

**AN EQUILIBRIUM ANALYSIS OF THE INSURANCE MARKET  
WITH VERTICAL DIFFERENTIATION**

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## **Abstract**

Each insurance product per se is identical but the insurance companies that sell this identical insurance product are not necessarily identical. Because the nature of insurance is to cover damages from accidents, for example, consumers hope to get their insurance moneys quickly from their insurance companies when accidents happen. In this study, this is interpreted as “aftercare” and all insurance companies incorporate it into their strategies. The insurance companies in the market are assumed to compete not only on price (insurance rate) but also on quality (level of aftercare). Thus, it is natural that the insurance market is assumed to have vertical differentiation.

## **Introduction**

Consumers have to decide from which insurance company they purchase an insurance product. These decisions can be affected by several factors.

The most common factor is price. If a decision on insurance, in a competitive market, can be explained by the level of price alone, all insurance companies would set the same price, as a result of Bertrand competition. However, previous studies have confirmed the existence of price dispersion in the insurance market. For example, Jung (1978) has proved that there is price dispersion on identical automobile liability insurance in Chicago, Illinois and there are statistically significant differences when insurance companies are classified by distribution system. Mathewson (1983), Dahlby and West (1986), and Posey and Yavas (1995) have argued that price dispersion stems from the search costs of consumers. Berger et al. (1989) presented empirical evidence on switching costs and differential prices. Schlesinger and Schulenburg (1991, 1993) explained price dispersion according to three factors: product differentiation, search cost, and switching cost simultaneously.

In previous studies, other factors in the insurance market are considered to explain price dispersion. One of these is coverage. Undoubtedly, all consumers prefer more coverage than less. However, other things being equal, it seems that coverage is not definitive to the existence of price dispersion. This can be explained by the following three reasons. First, in general, almost all of the insurance companies offer identical insurance products because the regulator will not be allowed to offer any other type of insurance product. Even if there were different insurance products, courts have frequently said that they are same (McDowell, 1989). Second, as McDowell (1989) has noted, even if there were total deregulation of insurance products, the similarity of insurance products would continue

because it is much more expensive to custom-make or tailor an insurance product for each consumer than to sell a standard one. Third, because insurance products are invisible and defined prior to purchase, it may be too difficult for common consumers to understand their contents completely. Thus, many consumers may not be fully aware of the contents of insurance products when they purchase them.

Another factor is the quality level. Schlesinger and Schulenburg (1991, 1993) have pointed out that all insurance companies are not necessarily identical even if all the insurance products are identical because it is possible to set different quality levels among insurance companies.<sup>1</sup> Furthermore, Schlesinger and Schulenburg (1991, 1993) have provided quality for insurance companies' reputation, solvency characteristics, marketing methods, claim-handling procedures, and so on. In the real world, all the above attitudes are compounded. However, to the extent that the scope of this paper is restricted to the Japanese insurance market, it can be considered that claim-handling procedures are the most meaningful factor. Because the nature of insurance is to cover the damage arising from an accident, consumers hope to get insurance money promptly from their insurance companies when an accident happens. This can be interpreted as "aftercare".<sup>2</sup>

In contrast, at least until now, when Japanese consumers purchase an insurance product, they may not regard other attitudes, such as solvency, as important because perhaps only a small number of insurance companies have gone bankrupt since World War II. In Japan, the marketing of non-life insurance is mostly through agents and the marketing of life insurance is through insurance practitioners, except for some foreign insurance affiliates. Then because both non-life and life insurance policies are sold through a single distribution system, marketing methods can be considered as identical.

In this paper, an attempt is made to analyze an insurance market where insurance companies not only offer insurance products but also provide aftercare if an accident happens. The model involved has two important features.

First, Schlesinger and Schulenburg (1991), Gravelle (1994) and Tsutsui et al. (2000) have analyzed a horizontal differentiated insurance market that is associated with "variety". However, because consumers may prefer more aftercare than less, this paper indicates the vertical differentiation that is associated with "quality".

Second, my model differs from general vertical differentiated models in the following points.

(1) The price in an insurance market is represented by the insurance rate, which equals the insurance premium divided by the insurance amount.<sup>3</sup> Thus, unlike other markets, to set a price in an insurance market, each insurance company controls simultaneously the

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<sup>1</sup> In particular, Japanese non-life insurance products, such as fire, automobile and personal accident, are provided by identical clauses. The reason for this is to purchase reinsurance from reinsurance companies and to purchase coinsurance with other insurance companies.

<sup>2</sup> In the real world, the level of aftercare can be measured by the complaint ratio, which is based on the number of complaints received. See Doerpinghaus (1991), Wells and Stafford (1995) and Hoyt and Query (1999).

<sup>3</sup> Hereafter, in this paper "price" and "insurance rate" are used interchangeably.

two variables, insurance premium and insurance amount.

(2) In general, though an insurance company can sell an insurance product at different prices to different consumers, it cannot set at different levels of quality to different consumers. For example, it is impossible to perform the differentiation that low-risk consumers receive their insurance money quickly and high-risk consumers receive their insurance money slowly. Furthermore, such a differentiation is prohibited by Japanese insurance law. In other words, regardless of the consumer's risk type, all consumers who have the same insurance company receive same level of quality if an accident happens.

(3) A consumer can receive aftercare only if an accident happens. This fact suggests that contrary to previous studies, two parameters, perception for quality and probability of an accident, are directly related to a consumer's decision.

This paper has two purposes: first, to analyze how a vertical differentiation in a competitive insurance market will affect price and secondly, to examine social welfare which is sum of consumer welfare and producer welfare as the regulator imposes minimum quality standards.

