Heat transfer for modified power law fluids in concentric annuli with heated moving cores

by

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The fully developed laminar heat transfer of modified power-law fluids in a concentric annulus with an axially moving core was analyzed taking into account the viscous dissipation of the flowing fluid. Applying the shear stress described by the modified power-law model, the energy equation including the viscous dissipation term is solved numerically for the thermal boundary conditions of constant heat flux at the moving core with the fixed outer tube insulated. The effects of the radius ratio, the flow index, the relative core velocity, dimensionless shear rate parameter and Brinkman number on the temperature distribution and Nusselt numbers are discussed.

1. Introduction

In the previous report(1), numerical solutions of the momentum equation were obtained for fully developed laminar flow of non-Newtonian fluids flowing in a concentric annulus with an axially moving core. The shear stress for non-Newtonian fluids was described by the modified power law model.

In the this study, the fully developed laminar heat transfer of non-Newtonian fluids in a concentric annulus with an axially moving core is studied. Applying the shear stress described by the modified power-law model and the fully developed velocity profile obtained from the previous report(1), the energy equation including the viscous dissipation term is solved numerically for the thermal boundary conditions of constant heat flux at the moving core with the fixed tube insulated (Case A). The effects of the radius ratio, relative velocity of the core, the flow index and dimensionless shear rate parameter on the velocity distribution and friction factor are discussed.

Nomenclature

A	area normal to the flow direction
Br	Brinkman number
c_p	specific heat at constant pressure
D_h	hydraulic diameter $\equiv 2(R_o - R_i)$
k	thermal conductivity
m	consistency index
n	flow index
Nu	Nusselt number
r	radial coordinate
r^*	dimensionless radial coordinate $\equiv r/D_h$
R	radius
q	wall heat flux
T	temperature
u	axial velocity of the fluid
u_m	average velocity of the fluid
u^*	dimensionless velocity $\equiv u/u_m$
U	axial velocity of the moving core
U^*	dimensionless relative velocity of the moving core $\equiv U/u_m$
V	dimensionless parameter
z	axial coordinate

Greek Symbols

α	radius ratio $\equiv R_i/R_o$
β	dimensionless shear rate parameter
η_a	apparent viscosity
η_a^*	dimensionless apparent viscosity $\equiv \eta_a/\eta^*$
η_0	viscosity at zero shear rate
η^*	reference viscosity

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ρ density
τ shear stress
θ dimensionless temperature
ξ transformed dimensionless radial coordinate $\equiv [2(1-\alpha)r^* - \alpha] / (1-\alpha)$

Subscripts
b bulk
i inner tube
o outer tube

2. Analysis

The physical model for the analysis is shown in Fig. 1. The inner core tube is axially moving at a constant velocity, U. The assumptions used in the analysis are:

1. The flow is incompressible, steady-laminar, and fully developed, hydrodynamically and thermally.
2. The fluid is non-Newtonian and the shear stress may be described by the modified power-law model[2], and physical properties are constant except viscosity.
3. The body forces and axial heat conduction are neglected.

Heat transfer

The energy equation together with the assumptions above is written as

$$k\frac{1}{r}\frac{d}{dr}\left[r\frac{dT}{dr}\right] + \tau\left(\frac{du}{dr}\right) = \rho\, c_p\, u\frac{dT_b}{dz}. \quad (1)$$

The velocity, u, and its gradient, $\frac{du}{dy}$, have been obtained in the previous report[1].

