Fluid flow and heat transfer to modified power law fluids in plane Couette-Poiseuille laminar flow between parallel plates

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The fully developed laminar heat transfer to modified power-law fluids flowing between parallel plates with one moving plate was analyzed taking into account the viscous dissipation of the flowing fluid. Applying the shear stress described by the modified power-law model, the energy equation together with the fully developed velocity profile is solved numerically for thermal boundary conditions of constant wall heat flux at one wall with the other insulated.The effects of the flow index, relative velocity of the moving plate, dimensionless shear rate parameter and Brinkman number on Nusselt numbers at the plate walls were discussed.

1. Introduction

Problems involving fluid flow and heat transfer with an axially moving core of solid body or fluid in an annular geometry can be found in many manufacturing processes, such as extrusion, drawing and hot rolling, etc. In such processes, a hot plate or cylindrical rod continuously exchanges heat with the surrounding environment. For such cases, the fluid involved may be Newtonian or non-Newtonian and the flow situations encountered can be either laminar or turbulent.

In the previous study(1), fully developed laminar heat transfer of a Newtonian fluid flowing between parallel plates with one moving plate was analyzed taking into account the viscous dissipation of the flowing fluid.

In the previous report(2), an exact solution of the momentum equation was obtained for fully developed laminar flow of a non-Newtonian fluid flowing between two parallel plates with one moving plate. The constitutive equation (i.e., the shear stress-shear rate relation) for the non-Newtonian fluid was described by the power-law model most frequently used in non-Newtonian fluid flow and heat transfer.

In the previous study(3), fully developed laminar heat transfer of a non-Newtonian fluid flowing between parallel plates with one moving plate was analyzed taking into account the viscous dissipation of the flowing fluid. Applying the velocity distribution obtained for the plane Couette-Poiseuille laminar flow, the energy equation with the viscous dissipation term was exactly solved for the boundary conditions of constant wall heat flux at one wall with the other insulated.

In this report, fully developed laminar heat transfer of a modified power-law fluid flowing between parallel plates with one moving plate was analyzed taking into account the viscous dissipation of the flowing fluid. Applying the shear stress described by the modified power-law model proposed by Capobianchi and Irvine(4), the energy equation together with the fully developed velocity profile is solved numerically for thermal boundary conditions of constant wall heat flux at one wall with the other insulated.The effects of the relative velocity of the moving plate, flow index, fluid consistency, dimensionless shear rate parameter and Brinkman number on Nusselt numbers at the plate walls were discussed.

Nomenclature

A	area normal to the flow direction
Br	Brinkman number
c_p	specific heat at constant pressure
D_h	hydraulic diameter $\equiv 2L$
f	friction factor
k	thermal conductivity
L	distance between the parallel plates
m	consistency index
n	flow index
Nu	Nusselt number
P	pressure
q	wall heat flux
Re_M	modified Reynolds number
T	temperature

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u	axial velocity of the fluid
u_m	average velocity of the fluid
u^*	dimensionless velocity $\equiv u / u_m$
U	axial velocity of the moving plate
U^*	dimensionless relative velocity of the moving plate $\equiv U / u_m$
V	dimensionless parameter
y	coordinate normal to the fixed plate
y^*	dimensionless coordinate $\equiv y / D_h$
z	axial coordinate

Greek Symbols

β	dimensionless shear rate parameter
η_a	apparent viscosity
η_a^*	dimensionless apparent viscosity $\equiv \eta_a/\eta^*$
η_0	viscosity at zero shear rate
η^*	reference viscosity
ρ	density
τ	shear stress
θ	dimensionless temperature

Subscripts

B	bulk
j	j = L for Case A, j = 0 for Case B
L	moving plate
0	fixed plate

2. Analysis

The physical model for the analysis is shown in Fig.1. The lower plate is axially moving at a constant velocity, U. The assumptions used in the analysis are:

1. The flow is incompressible, steady-laminar, and fully developed, hydrodynamically and thermally.
2. The fluid is non-Newtonian and the shear stress may be described by the modified power-law model[4], and physical properties are constant.
3. The body forces and axial heat conduction are neglected.