## The Model

In this section, the basic model is defined. There are two insurance companies, A and B. Suppose that two types of consumer exist in an insurance market, which include high-risk consumers with accident probability  $\pi_H$  and low-risk consumers with accident probability  $\pi_L$ , and assume that  $0 < \pi_L < \pi_H < 1/2$ . Furthermore, assume that both insurance companies can observe the type of consumer without cost. Denote that  $N_H$  is the number of high-risk consumers and  $N_L$  is the number of low-risk consumers, where both are strictly positive.

Let  $P_i^j$  and  $S_i^j$  represent the insurance premium and the insurance amount, respectively, of a consumer of type  $i$  purchased from the insurance company  $j$ . However, each insurance company cannot distinguish quality in accordance with each consumer. Thus, insurance company  $j$  offers only one quality,  $q^j \in [q_{\min}, q_{\max}]$ , regardless of type. Let  $q_{\min} \in [0, q_{\max})$  represent the minimum quality and  $q_{\max} \in (q_{\min}, \infty)$  represent the maximum quality. Assume that both insurance companies have the same form of the cost function and the level of quality has no influence on

quantities  $C(q^j) = aq^{j^2}/2$  where  $a > 0$ . Some examples of such quality features are the investment made educating employees and paying out insurance money more rapidly. It is assumed that both insurance companies cannot avoid paying this investment cost, even if they sell nothing.

The utility of consumers is assumed to be separable in income and aftercare. Thus, the utility function is

$$U = u(\bullet) + v(\bullet) \quad (1)$$

where  $u(\bullet)$  is the utility that is related to the income level and  $v(\bullet)$  is the utility that is related to the aftercare. Each consumer has to choose the purchase of one unit from the more desirable insurance company. Now, applying Pratt (1964), the consumer's utility, which is related to his income, becomes

$$CE^{INCO} = EP^{INCO} - \frac{rVar(P^{INCO})}{2} \quad (2)$$

where  $CE$  represents the certainty equivalent,  $EP$  represents the expected payoff, and  $P$  represents the (variable) payoff. Each variable that is indexed "INCO" is included in the utility  $u(\bullet)$ .  $r$  is the degree of absolute risk aversion and it is assumed to be constant regardless of the consumer's type and wealth.  $Var(\bullet)$  denotes the variance.

Consider  $EP^{INCO}$  in Equation (2). Let  $W$  be the initial wealth. Denote  $D$  as the value of an insurable asset and, for simplicity, there are only two states of the world, total loss and no loss. If a consumer of type  $i$  purchases an insurance product  $\delta_i^j \equiv \{P_i^j, S_i^j\}$  from insurance company  $j$ , this consumer is faced with the situation where his payoff becomes  $W - P_i^j - D + S_i^j$  if an accident happens and  $W - P_i^j$  if an accident does not happen. Then  $EP^{INCO}$  can be written as

$$EP^{INCO} = W - P_i^j - \pi_i(D - S_i^j). \quad (3)$$

Using Equation (3),  $Var(P^{INCO})$  in Equation (2) is given by

$$Var(P^{INCO}) = \pi_i(1 - \pi_i)(D - S_i^j)^2. \quad (4)$$

Substituting Equations (3) and (4) in Equation (2) gives

$$CE^{INCO} = W - P_i^j - \pi_i(D - S_i^j) - \frac{r}{2}\pi_i(1 - \pi_i)(D - S_i^j)^2. \quad (5)$$

Let us next consider  $v(\bullet)$ . First, assume that  $v(\bullet)$  is a linear function. Thus, the certainty equivalent,  $CE^{CARE}$ , which is related to the aftercare, is consistent with the expected utility,  $EP^{CARE}$ . Both variables that are indexed ‘‘CARE’’ are included in the utility  $v(\bullet)$ . Further suppose that all consumers have their own quality valuation, say  $\theta$ , which has uniform distribution on  $[0,1]$ .<sup>4</sup> Given these assumptions, it can be written as

$$CE^{CARE} = EP^{CARE} = \pi_i\theta q^j. \quad (6)$$

Using Equations (1), (5), and (6), the maximization problem for consumer  $i$  can be obtained by

$$Max_j CE_i^j = W - P_i^j - \pi_i(D - S_i^j) - \frac{r}{2}\pi_i(1 - \pi_i)(D - S_i^j)^2 + \pi_i\theta q^j. \quad (7)$$

This paper sets out a two-stage game as follows: In the first stage, both insurance companies decide  $q^j$  simultaneously. Thus, this stage can be considered ‘‘choice of quality’’. Without loss of generality, the quality that is offered by insurance company B is higher than that submitted by insurance company A. That is,  $q^B > q^A$ . In the second stage, both insurance companies decide the insurance product  $\delta_i^j$  simultaneously after they observe their qualities. Thus, this stage can be considered the ‘‘choice of price’’.

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<sup>4</sup> According to Stafford and Wells (1996) and Hoyt and Query (1999), the different valuations among consumers are a

Let the marginal consumer who does not differentiate between two insurance companies be denoted by  $\theta_i^*$ . For simplicity, the reservation prices for consumers for an insurance product are sufficiently high to ensure that all consumers are willing to purchase.<sup>5</sup> Then

$$\theta_i^* = \frac{P_i^B - P_i^A}{\pi_i(q^B - q^A)} + \frac{S_i^A - S_i^B}{q^B - q^A} + \frac{r(1 - \pi_i)(2D - S_i^A - S_i^B)(S_i^A - S_i^B)}{2(q^B - q^A)} \quad (8)$$

where the first term shows the effect of the difference between their insurance premiums, the second term shows the effect of the difference between their insurance amounts, and the third term shows the effect of uncertainty of consumer  $i$  after purchasing an insurance product. Further, all terms in the denominator include  $(q^B - q^A)$ . Thus, all of the above effects decrease with increasing the value of  $(q^B - q^A)$ .

