ) horizontal cable that is discussed in this paper. The inclination angle  $\theta$  is changed in order to maintain the same span length  $L$ . The profile of the inclined cable varies with the inclination angle  $\theta$ .

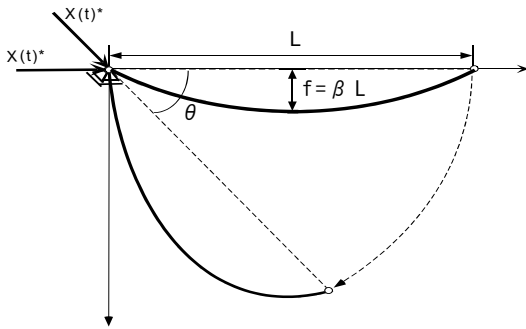


Figure 2 Analytical model and longitudinal excitation at the upper end

This paper describes the nonlinear parametric vibration of the cables. The parametric excitation is given by the longitudinal support displacement  $X(t)^*$  at the upper end, as shown in Figure 2. The loads  $p_{x^*}(x^*, t)$  and  $p_{z^*}(x^*, t)$  are zero. The support excitation is given by the longitudinal displacement  $u^*$  at the upper end of the inclined cable, as described by the following equation,

$$\bar{u}^*(0, \tau) = \bar{X}(\tau)^* = \bar{X}^* \sin \bar{\Omega} \tau \quad (9)$$

where  $\bar{X}(\tau)^* = X(t)^* / L$  is the non-dimensional support excitation,  $\bar{X}^*$  is the non-dimensional amplitude of the support excitation and  $\bar{\Omega}$  is the non-dimensional circular frequency of the parametric excitation.

In this paper, the frequency  $\bar{\Omega}$  is assumed to be  $\bar{\omega}_1$  or  $2\bar{\omega}_1$ . If  $\bar{\Omega} = \bar{\omega}_1$  is used, the excitation corresponds to the parametric excitation of the second unstable region. If  $\bar{\Omega} = 2\bar{\omega}_1$  is used, it corresponds to that of the principal unstable region [17].

In a nonlinear vibration analysis that considers cable loosening, when the value of the total tension is less than zero, it is considered to be zero, as described by the following equation.

$$\begin{aligned}
& \frac{\cos \theta}{\frac{dx^*}{ds} \cos \theta - \frac{dz^*}{ds} \sin \theta} + \Delta \bar{T} = 0 \\
& \text{when } \frac{\cos \theta}{\frac{dx^*}{ds} \cos \theta - \frac{dz^*}{ds} \sin \theta} + \Delta \bar{T} < 0. \quad (10)
\end{aligned}$$

In the case of inclined cables, the loosening must be evaluated at all points since the total tension is different at every point.

In order to solve equations (6), (7), and (8) while evaluating equation (10), the numerical method should be used. The explicit formula of the finite difference method [16, 17] is also employed in this study. The time interval for the numerical analysis should be defined so as to satisfy the stability condition of the scheme that is used.

The parameter  $\beta$  is set to less than 1/8 for small-sag cables. The ratio of the elongation stiffness to the horizontal tension  $k^2$  is set to 900. The number of divisions of the cable is set to 100. In other words, the non-dimensional length  $\Delta \bar{x}^*$  is 0.01. In order to satisfy the stability conditions, the time interval  $\Delta \tau$  must be less than  $1 / 4\Delta \bar{x}^{*2}$ ; in the present case,  $1.0 \times 10^{-5}$  is used. The parameters  $\delta$  and  $h$  needed to solve the divergence