2.1 Fluid Flow

The momentum equation together with the assumptions described above is

$$\frac{d\tau}{dy} = \frac{dP}{dz} \tag{1}$$

where

$$\tau = \eta_a \frac{du}{dy}. \tag{2}$$

The boundary conditions are:

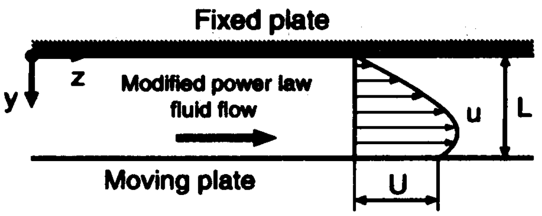

Fig.1 Schematic of parallel plates with one moving plate

$$\text{B.C.}: \begin{cases} u = 0 & \text{at} \quad y = 0 \\ u = U & \text{at} \quad y = L. \end{cases} \tag{3}$$

Here η_a is the apparent viscosity defined by

$$\eta_a = \frac{\eta_0}{1 + \frac{\eta_0}{m}\left|\frac{du}{dy}\right|^{1-n}} \qquad \text{for} \quad n < 1, \tag{4}$$

$$\eta_a = \eta_0\left(1 + \frac{m}{\eta_0}\left|\frac{du}{dy}\right|^{n-1}\right) \qquad \text{for} \quad n > 1. \tag{5}$$

Average fluid velocity, u_m, is defined as

$$u_m \equiv \frac{1}{L}\int_0^L u dy. \tag{6}$$

The momentum equation and its boundary conditions are reduced to

$$\frac{d}{dy^*}\left(\eta_a^* \frac{du^*}{dy^*}\right) = -2f \cdot Re_M, \tag{7}$$

$$\text{B.C.}: \begin{cases} u^* = 0 & \text{at} \quad y^* = 0 \\ u^* = U^* & \text{at} \quad y^* = \frac{1}{2}. \end{cases} \tag{8}$$

Friction factor, f, and modified Reynolds number, Re_M are defined as

$$f \equiv \frac{D_h}{2\rho u_m^2}\left(-\frac{dP}{dz}\right), \tag{9}$$

$$Re_M \equiv \frac{\rho u_m D_h}{\eta^*}. \tag{10}$$

Dimensionless apparent viscosity η_a^* is defined as

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{1+\beta}{1+\beta\left|\frac{du^*}{dy^*}\right|^{1-n}} \qquad \text{for} \quad n < 1, \tag{11}$$

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{\beta + \left|\frac{du^*}{dy^*}\right|^{n-1}}{\beta + 1} \qquad \text{for} \quad n > 1, \tag{12}$$

$$\eta^* = \frac{\eta_0}{1+\beta} \qquad \text{for} \quad n < 1, \tag{13}$$

$$\eta^* = \eta_0\left(1 + \frac{1}{\beta}\right) \qquad \text{for} \quad n > 1, \tag{14}$$

$$\beta = \frac{\eta_0}{m}\left(\frac{u_m}{D_h}\right)^{1-n}. \tag{15}$$

2.2 Heat transfer

The energy equation together with the assumptions above is written as

$$k\frac{d^2T}{dy^2} + \tau\left(\frac{du}{dy}\right) = \rho c_p u \frac{dT_B}{dz}. \tag{16}$$

The following two types of the thermal boundary conditions are specified:

Case A (constant heat flux at the moving plate with the fixed plate insulated):

$$\begin{cases} -k\frac{\partial T}{\partial y} = 0 & \text{at} \quad y = 0 \\ k\frac{\partial T}{\partial y} = q_L & \text{at} \quad y = L, \end{cases} \tag{17}$$

Case B (constant heat flux at the fixed plate with the moving plate insulated):

$$\begin{cases} -k\frac{\partial T}{\partial y} = q_0 & \text{at} \quad y = 0 \\ k\frac{\partial T}{\partial y} = 0 & \text{at} \quad y = L \end{cases} \tag{18}$$

where the wall heat fluxes, q_L and q_0, are taken as positive into the fluid.

Bulk temperature is defined as

$$T_B \equiv \frac{\iint_A uTdA}{\iint_A udA}. \tag{19}$$

dT_B / dz in Eq.(16) is evaluated, from an energy balance, as

$$\frac{dT_B}{dz} = \frac{q_j}{\rho c_p u_m L}\left[1 + \frac{\int_0^L \tau\left(\frac{du}{dy}\right)dy}{q_j}\right] \tag{20}$$

where j = L stands for Case A and j = 0 for Case B.