Equation (8) means that the consumers in the interval  $[0, \theta_i^*]$  purchase from insurance company A while those in the interval  $[\theta_i^*, 1]$  purchase from insurance company B. Because  $\theta$  is uniformly distributed on  $[0, 1]$ , the aggregate demand for the insurance product A in the type  $i$  market is  $\theta_i^*$  and for the insurance product B is  $1 - \theta_i^*$ .

Two points should be noted. First, a difference in the accident probability leads to not only a difference in demand for aftercare but also a difference in the insurance product. Thus, though consumers have the same distributional form regardless of their type, it does not necessarily mean that  $\theta_H^*$  and  $\theta_L^*$  are identical. Second, in equilibrium,  $0 < \theta_i^* < 1$  always holds. In other words, in equilibrium, both insurance companies have strictly positive demand.<sup>6</sup>

Each insurance company is assumed to be risk neutral and the expected profit functions take the form

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function of age, education level, and claim record.

<sup>5</sup> More precisely, see Appendix A.

<sup>6</sup> Proof of this is deferred to the Appendix B.

$$E(\Pi^A) = N_H \theta_H^* (P_H^A - \pi_H S_H^A) + N_L \theta_L^* (P_L^A - \pi_L S_L^A) - \frac{aq^{A^2}}{2}, \quad (9)$$

$$E(\Pi^B) = N_H (1 - \theta_H^*) (P_H^B - \pi_H S_H^B) + N_L (1 - \theta_L^*) (P_L^B - \pi_L S_L^B) - \frac{aq^{B^2}}{2}. \quad (10)$$

## Analysis of the Equilibrium

In this section, a subgame perfect Nash equilibrium is derived. This equilibrium is obtained by backward induction. At the beginning of the equilibrium analysis, it is shown that, given each quality, both insurance companies set the price simultaneously.

Substituting Equation (8) in Equations (9) and (10), the expected profit functions for the insurance companies become:

$$E(\Pi^A) = \sum_{i=H,L} \left[ N_i \left\{ \frac{P_i^B - P_i^A}{\pi_i (q^B - q^A)} + \frac{S_i^A - S_i^B + \frac{r}{2} (1 - \pi_i) (2D - S_i^A - S_i^B) (S_i^A - S_i^B)}{q^B - q^A} \right\} (P_i^A - \pi_i S_i^A) \right] - \frac{aq^{A^2}}{2}, \quad (11)$$

$$E(\Pi^B) = \sum_{i=H,L} \left[ N_i \left\{ 1 - \frac{P_i^B - P_i^A}{\pi_i (q^B - q^A)} - \frac{S_i^A - S_i^B + \frac{r}{2} (1 - \pi_i) (2D - S_i^A - S_i^B) (S_i^A - S_i^B)}{q^B - q^A} \right\} (P_i^B - \pi_i S_i^B) \right] - \frac{aq^{B^2}}{2}. \quad (12)$$

The first-order optimality conditions are

$$\frac{\partial E(\Pi^A)}{\partial P_i^A} = -2P_i^A + P_i^B + \pi_i (2S_i^A - S_i^B) + \frac{r}{2} \pi_i (1 - \pi_i) (2D - S_i^A - S_i^B) (S_i^A - S_i^B) = 0, \quad (13)$$



$$\begin{aligned}\frac{\partial E(\Pi^A)}{\partial S_i^A} &= \{1+r(1-\pi_i)(D-S_i^A)\}(P_i^A - \pi_i S_i^A) - (P_i^B - P_i^A) - \pi_i(S_i^A - S_i^B) \\ &\quad - \frac{r}{2}\pi_i(1-\pi_i)(2D-S_i^A-S_i^B)(S_i^A-S_i^B) = 0,\end{aligned}\quad (14)$$

$$\begin{aligned}\frac{\partial E(\Pi^B)}{\partial P_i^B} &= -2P_i^B + P_i^A + \pi_i(2S_i^B - S_i^A) + \pi_i(q^B - q^A) \\ &\quad - \frac{r}{2}\pi_i(1-\pi_i)(2D-S_i^A-S_i^B)(S_i^A-S_i^B) = 0,\end{aligned}\quad (15)$$

$$\begin{aligned}\frac{\partial E(\Pi^B)}{\partial S_i^B} &= \{1+r(1-\pi_i)(D-S_i^B)\}(P_i^B - \pi_i S_i^B) - \pi_i(q^B - q^A) + (P_i^B - P_i^A) \\ &\quad + \pi_i(S_i^A - S_i^B) + \frac{r}{2}\pi_i(1-\pi_i)(2D-S_i^A-S_i^B)(S_i^A-S_i^B) = 0.\end{aligned}\quad (16)$$

By combining these equations, then<sup>7</sup>

$$P_i^{A*} = \pi_i D + \frac{\pi_i(q^B - q^A)}{3}, \quad (17)$$

$$S_i^{A*} = D, \quad (18)$$

$$P_i^{B*} = \pi_i D + \frac{2\pi_i(q^B - q^A)}{3}, \quad (19)$$

$$S_i^{B*} = D. \quad (20)$$

Each insurance rate  $p_i^j$  can be denoted by  $\frac{P_i^j}{S_i^j}$ . Thus, using the above equations, each

equilibrium insurance rate may be written as

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<sup>7</sup> There are three solutions for Equations (13) to (16). However, only Equations (17) to (20) satisfy the second-order

$$p_i^{A*} = \pi_i + \frac{\pi_i(q^B - q^A)}{3D}, \quad (21)$$

$$p_i^{B*} = \pi_i + \frac{2\pi_i(q^B - q^A)}{3D}. \quad (22)$$

The implications of the above equations are as follows: First, from Equations (21) and (22), each insurance rate includes not only the accident probability but also the additional costs of differentiating quality. In other words, both insurance companies offer an actuarially “unfair” price. Second, as compared with Equations (21) and (22), since  $q^B > q^A$  then  $p_i^{B*} > p_i^{A*}$ . Therefore, for all  $i \in \{H, L\}$ , the price of insurance product B is higher than the price of insurance product A. Third, Equations (18) and (20) show clearly that, in equilibrium, both insurance companies sell the full insurance. Ehrlich and Becker (1972) and Stiglitz (1977) have proved that with perfect information, in equilibrium, all insurance companies sell the full insurance. In this model, it is clear that this proposition is also satisfied even if product differentiation prevails.