The thermal boundary conditions:

Case A (constant heat flux at the moving core with the fixed outer tube insulated):

$$\begin{cases} -k\dfrac{\partial T}{\partial r} = q_i & \text{at } r = R_i \\ k\dfrac{\partial T}{\partial r} = 0 & \text{at } r = R_0 \end{cases} \quad (2)$$

τ in Eq.(1) is the shear stress defined by

$$\tau = \eta_a\frac{du}{dr} \quad (3)$$

where η_a is the apparent viscosity defined by

$$\eta_a = \frac{\eta_0}{1+\frac{\eta_0}{m}\left|\frac{du}{dr}\right|^{1-n}} \quad \text{for } n<1, \quad (4)$$

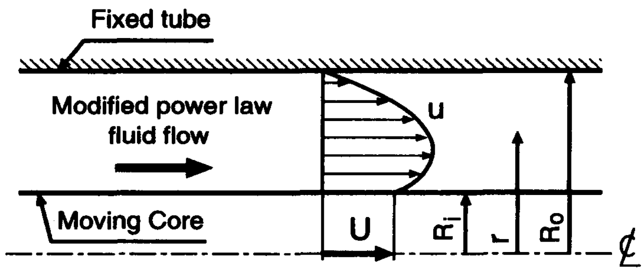

Fig.1 Schematic of a concentric annulus with an axially moving core

$$\eta_a = \eta_0\left(1+\frac{m}{\eta_0}\left|\frac{du}{dr}\right|^{n-1}\right) \quad \text{for } n>1. \quad (5)$$

Dimensionless apparent viscosity, η_a^*, is defined as

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{1+\beta}{1+\beta\left|\frac{du^*}{dr^*}\right|^{1-n}} \quad \text{for } n<1, \quad (6)$$

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{\beta+\left|\frac{du^*}{dr^*}\right|^{n-1}}{\beta+1} \quad \text{for } n>1, \quad (7)$$

where

$$\eta^* = \frac{\eta_0}{1+\beta} \quad \text{for } n<1, \quad (8)$$

$$\eta^* = \eta_0\left(1+\frac{1}{\beta}\right) \quad \text{for } n>1, \quad (9)$$

$$\beta = \frac{\eta_0}{m}\left(\frac{u_m}{D_h}\right)^{1-n}. \quad (10)$$

Bulk temperature, T_b, is defined as

$$T_b \equiv \frac{\iint_A uTdA}{\iint_A udA} = \frac{2}{u_m(R_0^2-R_i^2)}\int_{R_i}^{R_0} uTrdr. \quad (11)$$

By integrating Eq.(1) with Eq.(2), dT_b / dz is obtained as:

$$\frac{dT_b}{dz} = \frac{2R_i q_i}{\rho c_p u_m(R_0^2-R_i^2)}\left[1+\frac{\int_{R_i}^{R_0}\tau r\left(\frac{du}{dr}\right)dr}{R_i q_i}\right] \quad (12)$$

The average fluid velocity, u_m, is defined as

$$u_m \equiv \frac{1}{\pi(R_0^2-R_i^2)}\int_{R_i}^{R_0} u\, 2\pi\, rdr. \quad (13)$$

Introducing dimensionless temperature, θ, defined as

$$\theta \equiv T/[q_i D_h / k], \quad (14)$$

the energy equation and the boundary conditions may be expressed in the dimensionless forms as

$$\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d\theta}{dr^*}\right) = \frac{4\alpha}{(1+\alpha)}u^* + Br_i\cdot V. \quad (15)$$

$$\textbf{Case A:} \begin{cases} \dfrac{d\theta}{dr^*} = -1 & \text{at } r^* = \dfrac{\alpha}{2(1-\alpha)} \\ \dfrac{d\theta}{dr^*} = 0 & \text{at } r^* = \dfrac{1}{2(1-\alpha)} \end{cases} \quad (16)$$

$$Br_i \equiv \frac{\eta^* u_m^2}{D_h q_i} \tag{17}$$

$$V = \left\{ \frac{8(1-\alpha)}{(1+\alpha)} \int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} r^* \eta_a^* \left(\frac{du^*}{dr^*} \right)^2 dr^* \right\} u^* - \eta_a^* \left(\frac{du^*}{dr^*} \right)^2. \tag{18}$$