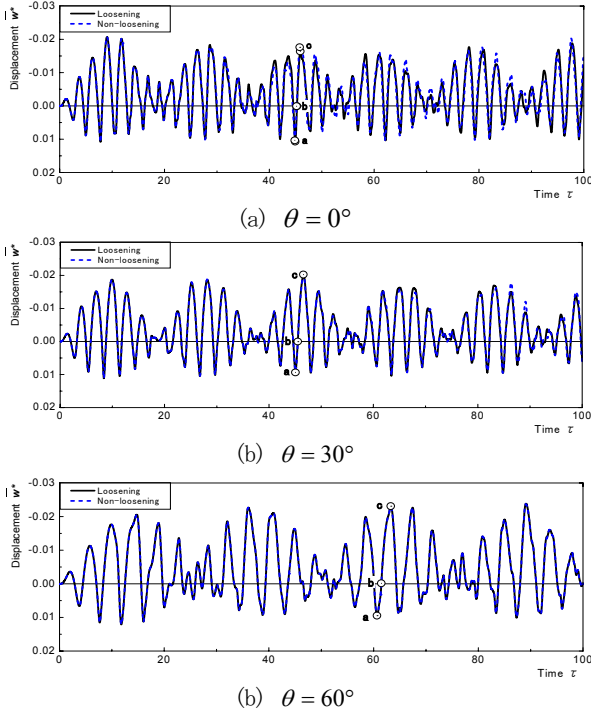


Figure 3 Time histories of transverse displacement in the second unstable region ( $\beta=0.04$ ,  $\bar{X}^*=0.000338$ )

problem, have the same values that were used in previous papers [16, 17]:  $\delta = 10^{-7}$  and  $h=0.001$ .

#### 4. Parametric Responses of the Second Unstable Region

Figures 3, 4, 5, and 6 show the nonlinear parametric responses of the second unstable region ( $\bar{\Omega}=\bar{\omega}_1$ ) when the cable is subjected to the support excitation at the upper end ( $\bar{X}^*=0.000338$ ). Figures 3 and 4 show the time histories of the transverse displacement and the total tension at the center of the inclined cable. Figure 5 shows the space shapes of the three cables. The corresponding maximum transverse displacement and total tension during nonlinear parametric vibration are shown in Figure 6. Notations  $a$ ,  $b$ , and  $c$  correspond to the maximum, zero, and minimum displacements at the center of the horizontal span length.

From Figure 3, the effect of loosening on the responses of inclined cables under parametric excitation of the second unstable region is small because the regions that generate compression are narrowing, which is also true for horizontal cables [17].

However, loosening appears at a different point in an inclined cable than in a horizontal cable. The initial tension of an inclined cable decreases from the upper end to the lower end and the minimum initial tension appears at the lower end of the inclined cable (see Figure 6). Therefore,

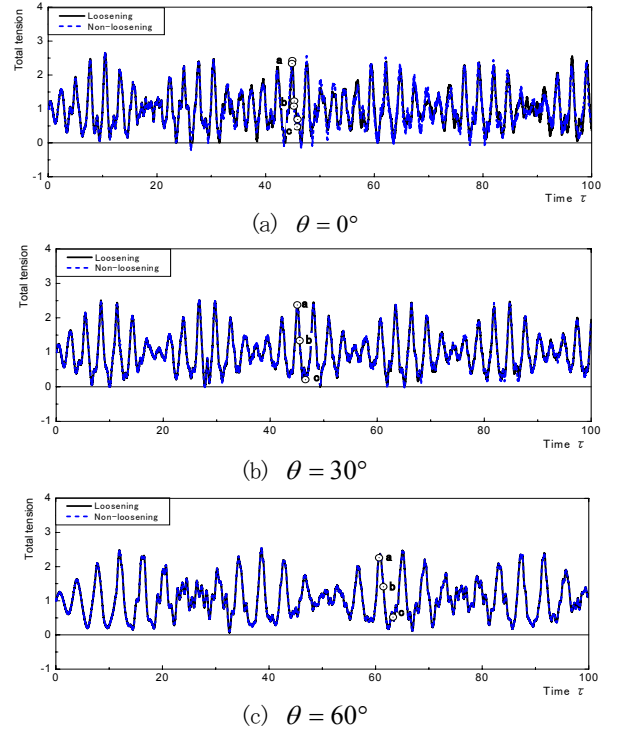


Figure 4 Time histories of total tension in the second unstable region ( $\beta=0.04$ ,  $\bar{X}^*=0.000338$ )