To consider the quality choices in the first stage of the game, substituting Equations (17) to (20) in Equations (11) and (12), each expected profit function can be written as

$$E(\Pi^A) = \frac{1}{9} N_c (q^B - q^A) - \frac{aq^{A^2}}{2}, \quad (23)$$

$$E(\Pi^B) = \frac{4}{9} N_c (q^B - q^A) - \frac{aq^{B^2}}{2} \quad (24)$$

where  $N_c \equiv \pi_H N_H + \pi_L N_L$ . Here,  $N_c$  denotes the expected number in case of an accident.

Suppose that  $\bar{q} \equiv \frac{4N_c}{9a} < q_{\max}$  and  $q_{\min} < \bar{q}$ . Then one asymmetric subgame perfect equilibrium exists. Hence, the insurance companies set their qualities as follows:<sup>8</sup>

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conditions for the maximization problem. Proof of this is deferred to Appendix C.

$$q^{A*} = q_{\min}, \quad (25)$$

$$q^{B*} = \bar{q}. \quad (26)$$

Substituting Equations (25) and (26) in Equations (21) and (22), they can be seen that

$$p_i^{A*} = \pi_i + \frac{\pi_i}{3D}(\bar{q} - q_{\min}), \quad (27)$$

$$p_i^{B*} = \pi_i + \frac{2\pi_i}{3D}(\bar{q} - q_{\min}). \quad (28)$$

Equations (27) and (28) can explain why the price of the low-risk consumer is sometimes higher than that of the high-risk consumer in the (Japanese) insurance market. In other words, even if a consumer is low risk, he or she may face a high price to purchase from insurance company B that offers high quality, and vice versa. In order to deal with this problem, circumstances are examined under which the above situation occurs.

Mathematically, it needs to confirm the exogenous condition that satisfies  $p_L^{B*} > p_H^{A*}$ .

Thus, subtracting  $p_H^{A*}$  from  $p_L^{B*}$ , it can be obtained that

$$p_L^{B*} - p_H^{A*} = -(\pi_H - \pi_L) + \frac{1}{3D}(2\pi_L - \pi_H)(\bar{q} - q_{\min}). \quad (29)$$

The first term shows the difference between the insurance rates; the second term is related to the difference between the quality levels. The first term is obviously strictly negative. Hence, it must be satisfied that the second term is strictly positive whenever  $p_L^{B*} > p_H^{A*}$ . Simplification yields the condition  $2\pi_L - \pi_H > 0$ . This condition means that the difference between the two accident probabilities is relatively small.

However, the condition  $2\pi_L - \pi_H > 0$  is merely the necessary condition but not the sufficient condition for  $p_L^{B*} > p_H^{A*}$ . Consequently, even if this condition is satisfied, it is possible that  $p_L^{B*} > p_H^{A*}$  is not satisfied. From (29),  $p_L^{B*} > p_H^{A*}$  is easily obtained when

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<sup>8</sup> The following solutions are proved in Appendix D.

$(\bar{q} - q_{\min})$  is large. This implies that the more the quality is differentiated, the greater the probability that  $p_L^{B*} > p_H^{A*}$  is satisfied.

## Comparative Statics

In this section, several comments are made on the equilibrium prices that are shown in Equations (27) and (28). By Equations (27) and (28), all equilibrium prices are affected by five kinds of exogenous variable: 1) accident probability,  $\pi_i$ ; 2) value of an insurable asset,  $D$ ; 3) number of consumers,  $N_i$ ; 4) cost function form,  $a$ ; and 5) minimum quality level,  $q_{\min}$ . Now, consider the case where only one exogenous variable changes.

### 1) *accident probability* ( $\pi_i$ )

Partially differentiating Equations (27) and (28) with respect to  $\pi_i$ ,

$$\frac{\partial p_i^{j*}}{\partial \pi_i} > 0, \quad (30)$$

$$\frac{\partial^2 p_i^{j*}}{\partial \pi_i^2} > 0, \quad (31)$$

$$\frac{\partial p_i^{j*}}{\partial \pi_k} > 0 \quad \text{for } i \neq k. \quad (32)$$

These derivatives have a number of interesting features.

First, from Equations (30) and (31), all equilibrium prices are monotone increasing and convex functions of their own accident probability. This result implies that each price is a non-linear (convex) function of its own accident probability in spite of a perfectly informed insurance market. An intuitive explanation for this is provided. An increase in  $\pi_i$  increases both the expected insurance money and the total aftercare cost. Expected insurance money is a linear function of its own accident probability whose slope is equal

to unity, while the aftercare cost is a convex function of its own accident probability because it leads to an increase, not only in the expected number in case of an accident, but also the equilibrium quality of insurance company B.