Nusselt number, Nu_{ii}, on the inner tube is calculated as

$$Nu_{ii} \equiv \frac{[q_i / (T_i - T_b)] D_h}{k} = \frac{1}{\theta_i - \theta_b} \tag{19}$$

where dimensionless bulk temperature, θ_b, is defined as

$$\theta_b \equiv \frac{T_b k}{q_i D_h} = \frac{8(1-\alpha)}{1+\alpha} \int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} u^* \theta r^* dr^* \tag{20}$$

3. Results and discussion

The parameter V governs the heat transfer with viscous dissipation through Eq.(15). The behaviors of parameter V are shown in Fig.2 for the nine combinations of $n = 0.5$, 1.0, 1.5 and $U^* = -1.0$, 0.0, 1.0 respectively. Here V is shown as a function of transformed dimensionless radial coordinate ξ with β as a parameter. $\xi = 0$ is the surface of moving inner core. $\xi = 1$ is the surface of fixed outer tube. For pseudoplastic fluids ($n < 1$) the extreme at $\beta \to 0$ corresponds to a Newtonian fluid and that at $\beta \to \infty$ to a power law fluid. For dilatant fluids ($n > 1$) the extreme at $\beta \to \infty$ corresponds to a Newtonian fluid and that at $\beta \to 0$ to a power law fluid. $n = 1$ is a Newtonian fluid. The absolute values of V become larger near the moving core with an increases n. However it become smaller near the both tubes with an increase U^*.

Temperature difference $(\theta - \theta_b)$ for Case A is shown in Fig.3 for $\beta = 1.0$. $\xi = 0$ corresponds to the heated inner tube and $\xi = 1$ the insulated outer tube. The temperature difference $(\theta - \theta_b)$ increases with an increase in Br_i near the walls. But it decreases in the middle section. This is attributed to that near the walls the absolute values parameter V is large (see Fig.2) and that the viscous dissipation effect is large. The heat transfered axially by convection is large in the middle section. For $U^* = -1.0$, $(\theta - \theta_b)$ greatly increases with an increase in Br_i near the moving core, but increases a little near the fixed tube. In the middle section it decreases. For $U^* = 1.0$, $(\theta - \theta_b)$ increases with an increase in Br_i near the fixed tube and decreases near the moving core. It can be seen that $(\theta - \theta_b)$ decreases with an increase in U^* near the moving core. In this paper the typical results for parameter V and $(\theta - \theta_b)$ have been illustrated for $\alpha = 0.5$.

Nusselt numbers Nu_{ii}, are shown in Figs.4(a), 4(b) and 4(c). Nu_{ii} decreases with an increase in Br_i for $U^* = -1.0$ and $U^* = 0.0$ and increases with an increase in Br_i for $U^* = 1.0$. These behaviors of Nusselt numbers Nu_{ii} can be explained by the viscous dissipation effects on temperature difference $(\theta - \theta_b)$, as mentioned above.

4. Conclusions

The fully developed laminar heat transfer of modified power-law fluids in a concentric annulus with an axially moving core was analyzed taking into account the viscous dissipation of the flowing fluid. The effects of the radius ratio, the flow index, the relative core velocity, dimensionless shear rate parameter and Brinkman number on the temperature distribution and Nusselt numbers were studied.

In this report the heat transfer results for Case A only are discussed. The counterpart for Case B will be shown in another report.

Reference

1. G. Davaa, T. Shigechi and S. Momoki "Fluid flow for modified power law fluids in concentric annuli with axially moving cores" *Reports of the Faculty of Engineering, Nagasaki University*, vol. **32**, No. **58**, p.83-90 (2002).
2. M. Capobianchi and T. F. Irvine, "Predictions of pressure drop and heat transfer in concentric annular ducts with modified power law fluids" *Wärme-und Stoffübertragung*, **27**, p.209-215 (1992)