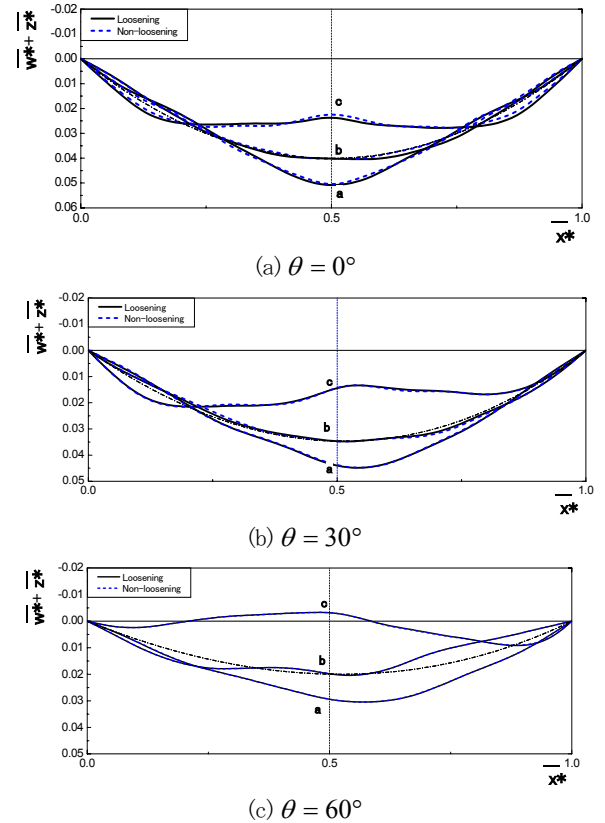


Figure 5 Space shapes in the second unstable region ( $\beta=0.04$ ,  $\bar{X}^*=0.000338$ )

in the case of inclined cables with small sags, the compressive force is generated at the lower end of the cable,

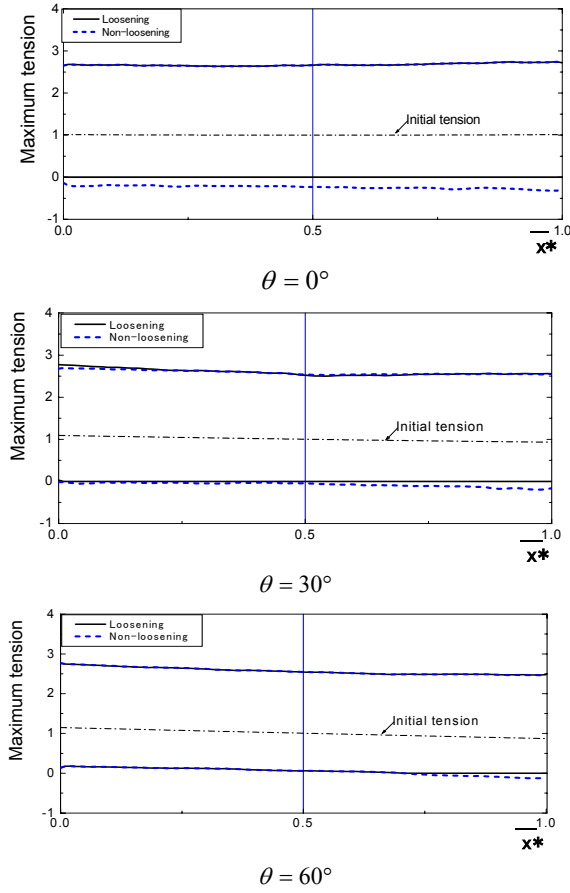


Figure 6 Maximum total tension in the second unstable region ( $\beta = 0.04$ ,  $\bar{X}^* = 0.000338$ )

and the loosening appears first at the lower end.

Figures 7 and 8 show the time histories, space shapes, and maximum responses when the amplitude of the parametric excitation is about 1.5 times the amplitude used in Figures 3, 4, 5, and 6. Comparing Figure 7(a) with Figure 5(b), the cable has a higher order of modal shape in the compressive force region and maintains space shapes that do not easily generate compressive forces when loosening appears. This is also characteristic of horizontal cable [16, 17].

Figure 8(b) illustrates how loosening affects the negative transverse displacements of inclined cables and how the effect of cable loosening on transverse displacement changes with the position. Loosening affects the negative maximum transverse response but scarcely affects the positive maximum transverse response, which are also characteristics of horizontal cables [17].