Second, using Equations (30) and (31), then  $\frac{\partial p_i^{B*}}{\partial \pi_i} > \frac{\partial p_i^{A*}}{\partial \pi_i}$  and  $\frac{\partial^2 p_i^{B*}}{\partial \pi_i^2} > \frac{\partial^2 p_i^{A*}}{\partial \pi_i^2}$ . The

change in the insurance rates of insurance company B is larger than that of insurance company A because it only affects  $q^{B*}$  while insurance company A always sets its quality  $q_{\min}$ .

Third, by Equation (32), all equilibrium prices are monotone increasing functions of the other accident probability. This result can be explained as follows. An increase in  $\pi_k$  implies that a consumer of type  $k$  would be more likely to desire high quality. Insurance company B raises its quality. This increase leads to an increase in the equilibrium prices of both types because of the additional quality cost. Moreover, its increase also increases the insurance rates of insurance company A. However, this feature stems not from the existence of the quality variable, but from the impossibility of quality differentiation between consumer types. If it were possible to set the quality in accordance with consumer types, an increase in  $\pi_k$  would only affect the price and quality of type  $k$ .

## 2) value of an insurable asset ( $D$ )

Differentiating Equations (27) and (28) with respect to  $D$ ,

$$\frac{\partial p_i^{j*}}{\partial D} < 0, \quad (33)$$

$$\frac{\partial^2 p_i^{j*}}{\partial D^2} > 0. \quad (34)$$

Equations (33) and (34) indicate that all equilibrium prices are monotone decreasing and convex functions of  $D$ . The quality cost per insurance product is constant regardless of the insurance amount. Accordingly, the more the insurance amount increases, the more the quality cost per insurance amount decreases. Thus, an increase in the value of an insurable asset brings about a decrease in all equilibrium prices because, in equilibrium,

$S_i^{j*} = D$ . In other words, there are economies of scale with respect to quality cost. It can be also interpreted as “band grading” in the insurance institution.

### 3) *number of consumers* ( $N_i$ )

The following property is readily verified:

$$\frac{\partial p_i^{j*}}{\partial N_i} > 0. \quad (35)$$

The intuition underlying this derivative (35) is readily shown. An increase in the number of consumers increases the fraction of the consumers who will be pressing a claim. Then, insurance company B increases its quality and the insurance rates of both insurance companies also increase.

### 4) *cost function form* ( $a$ )

From Equations (27) and (28), the following derivative can be derived:

$$\frac{\partial p_i^{j*}}{\partial a} < 0. \quad (36)$$

An increase in  $a$  implies that the cost function becomes steeper. It leads to a decrease in the quality of insurance company B and both insurance rates also decrease.

### 5) *minimum quality level* ( $q_{\min}$ )

Equation (27) and (28) is partially differentiated with respect to  $q_{\min}$ :

$$\frac{\partial p_i^{j*}}{\partial q_{\min}} < 0. \quad (37)$$

Thus, an increase in the minimum quality level decreases all equilibrium prices because it limits the range in which the insurance companies can differentiate quality and

then they face an intensified price competition.

## The Effect of Minimum Quality Standards

In this paper thus far, the situation where minimum quality level is an exogenous variable has been considered. However, in real life, the regulator often sets the minimum quality level endogenously.

For example, consider the Japanese life insurance contract. In principle, it binds the insurance company to pay insurance money within five days after the procedural conditions have been completed. These contracts are ruled by the regulator and so all insurance contracts are almost identical. Namely, the time of payment, which is contained in the claims handling procedures, is the regulator's endogenous variable.

To cite another example, since 1996, a life insurance company would be permitted to enter the non-life insurance market in Japan, and vice versa. However, the regulator requested life insurance companies that are going to enter the non-life insurance market to employ their own assessors. This requirement means that a life insurance company must represent a certain quality level when entering in the non-life insurance market.

In this section, the effect of the minimum quality standards on social welfare is analyzed. Several theoretical models of minimum quality standards have been developed, including Leland (1979), Ronnen (1991) and Valletti (2000). A common theme of these researchers is that minimum quality standards can reduce an extreme quality differentiation and raise quality. Imposing minimum quality standards necessarily improves consumer welfare. However, at the same time, producer welfare decreases because not only the price decreases but also the quality cost increases. Thus, it is not clear whether the imposition of minimum quality standards is desirable or not.

First, consider consumer welfare. By Equations (18) and (20), all consumers purchase full insurance. Thus, a consumer of type  $i$  derives the following utility from buying the insurance product  $\delta_i^j$ .

$$U = u(W - P_i^{j*}) + \pi_i \theta q^{j*}. \quad (38)$$

Substituting Equations (17) to (20) in Equation (8), it can be written as

$$\theta_i^* = \frac{1}{3} \quad \text{for } i \in \{H, L\}. \quad (39)$$

Let  $CW$  be consumer welfare. Using Equations (38) and (39), then

$$CW = \sum_{i=H,L} N_i \left[ \int_0^{1/3} \{u(W - P_i^{A*}) + \pi_i \theta q_{\min}\} d\theta + \int_{1/3}^1 \{u(W - P_i^{B*}) + \pi_i \theta \bar{q}\} d\theta \right]. \quad (40)$$

In contrast, let  $PW$  be producer welfare. It is equal to

$$PW = \frac{1}{3} \{N_H (P_H^{A*} - \pi_H D) + N_L (P_L^{A*} - \pi_L D)\} - \frac{aq_{\min}^2}{2} \\ + \frac{2}{3} \{N_H (P_H^{B*} - \pi_H D) + N_L (P_L^{B*} - \pi_L D)\} - \frac{a\bar{q}^2}{2}. \quad (41)$$

Further, denote that the social welfare is  $SW$ . The form of  $SW$  is

$$SW = CW + PW. \quad (42)$$

Differentiating Equation (42) with respect to  $q_{\min}$

$$\frac{dSW}{dq_{\min}} = \frac{1}{9} \left[ \pi_H N_H \{u'(W - P_H^{A*}) + 4u'(W - P_L^{A*})\} + \pi_L N_L \{u'(W - P_H^{B*}) + 4u'(W - P_L^{B*})\} \right] \\ - \frac{N_C}{2} - aq_{\min}. \quad (43)$$

where  $u' \equiv \frac{\partial u}{\partial q_{\min}}$ .