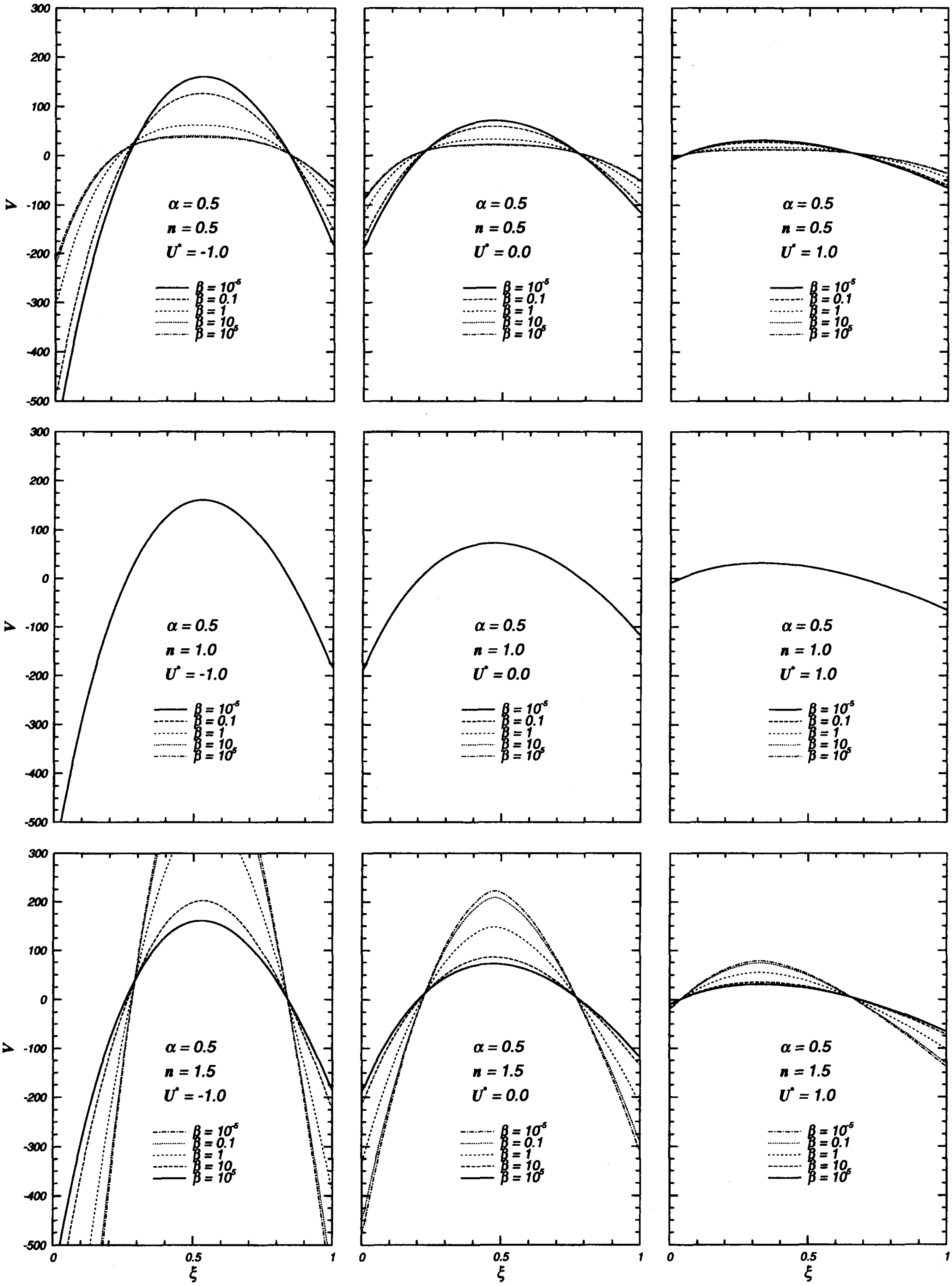

Fig.2 Distribution of parameter V for $\alpha = 0.5$