Introducing dimensionless temperature, θ, defined as

$$\theta \equiv T\big/[q_j D_h/k], \tag{21}$$

the energy equation and the boundary conditions may be expressed in the dimensionless forms as

$$\frac{d^2\theta}{dy^{*2}} = 2u^* + Br \cdot V, \tag{22}$$

$$\textbf{Case A}: \begin{cases} \frac{d\theta}{dy^*} = 0 & \text{at} \quad y^* = 0 \\ \frac{d\theta}{dy^*} = 1 & \text{at} \quad y^* = \frac{1}{2}, \end{cases} \tag{23}$$

$$\textbf{Case B}: \begin{cases} \frac{d\theta}{dy^*} = -1 & \text{at} \quad y^* = 0 \\ \frac{d\theta}{dy^*} = 0 & \text{at} \quad y^* = \frac{1}{2} \end{cases} \tag{24}$$

where

$$Br \equiv \frac{\eta^* u_m^2}{D_h q_j} \tag{25}$$

and

$$V = 2u^*\left\{\int_0^{1/2} \eta_a^*\left(\frac{du^*}{dy^*}\right)^2 dy^*\right\} - \eta_a^*\left(\frac{du^*}{dy^*}\right)^2. \tag{26}$$

Nusselt number, Nu_j, is defined as

$$Nu_j \equiv \frac{[q_j/(T_j - T_B)]D_h}{k} = \frac{1}{\theta_j - \theta_B} \tag{27}$$

where dimensionless bulk temperature, θ_B, is defined as

$$\theta_B \equiv T_B\big/[q_j D_h/k] \tag{28}$$

and $(\theta_j - \theta_B)$ is calculated as

$$\theta_j - \theta_B = \int_0^{1/2} u^*\,(\theta_j - \theta)\,dy^*. \tag{29}$$

3. Results and discussion

In order to examine the reliability of the numerical solutions obtained in this study, the solutions computed for the momentum and energy equations together with respective boundary conditions were compared with the analytical solutions determined at the extremes at $\beta \to 0$ and $\beta \to \infty$. Both solutions were in good agreement.

3.1 Fluid Flow

Parameter β defined by Eq.(15) represents dimensionless average shear rate under the specified fluid flow condition. For pseudoplastic fluids ($n < 1$) the extreme at to $\beta \to 0$ corresponds to a Newtonian fluid and that at $\beta \to \infty$ a power law fluid. For dilatant fluids ($n > 1$) the extreme at $\beta \to \infty$ corresponds to a Newtonian fluid and that at $\beta \to 0$ to a power law fluid.

Figure 2 shows the effect of parameter β on friction factor in terms of fRe_M.

The values of fRe_M at the extremes of $\beta \to 0$ and $\beta \to \infty$ approach, respectively, to the values of Newtonian fluid and power law fluid.

It is seen in Fig.2 that the values of fRe_M become greater with a decrease in U^*.

Figure 3 shows the effect of parameter β on velocity profile across the channel for the nine combinations of n = 0.5, 1.0, 1.5 and U^* = -1, 0, 1. For n = 0.5 and n = 1.5 the velocity profiles are different depending on the magnitude of β with a decrease in U^*.

Square of velocity gradient or parameter V given by Eq.(26) controls the heat transfer through Eq.(22) for the case with viscous dissipation. The magnitudes of them are shown in Figs.4 and 5. It is seen that the values of square of velocity gradient and V become larger near the walls with an increase in β for $U^* = -1$ and $U^* = 0$. However they remain small near the moving wall for $U^* = 1$.

3.2 Heat transfer for Case A

Temperature difference $(\theta - \theta_B)$ for Case A is shown in Fig.6 for $\beta = 1$. For $U^* = -1$ and $U^* = 0$, $(\theta - \theta_B)$ increases near the walls with an increase in Br_A and has a minimum value in the middle region of the channel. This is attributed to the heat generated by viscous dissipation near the wall as seen in Figs.4 and 5, and large axial heat convection in the middle region as seen in Fig.3. For the case of $U^* = 1$, $(\theta - \theta_B)$ increases with an increase in Br_A near the fixed wall. This is also owing to the heat generated by viscous dissipation near the fixed wall as seen in Figs.4 and 5.

Nusselt numbers for Case A, Nu_L, are shown in Fig.7. Nu_L decreases with an increase in Br_A for $U^* = -1$ and $U^* = 0$ and increases with an increase in Br_A for $U^* = 1$. These behaviors of Nusselt numbers Nu_L can be explained by the viscous dissipation effect on temperature difference $(\theta - \theta_B)$, as mentioned above.

4. Conclusions

Fully developed Couette-Poiseuille laminar flow between parallel plates was analyzed using the modified power-law model proposed by Capobianchi and Irvine[4]. Furthermore, applying the fully developed velocity distribution calculated numerically, the energy equation together with the boundary conditions of constant wall heat flux at one wall with the other insulated was solved numerically taking into account viscous dissipation effect.

In the analyses of fluid flow and heat transfer of non-Newtonian fluids, the results calculated by adopting the simple power-law fluid model do not predict correctly the values of friction factor and Nusselt number in the region of lower shear rate. In order to calculate the whole region of shear rate from zero to infinity, the modified power-law model as adopted in this study should be used.