Now, according to Leland (1979), to investigate whether the imposition of minimum quality standards is desirable or not, it needs to confirm whether Equation (43) at  $q_{\min} = 0$  is positive or not. In this case,

$$\left. \frac{dSW}{dq_{\min}} \right|_{q_{\min}=0} = \frac{1}{9} \left[ \pi_H N_H \left\{ u' \left( W - \pi_H \left( D + \frac{4N_C}{27a} \right) \right) + 4u' \left( W - \pi_L \left( D + \frac{4N_C}{27a} \right) \right) \right\} \right. \\ \left. + \pi_L N_L \left\{ u' \left( W - \pi_H \left( D + \frac{8N_C}{27a} \right) \right) + 4u' \left( W - \pi_L \left( D + \frac{8N_C}{27a} \right) \right) \right\} - \frac{N_C}{2} \right]. \quad (44)$$



The sign of Equation (44) is ambiguous. Consider the situation where Equation (44) is positive. When there is greater marginal utility (that is,  $u'(\bullet)$  is large), Equation (44) is positive. Thus, for the insurance market with small initial wealth ( $W$  is small), a slight slope for the cost function ( $a$  is small), and a high value of an insurable asset ( $D$  is large), to impose minimum quality standards may be socially desirable. In contrast, the effect of accident probability ( $\pi_i$ ) and the number of consumers ( $N_i$ ) have an ambiguous effect.

These results can be described in detail. First, a decrease in  $a$  leads to an increase in the quality of insurance company B and so both insurance rates rise. Then, some consumers, whose  $\theta$  is relatively low, have a decrease in their utilities but other consumers have an increase in their utilities. Further, both insurance companies increase their expected profits. However, if  $a$  becomes smaller than a certain level, the quality diversity becomes too large and many consumers may have their utilities decreased. Thus, minimum quality standards may be socially desirable when  $a$  is small.

Second, consider the case where  $W$  is small and/or  $D$  is large. By assuming that each consumer is risk averse, even if minimum quality standards cause insurance rates to fall a little, their utility increases drastically. Thus, in the situation where  $W$  is small and/or  $D$  is large, it is advantageous to impose minimum quality standards.

Third, consider why the effects of  $\pi_i$  and  $N_i$  are ambiguous. The reason for these is described as follows. An increase in  $\pi_i$  and/or  $N_i$  raises a consumer's marginal utility  $u'(\bullet)$  and so the first term in Equation (44) increases. However, at the same time, the second term in Equation (44) decreases because these increases lead to an increase in  $N_C$ .

Accordingly, whether or not minimum quality standards may be desirable is unclear.

Finally, the results presented in this section point out minimum quality standards may be desirable when each consumer is not wealthy. Many firms have more wealth than individuals. Thus, generally speaking, minimum quality standards may be desirable for individuals but not for firms. This interpretation coincides with the fact that the Japanese regulator imposes more severe regulations upon the insurance market for individuals than for firms.

## Conclusions and Remarks

According to Schlesinger and Schulenburg (1993), each insurance product per se is identical but the insurance companies that sell this identical insurance product are not

necessarily identical. In this study, this point can be interpreted as “aftercare” and each insurance company incorporates it into its own strategy. The Japanese insurance market is investigated where competition is not only on price (insurance rate) but also on quality (level of aftercare). Thus, this paper considers the insurance market with vertical differentiation.

In this discussion, some light is shed on several real life questions. For example, in the Japanese insurance market, there are two types of insurance company. One is “domestic”, the other is “foreign”. It is well known that domestic insurance companies set relatively higher prices than foreign insurance companies. Many foreign insurance companies compete aggressively on prices. In contrast, domestic insurance companies focus on service competition instead of price competition. Namely, domestic insurance companies tend to offer high prices and high quality, while foreign insurance companies tend to offer low prices and low quality. This fact coincides with the results of this paper.<sup>9</sup>

In another example, as described above, minimum quality standards may be desirable for individuals but not for firms. Hence, to impose a different type of regulations between individuals and firms may be rational. Such insistence can be seen in a considerable number of works in the insurance field. However, the results in this paper are different from other researchers. Particularly, others emphasize the weakness of individuals in terms of information gap, bargaining power, and level of knowledge. In this paper, whether minimum quality standards are desirable or not is decided by the level of wealth. Thus, even if there were the consumers who have perfect knowledge and strong bargaining power, the regulator should impose some regulations for individuals.

However, the above analysis is incomplete on several points. The following two points are particularly interesting.

First, in this model, both insurance companies and consumers have perfect information. Thus, all insurance companies do not need to deal with adverse selection. However, in the real world, because they cannot verify each consumer’s type, insurance companies may have to decide price and quality due to tell their true types. Furthermore, assuming perfect information is also related to the consumer side. Recently, price information has become clearer because major consumers can easily compare prices using the Internet. However, some uncertainty remains. That price cannot be decided as long as the policy term continues because consumers get policy dividends at least once a year. The level of quality in the timing of contracts does not necessarily coincide with that in the timing of accidents because the policy term is normally long.