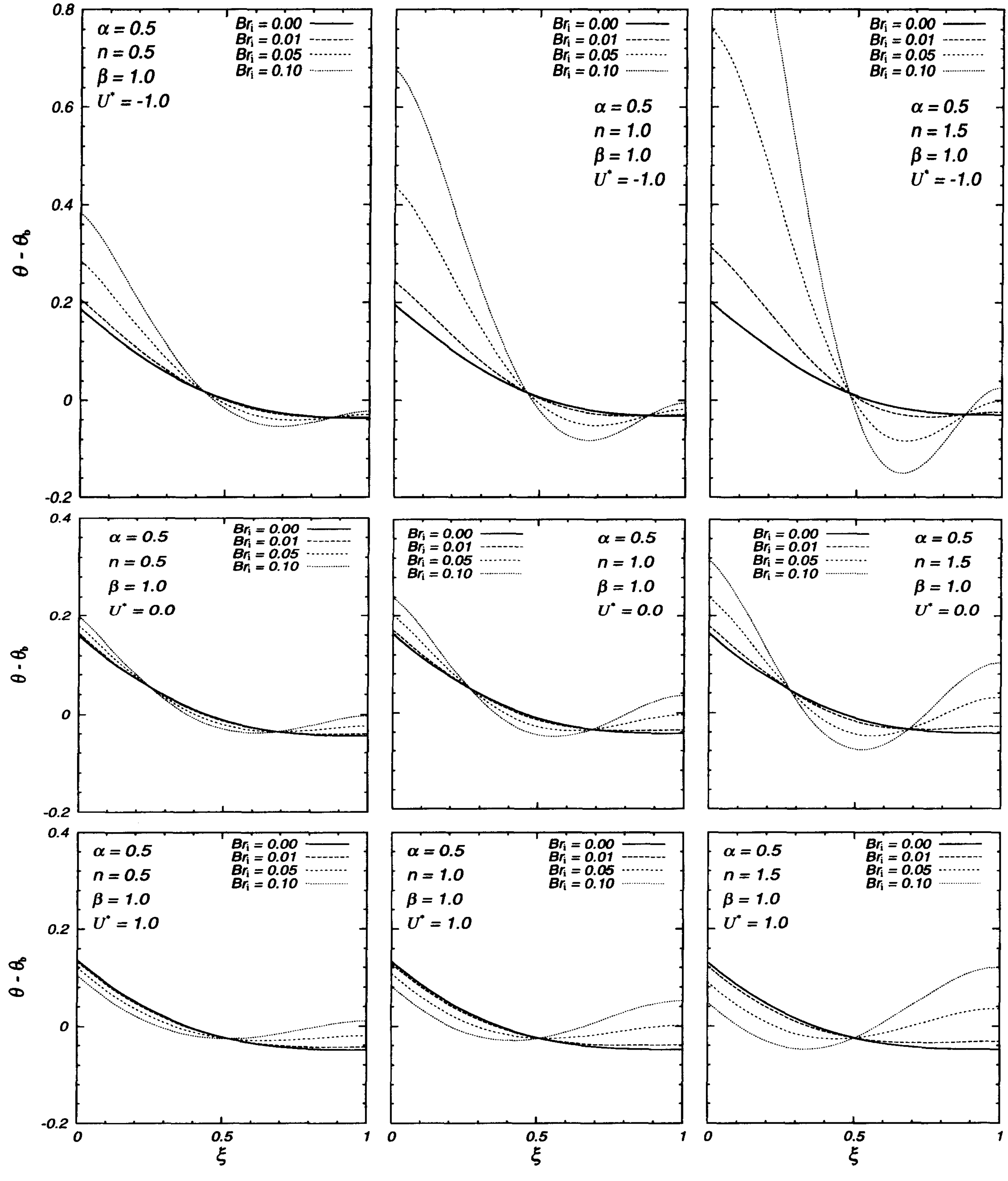

Fig.3 Dimensionless temperature difference for Case A ($\alpha = 0.5$, $\beta = 1.0$)