Figure 9 shows the relationship between the minimal amplitude  $\bar{X}^*$  of the support excitation that generates compressive forces in the cable and inclination angle  $\theta$  in the second unstable region for the three different cables. The corresponding first natural non-dimensional frequencies

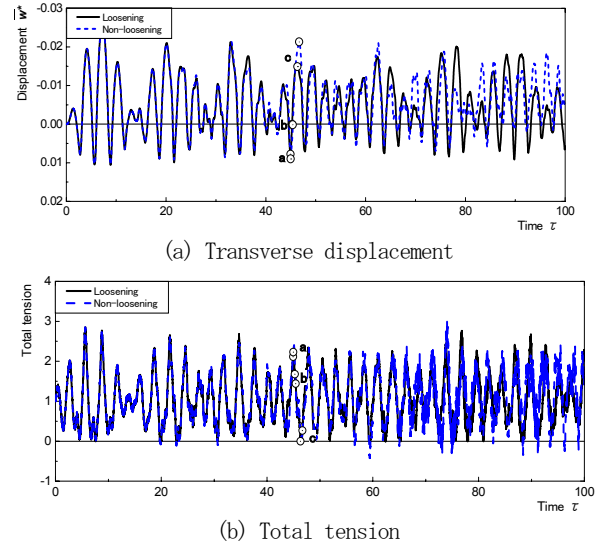


Figure 7 Time histories in the second unstable region ( $\theta = 30^\circ$ ,  $\beta = 0.04$ ,  $\bar{X}^* = 0.000557$ )

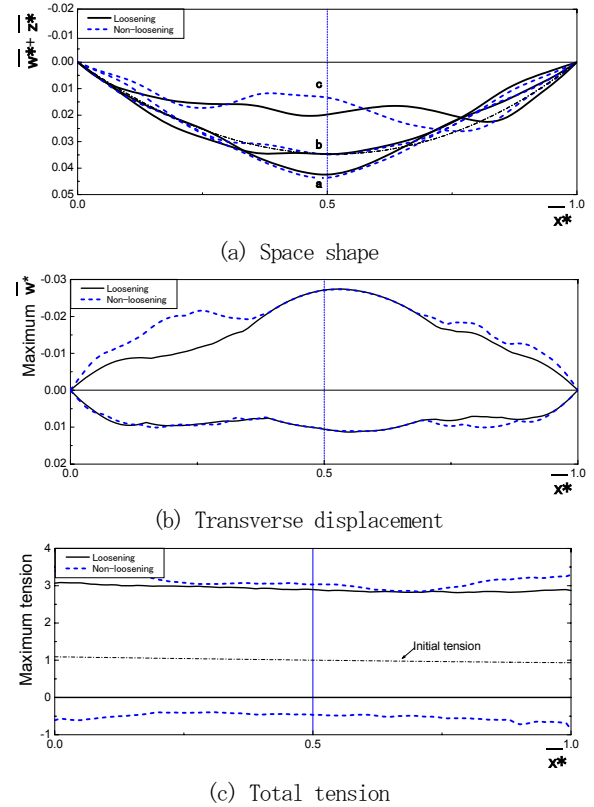


Figure 8 Space shape and maximum responses in the second unstable region ( $\theta = 30^\circ$ ,  $\beta = 0.04$ ,  $\bar{X}^* = 0.000557$ )

of the same cables are shown in Figure 10.

When the inclination angle  $\theta = 0^\circ \sim 30^\circ$ , cable loosening is easily generated by small amplitudes of support excitation, and the effect of parameter  $\beta$  is small. When the inclination angle  $\theta > 30^\circ$ , the minimal amplitude of support excitation increases, except when parameter  $\beta = 0.04$  and the inclination angle  $\theta = 30^\circ \sim 65^\circ$ , as the

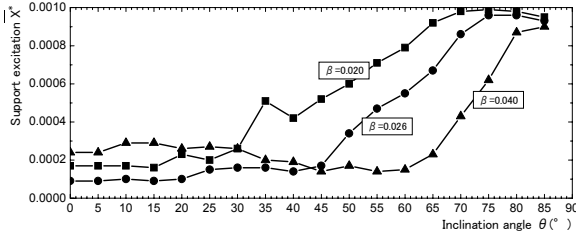


Figure 9 Relationship between minimal amplitude of support excitation and inclination angle in the second unstable region

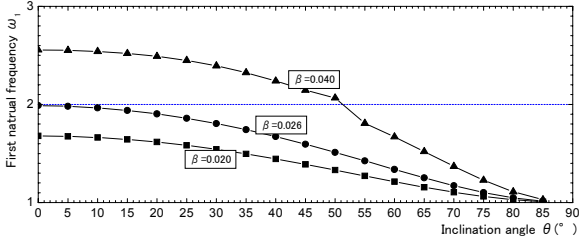


Figure 10 First natural non-dimensional frequencies of inclined cables

inclination angle becomes large and the effect of parameter  $\beta$  becomes conspicuous. This means that it is harder to induce loosening in inclined cables with large angles than in horizontal cables.