Second, this paper explicitly analyzes the insurance market with vertical differentiation where aftercare is the only quality variable. However, there are, in fact, other factors that can be considered as quality variables. For example, the service that is offered ex ante or

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<sup>9</sup> There is good evidence that domestic non-life insurance companies have far more service centers to provide claim-handling procedures than do foreign companies.

interim the policy term seems to be a disregarded factor.<sup>10</sup> Financial robustness (degree of solvency) will become important.<sup>11</sup> Further, horizontal differentiation may need to be incorporated into the model.

These points are still open questions. Much additional work is required to relax the above restrictions and they are left to possible further research. However, several results in this paper include important political implications for the insurance market.

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<sup>10</sup> For example, Crosby and Stephens (1987) have confirmed that consumers who had been receiving in high-level services before they purchase are willing to pay higher insurance premiums than consumers who had not receiving.

<sup>11</sup> This has been referred to in previous studies, for example, Schlesinger and Schulenburg (1991), Pritchett (1994) and Hoyt and Query (1999).

## Appendix A

If a consumer of type  $i$  does not purchase an insurance product, the consumer's certainty equivalent,  $CE_0$ , is

$$CE_0 = W - \pi_i D - \frac{r}{2} \pi_i (1 - \pi_i) D^2. \quad (\text{A1})$$

Thus, the consumer's purchasing (or participating) condition should be written in this form:

$$CE^{INCO} \geq CE_0 \quad (\text{A2})$$

or finally

$$-P_i^j + \pi_i S_i^j + \frac{r}{2} \pi_i (1 - \pi_i) (2D - S_i^j) S_i^j + \pi_i \theta q^j \geq 0. \quad (\text{A3})$$

To induce the condition where all consumers purchase an insurance product, one can confine attention to the consumer whose  $\theta$  is zero. Substituting  $\theta = 0$  into Equation (A3) and solving that for  $r$ :

$$r \geq \frac{2(P_i^j - \pi_i S_i^j)}{\pi_i (1 - \pi_i) (2D - S_i^j) S_i^j}. \quad (\text{A4})$$

Hence, all consumers purchase an insurance product if and only if Equation (A4) holds.

## Appendix B

Suppose that  $\theta_i^* \geq 1$ . Then, the demand for insurance company B in the type  $i$  market is zero. However, now insurance company B has an incentive to alter its own insurance product to  $\tilde{\delta}_i^B \equiv \{\tilde{P}_i^B, \tilde{S}_i^B\}$ , where  $\tilde{P}_i^B = P_i^A + \pi_i (q^B - q^A) - \varepsilon$  ( $\varepsilon$  is a very small positive number) and  $\tilde{S}_i^B = S_i^A$ , because it always gets additional expected profit. Thus,  $\theta_i^* \geq 1$  is

impossible and, hence,  $\theta_i^* < 1$ . Similarly, it is easy to prove  $\theta_i^* > 0$ .

## Appendix C

In this appendix, the simultaneous equations, shown by Equations (13) to (16), are solved. First using Equations (13) and (15), the following equations are derived.

$$P_i^{A^*} = \pi_i S_i^{A^*} + \frac{\pi_i \left\{ (q^B - q^A) + \frac{r}{2} (1 - \pi_i) (2D - S_i^{A^*} - S_i^{B^*}) (S_i^{A^*} - S_i^{B^*}) \right\}}{3}, \quad (C1)$$

$$P_i^{B^*} = \pi_i S_i^{B^*} + \frac{\pi_i \left\{ 2(q^B - q^A) - \frac{r}{2} (1 - \pi_i) (2D - S_i^{A^*} - S_i^{B^*}) (S_i^{A^*} - S_i^{B^*}) \right\}}{3}. \quad (C2)$$

Substituting Equations (C1) and (C2) in Equation (14), it can be expressed as

$$(D - S_i^{A^*}) \left\{ \frac{(q^B - q^A) + \frac{r}{2} (1 - \pi_i) (2D - S_i^{A^*} - S_i^{B^*}) (S_i^{A^*} - S_i^{B^*})}{3} \right\} = 0. \quad (C3)$$

Similar to above, substituting Equations (C1) and (C2) in Equation (16), then

$$(D - S_i^{B^*}) \left\{ \frac{2(q^B - q^A) - \frac{r}{2} (1 - \pi_i) (2D - S_i^{A^*} - S_i^{B^*}) (S_i^{A^*} - S_i^{B^*})}{3} \right\} = 0. \quad (C4)$$

In order to hold Equation (C3), either equation must satisfy the following:

$$D - S_i^{A^*} = 0, \quad (C5)$$

$$(q^B - q^A) + \frac{r}{2} (1 - \pi_i) (2D - S_i^{A^*} - S_i^{B^*}) (S_i^{A^*} - S_i^{B^*}) = 0. \quad (C6)$$

In order to hold Equation (C4), either equation must satisfy the following:

$$D - S_i^{B*} = 0, \quad (C7)$$

$$2(q^B - q^A) - \frac{r}{2}(1 - \pi_i)(2D - S_i^{A*} - S_i^{B*})(S_i^{A*} - S_i^{B*}) = 0. \quad (C8)$$

Accordingly, the combinations to be satisfied with both Equations (C3) and (C4) are four. However, the combination (C6) and (C8) is never satisfied because the sum of Equations (C6) and (C8) is  $3(q^B - q^A) \neq 0$ . Thus, the remainder of this appendix is to check the remaining three cases.

**Case 1: combination (C5) and (C7)**

It is easy to obtain that

$$S_i^{A*} = S_i^{B*} = D. \quad (C9)$$

Thus, both insurance companies offer full insurance.