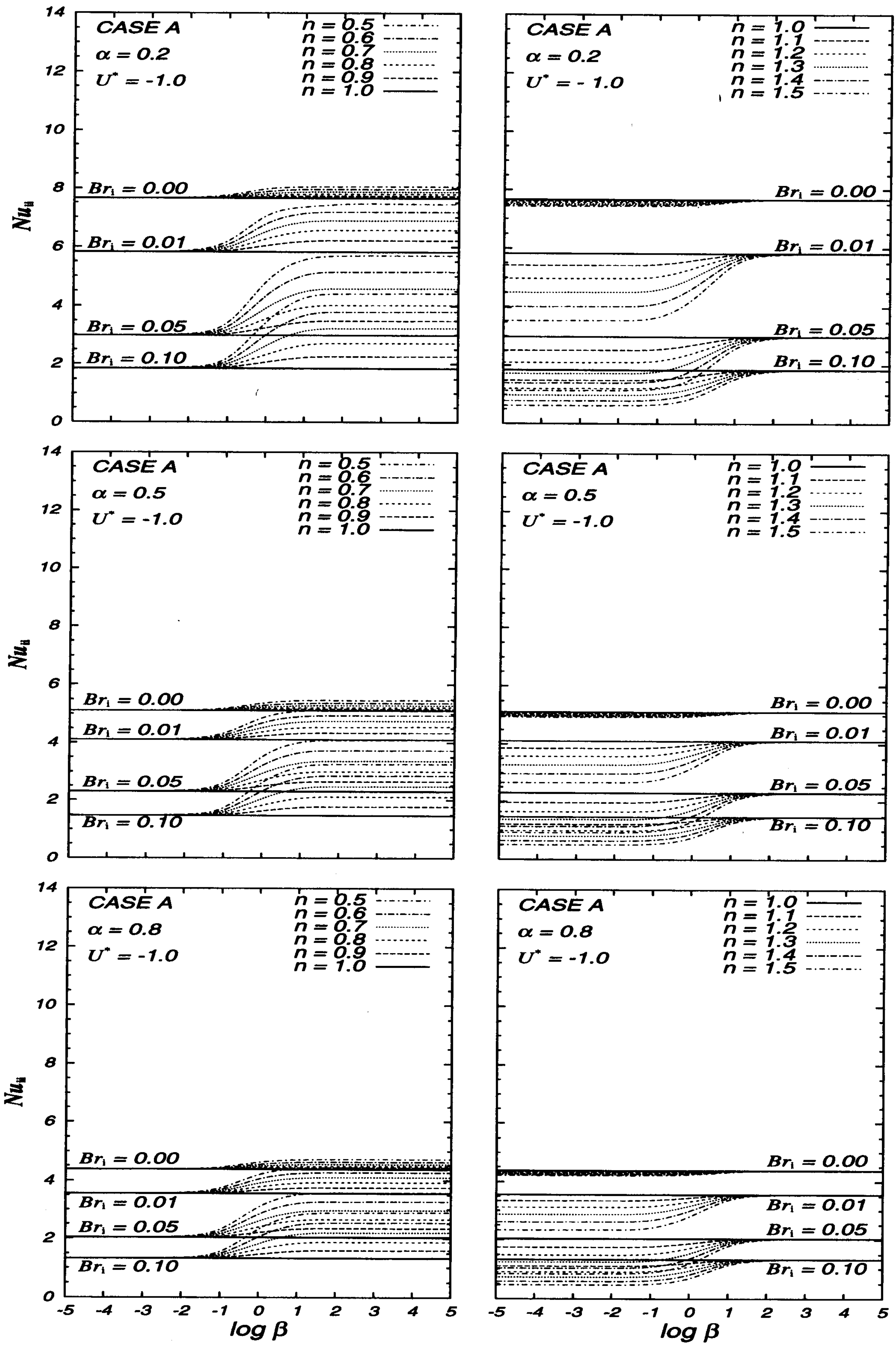

Fig.4(a) Nusselt numbers for $U^* = -1.0$ (Case A)