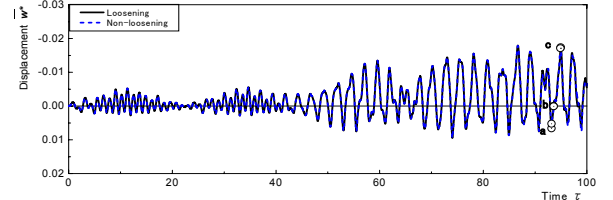
The effect of the inclination angle for parameter  $\beta=0.04$  differs from the other cables. This may be explained by fact that the modal shape of the inclined cable varies with the magnitude of inclination angles, as shown as Figure 10.

##### 5. Parametric Responses of the Principal Unstable Region

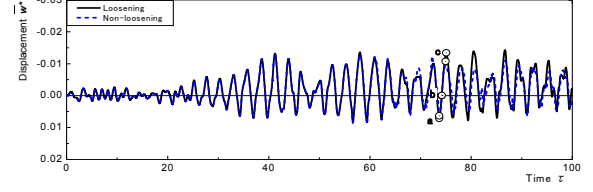
Figures 11, 12, 13, and 14 show the nonlinear parametric responses of the principal unstable region ( $\bar{\Omega} = 2\bar{\omega}_1$ ) when the cable is subjected to the support excitation at the upper end ( $\bar{X}^* = 0.000338$ ). Figures 15 and 16 show time histories, space shapes, and maximum responses when the amplitude of support excitation  $\bar{X}^* = 0.000557$ . Comparing Figure 12(b) with Figure 15(a), the cable maintains space shapes that do not easily generate compressive forces when loosening occurs, which is the same as those in the second unstable region. Loosening affects the negative maximum response but scarcely affects the positive maximum response, as shown as Figures 14 and 16(b). The results are similar to those obtained for the second unstable regions.

The relationship between the minimal amplitude  $\bar{X}^*$  of the parametric excitation that generates compressive forces in the cable and the inclination angle  $\theta$  in the principal unstable region is shown in Figure 17.

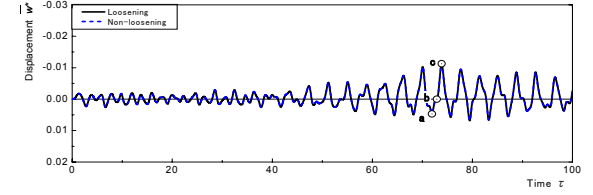
Comparing Figure 9 with Figure 17, the minimal amplitudes of support excitation in the principal unstable region are larger than those in the second unstable region, where the inclination angle  $\theta = 0^\circ \sim 20^\circ$ . Unlike those in the second



(a)  $\theta = 0^\circ$

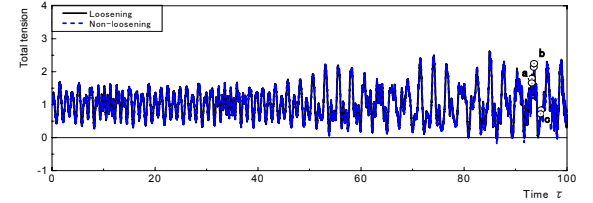


(b)  $\theta = 30^\circ$

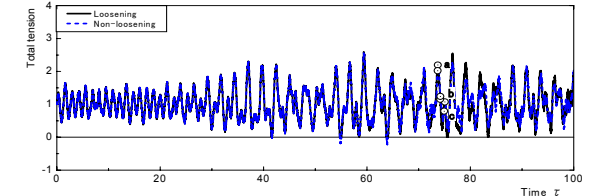


(c)  $\theta = 60^\circ$

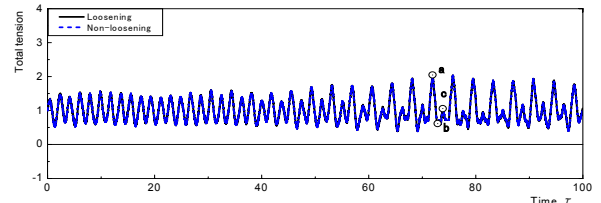
Figure 11 Time histories of transverse displacement in the principal unstable region ( $\beta=0.04$ ,  $\bar{X}^*=0.000338$ )