In this report the heat transfer study results for Case A only are discussed due to the limited space. The counterpart for Case B will be shown in another report.

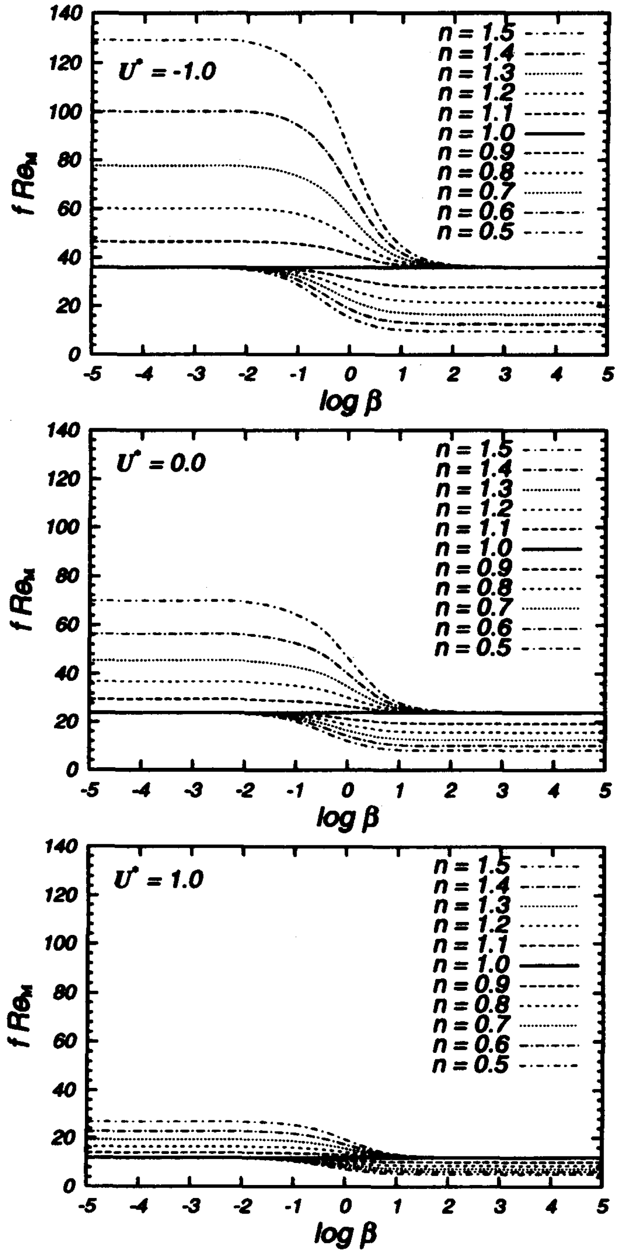

Fig.2 Friction factor

Reference

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2. G.Davaa, et al., "Plane Couette-Poiseuille flow of power-law non-Newtonian fluids" *Reports of the Faculty of Engineering, Nagasaki University* vol.30, No.54, 29-36 (2000).
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4. M. Capobianchi and T.F.Irvine, "Predictions of pressure drop and heat transfer in concentric annular ducts with modified power law fluids" *Wärme-und Stoffübertragung*, 27, 209-215 (1992).