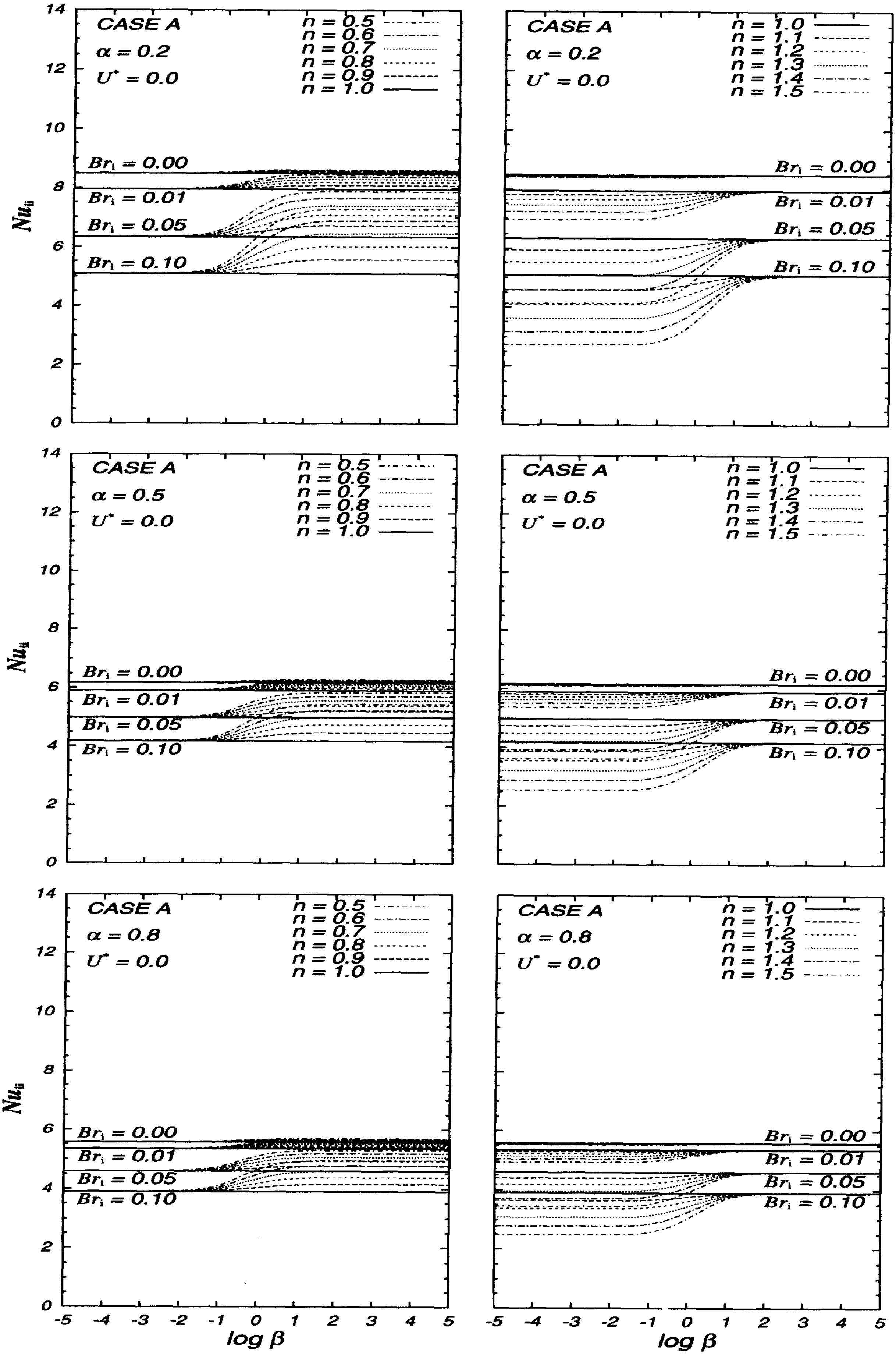

Fig.4(b) Nusselt numbers for $U^* = 0.0$ (Case A)