(a)  $\theta = 0^\circ$



(b)  $\theta = 30^\circ$



(c)  $\theta = 60^\circ$

Figure 12 Time histories of total tension in the principal unstable region ( $\beta=0.04$ ,  $\bar{X}^*=0.000338$ )

unstable region, the minimal amplitude of excitation in the principal unstable region decreases when the inclination angle  $\theta = 20^\circ \sim 60^\circ$ , except when  $\beta=0.04$ . The minimal amplitude of excitation then increases thereafter and approaches the same values as those in the second unstable region when the inclination angle  $\theta$  increases.

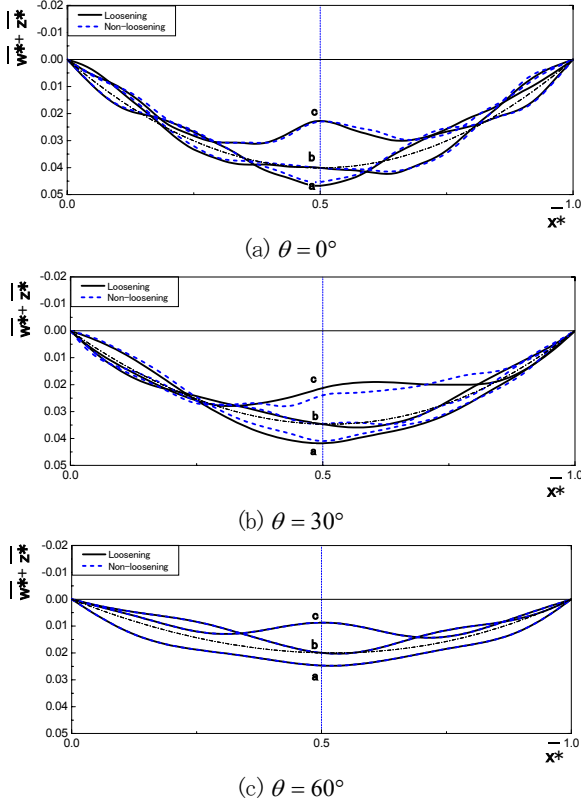


Figure 13 Space shapes in the principal unstable region ( $\beta = 0.04$ ,  $\bar{X}^* = 0.000338$ )

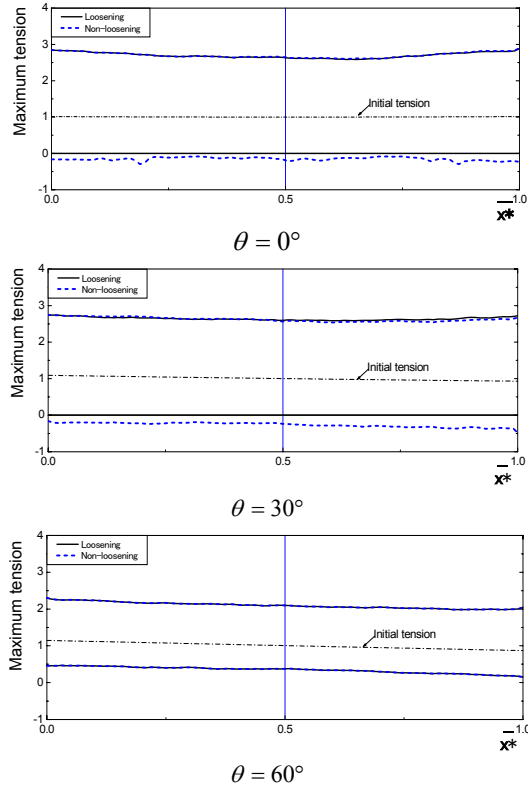


Figure 14 Maximum total tensions in the principal unstable region ( $\beta = 0.04$ ,  $\bar{X}^* = 0.000338$ )