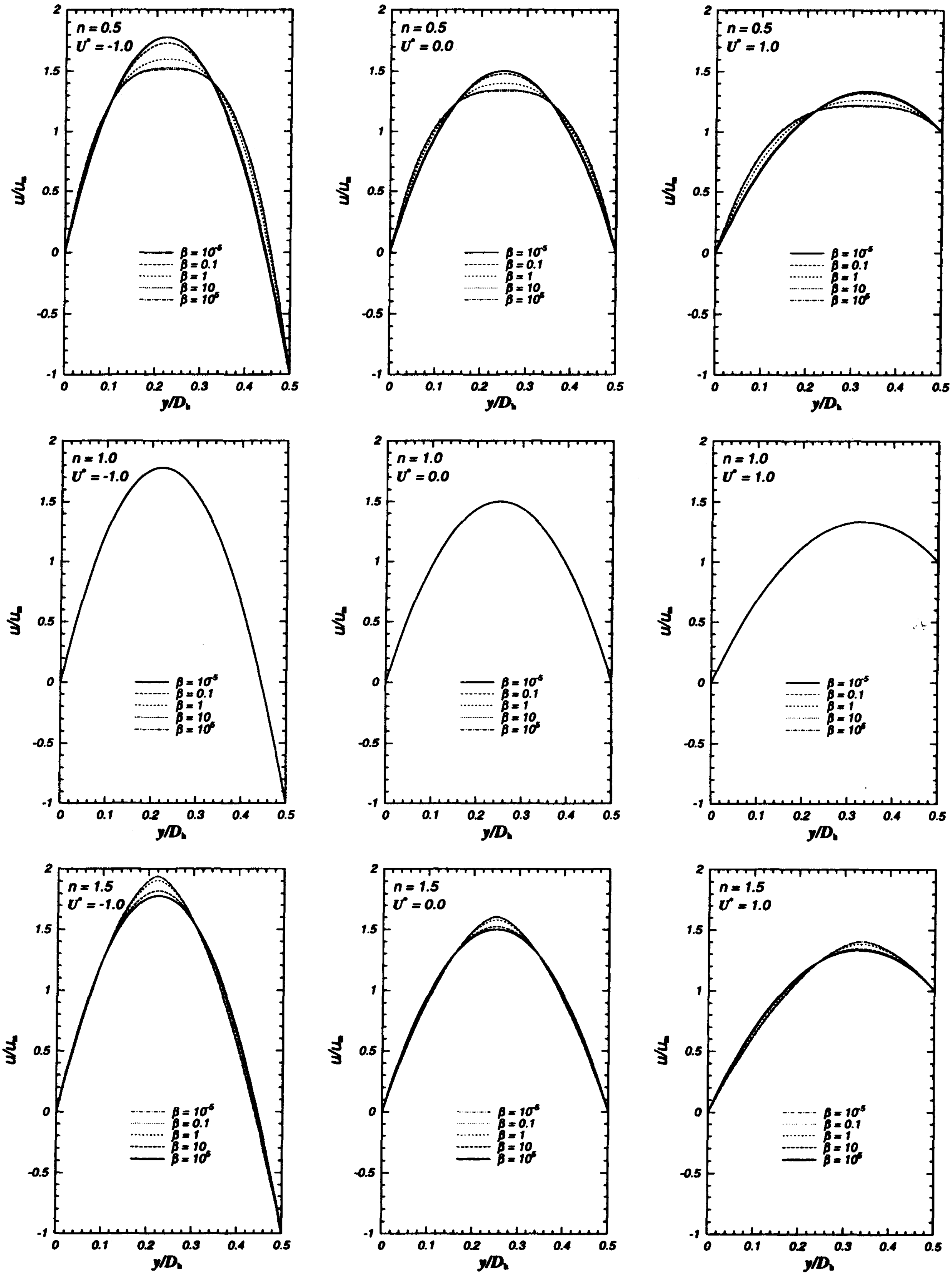

Fig.3 Velocity profile

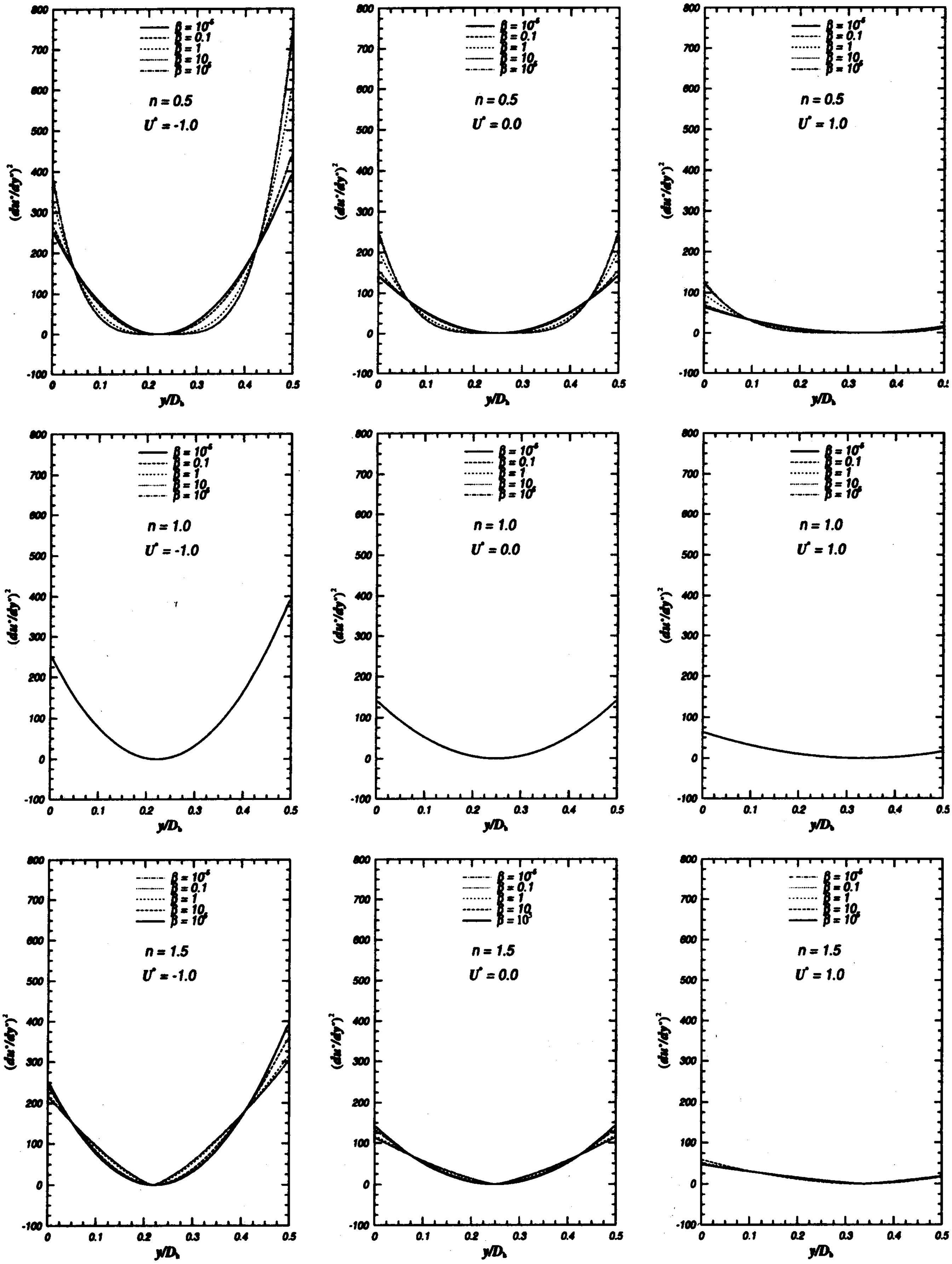