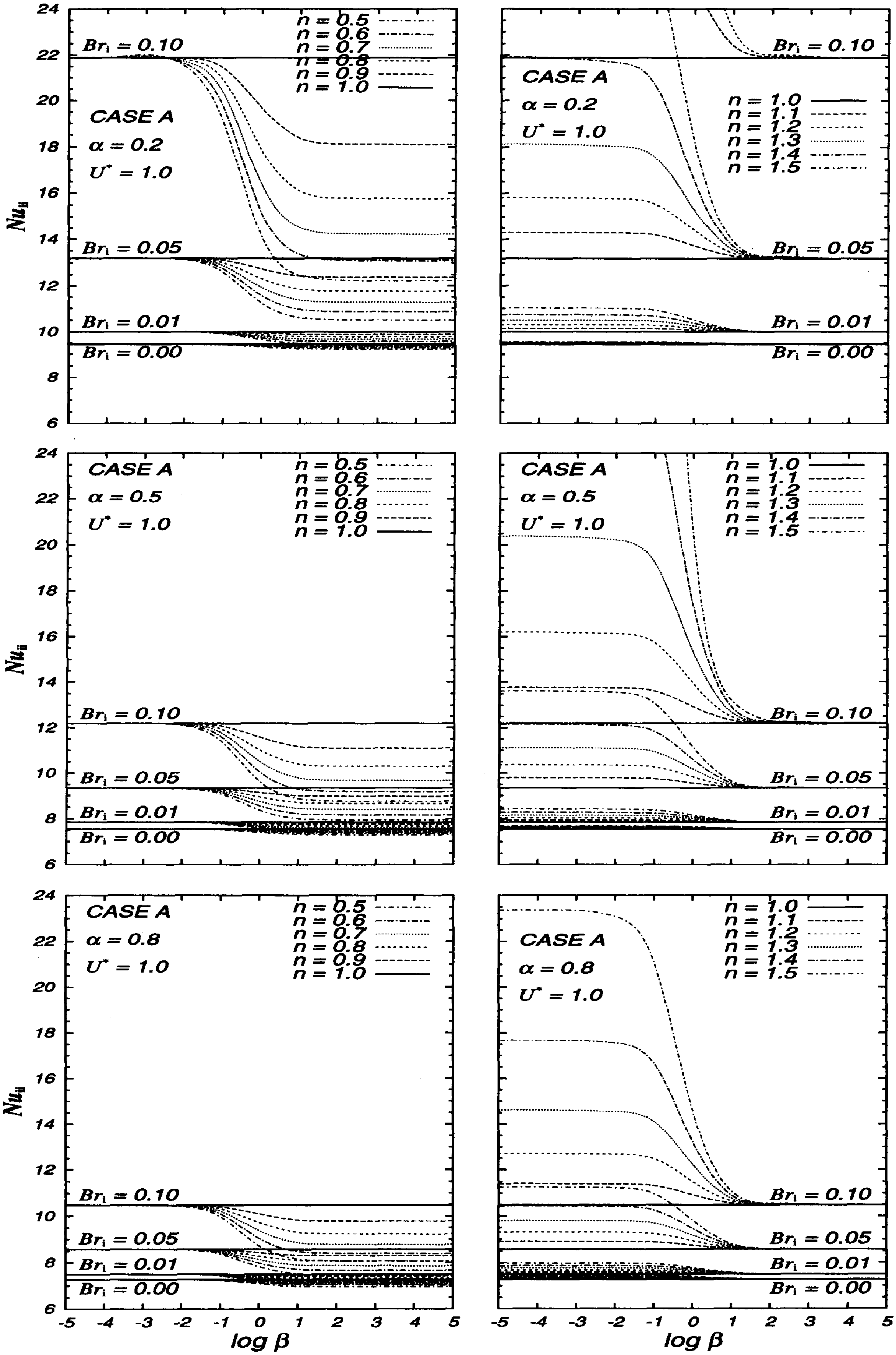

Fig.4(c) Nusselt numbers for $U^* = 1.0$ (Case A)