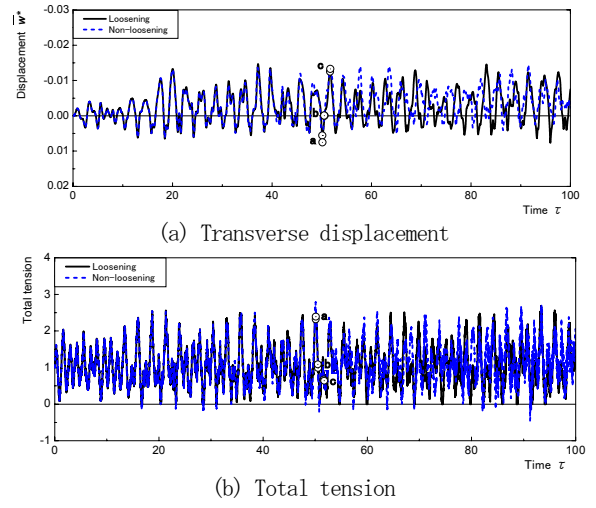


Figure 15 Time histories in the principal unstable region ( $\theta = 30^\circ$ ,  $\beta = 0.04$ ,  $\bar{X}^* = 0.000557$ )

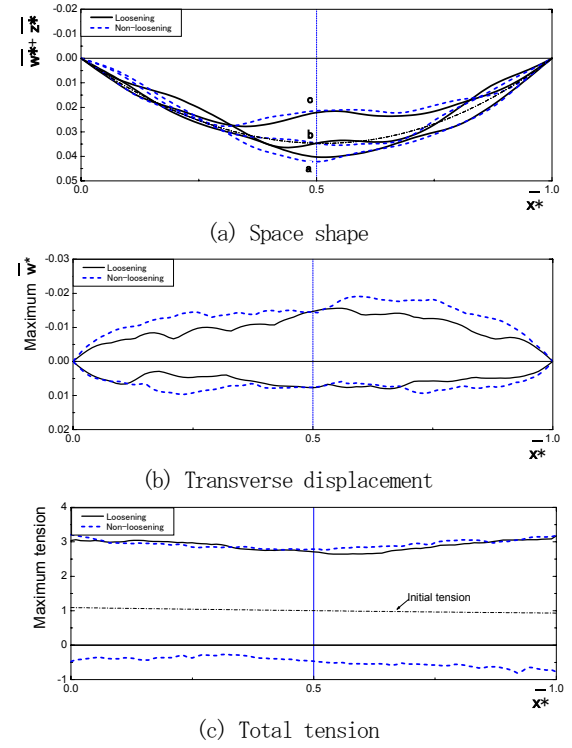


Figure 16 Space shapes and maximum responses in the principal unstable region ( $\theta = 30^\circ$ ,  $\beta = 0.04$ ,  $\bar{X}^* = 0.000557$ )

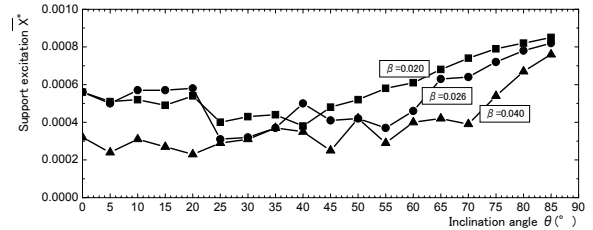


Figure 17 Relationship between minimal amplitude of support excitation and inclination angle in the principal unstable region

## 6. Conclusions

This paper examined the effect of cable loosening on the nonlinear parametric vibrations of inclined cables subjected to periodic the support excitation. In order to calculate the loosening of inclined cables without a small-sag limitation, it was necessary to first derive new equations of motion for an inclined cable. Regarding the effect of loosening on the nonlinear parametric vibrations of inclined cables with small sags, the main findings are as follows:

1. The total tension in the inclined cables during nonlinear parametric vibration is not constant and cable loosening is generated first at the lower end.
2. The minimum support excitation that generates a compressive force varies with the inclination angle. The influence of the inclination angle is dependent upon the initial profile of the cables.
3. Cable loosening affects the negative maximum transverse response but scarcely affects the positive maximum transverse response.
4. Cable loosening easily occurs in a cable with a small inclination angle under small amplitudes of parametric excitation in the second unstable region.

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