Fig.4 Square of velocity gradient

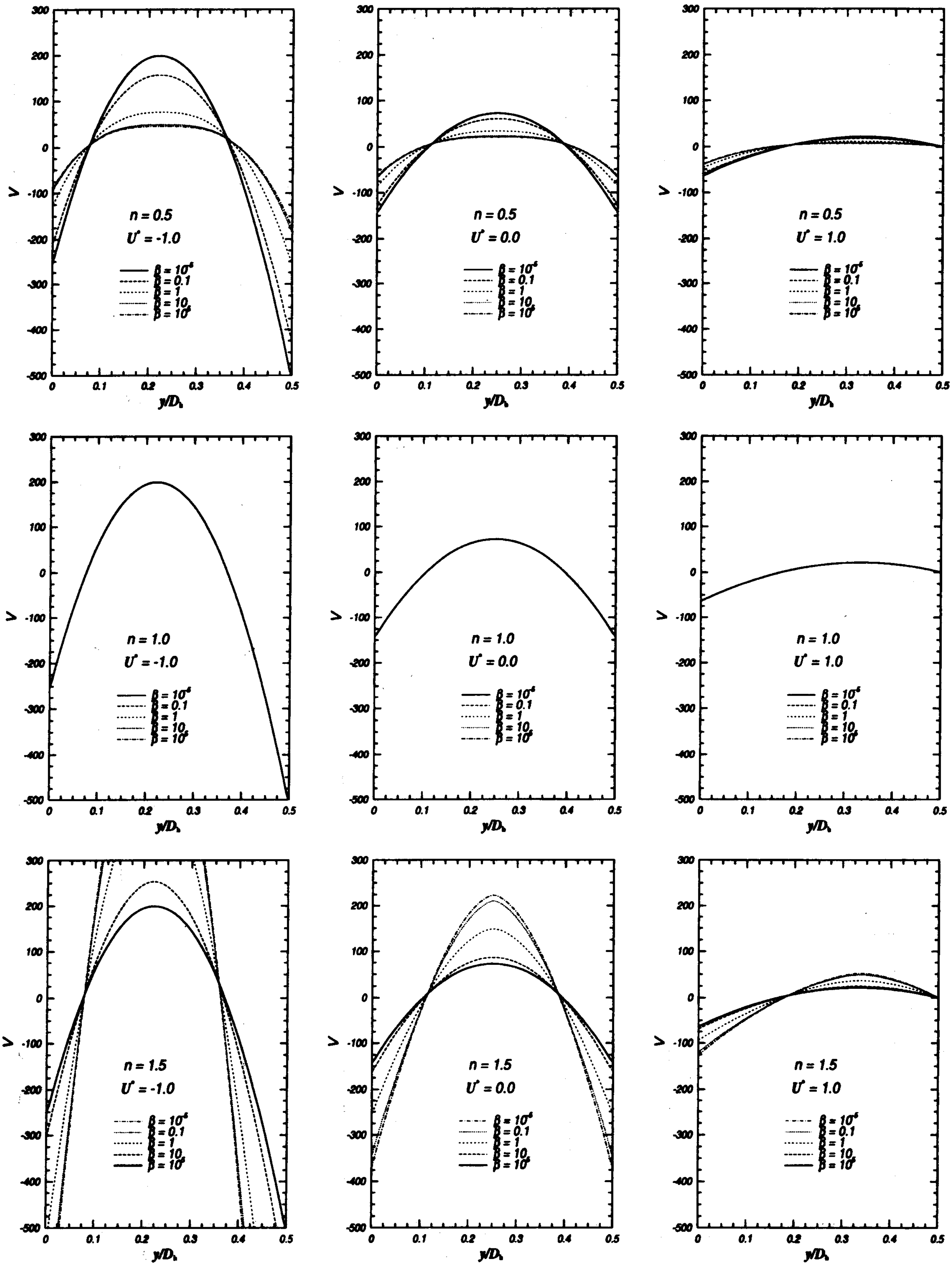

Fig.5 Distribution of parameter V

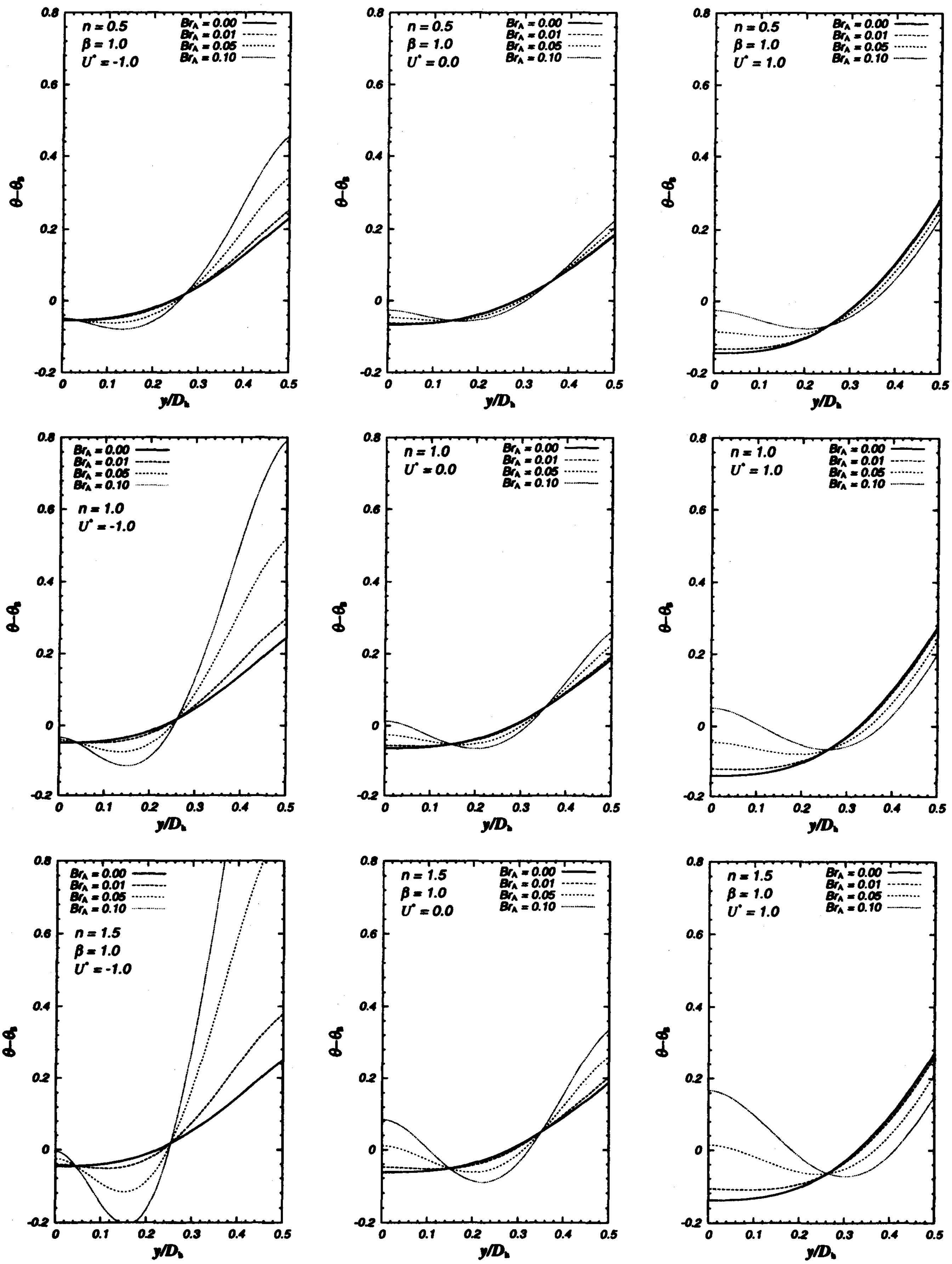

Fig.6 Dimensionless temperature difference for Case A ($\beta = 1$)

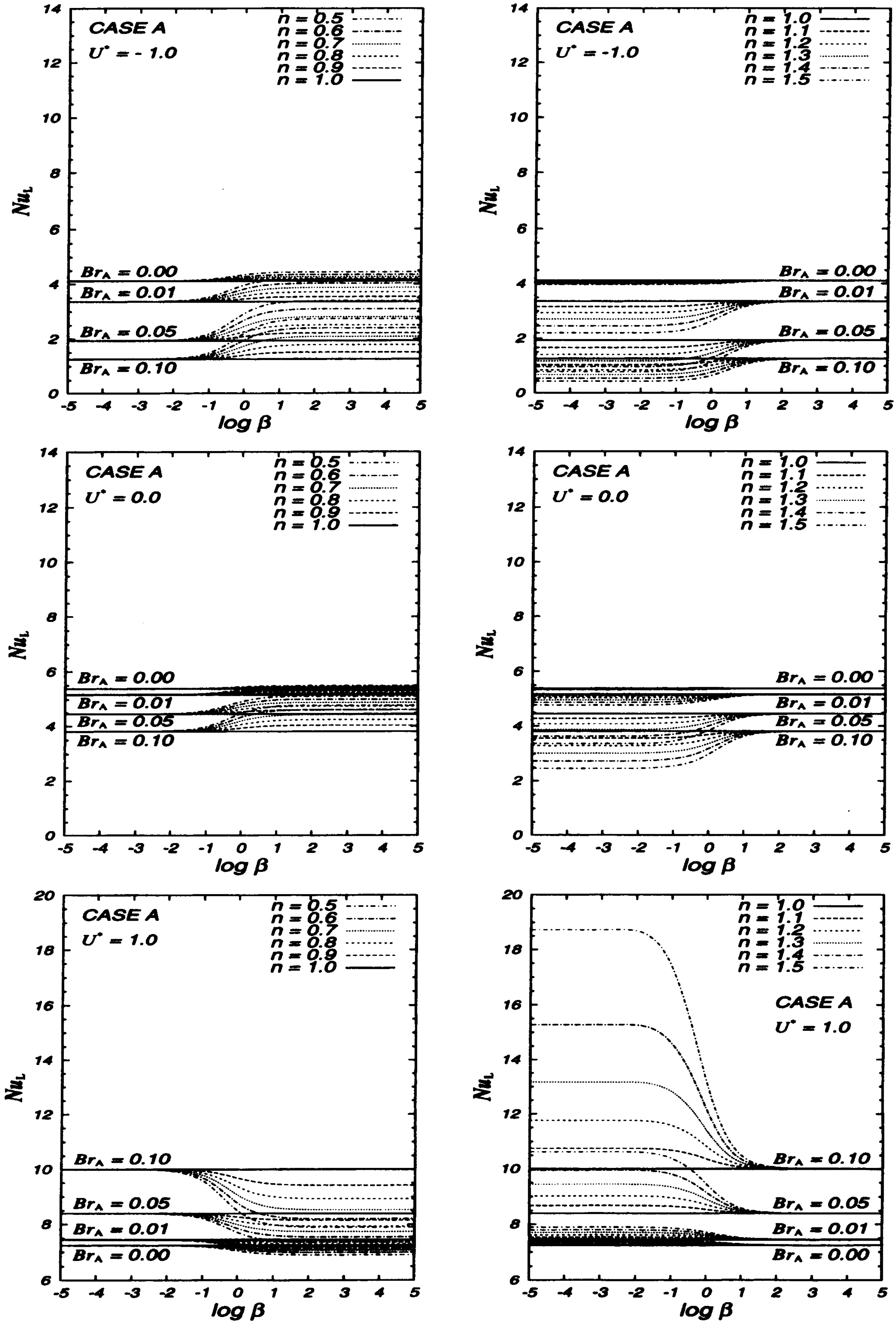

Fig.7 Nusselt numbers for Case A