**Case 2: combinations (C6) and (C7)**

From Equation (C7), it is given by

$$S_i^{B*} = D. \quad (C10)$$

Substituting Equation (C10) in Equation (C6), it becomes<sup>12</sup>

$$S_i^{A*} = D - \sqrt{\frac{2(q^B - q^A)}{r(1 - \pi_i)}}. \quad (C11)$$

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<sup>12</sup> Because  $S_i^{A*} \leq D$ ,  $S_i^{A*} = D + \sqrt{\frac{2(q^B - q^A)}{r(1 - \pi_i)}}$  is an improper solution.

**Case 3: combination (C5) and (C8)**

From Equation (C5), it is given by

$$S_i^{A*} = D. \quad (C12)$$

Substituting Equation (C12) in Equation (C8), it becomes

$$S_i^{B*} = D - \sqrt{\frac{4(q^B - q^A)}{r(1 - \pi_i)}}. \quad (C13)$$

Although there are three kinds of solutions in these simultaneous equations, Case 1 is the only solution for the maximization problem. The others are merely saddle-point solutions. To prove that, first define the second-order conditions for the maximization problem for insurance company A as follows:

$$\frac{\partial^2 E(\Pi^A)}{\partial P_i^{A^2}} < 0, \quad \frac{\partial^2 E(\Pi^A)}{\partial S_i^{A^2}} < 0, \quad \Delta > 0 \quad (C14)$$

$$\text{where } \Delta \equiv \left( \frac{\partial^2 E(\Pi^A)}{\partial P_i^{A^2}} \right) \left( \frac{\partial^2 E(\Pi^A)}{\partial S_i^{A^2}} \right) - \left( \frac{\partial^2 E(\Pi^A)}{\partial S_i^A \partial P_i^A} \right)^2.$$

Because insurance company B has the same form, its conditions are omitted. To confirm the second-order condition, the following equations are derived.

$$\frac{\partial^2 E(\Pi^A)}{\partial P_i^{A^2}} = -\frac{2N_i}{\pi_i(q^B - q^A)}, \quad (C15)$$

$$\frac{\partial^2 E(\Pi^A)}{\partial S_i^{A^2}} = -\frac{N_i}{q^B - q^A} \left\{ r(1 - \pi_i)(P_i^{A*} - \pi_i S_i^{A*}) + 2\pi_i + 2r\pi_i(1 - \pi_i)(D - S_i^{A*}) \right\}, \quad (C16)$$

$$\left(\frac{\partial^2 E(\Pi^A)}{\partial S_i^A \partial P_i^A}\right)^2 = \left[\frac{N_i}{q^B - q^A} \{2 + r(1 - \pi_i)(D - S_i^{A*})\}\right]^2. \quad (\text{C17})$$

Equations (C15) and (C16) are always strictly negative. However, the sign of  $\Delta$  is unclear. Let us first consider Case 1. The following equation can be written as

$$\Delta = \frac{2r(1 - \pi_i)N_i^2}{3(q^B - q^A)} > 0. \quad (\text{C18})$$

From Equation (C18), Case 1 is always satisfied with the second-order conditions for the maximization problem.

In contrast, consider Case 2. The following equation can be written as

$$\Delta = -\frac{2r(1 - \pi_i)N_i^2}{q^B - q^A} < 0. \quad (\text{C19})$$

From Equation (C19), Case 2 is never satisfied with the second-order conditions for the maximization problem. Similarly, it can be confirmed that Case 3 is also never satisfied. Hence, Case 1 is the only solution for the maximization problem. Finally, substituting Equation (C9) into Equations (C1) and (C2), all equilibrium insurance premiums (17) and (19) are derived.

## Appendix D

Suppose that both insurance companies choose the same quality level  $\hat{q}$ . Then in the second stage they must offer an actuarially fair rate. Thus, their expected profits are

$$E(\Pi^A(\hat{q})) = E(\Pi^B(\hat{q})) = -\frac{a\hat{q}^2}{2}. \quad (\text{D1})$$

For any quality level  $\hat{q} > q_{\min}$ , either insurance company has an incentive to lower its own quality because it leads not only to an increase in revenue but also to a decrease in quality cost.



For quality level  $\hat{q} = q_{\min}$ , their expected profits are

$$E(\Pi^A(q_{\min})) = E(\Pi^B(q_{\min})) = -\frac{aq_{\min}^2}{2}. \text{ Now, consider that insurance company B offers}$$

another quality level  $\tilde{q} = q_{\min} + \varepsilon$ , where  $\varepsilon$  is a very small positive number. Then, the expected profit of insurance company B is given by

$$E(\Pi^B(\tilde{q})) = \frac{4}{9}N_c(\tilde{q} - q_{\min}) - \frac{a\tilde{q}^2}{2}. \quad (\text{D2})$$

In order to prove the nonexistence of a symmetric equilibrium, it must be shown that  $E(\Pi^B(\tilde{q}))$  is larger than  $E(\Pi^B(q_{\min}))$ . Subtracting  $E(\Pi^B(q_{\min}))$  from  $E(\Pi^B(\tilde{q}))$ , it can be seen that

$$E(\Pi^B(\tilde{q})) - E(\Pi^B(q_{\min})) = \frac{1}{18}(\tilde{q} - q_{\min})\{8N_c - 9a(\tilde{q} + q_{\min})\}. \quad (\text{D3})$$

Thus, if Equation (D3) is strictly positive, then

$$8N_c - 9a(\tilde{q} + q_{\min}) > 0. \quad (\text{D4})$$

In addition, Equation (D4) becomes

$$\varepsilon < \frac{8N_c}{9a} - 2q_{\min}. \quad (\text{D5})$$

The inequality Equation (D5) is satisfied if  $q_{\min} < \bar{q}$ . Hence, in the case of  $\hat{q} = q_{\min}$ , either insurance company has an incentive to change own quality level.<sup>13</sup>

From the above description, attention can be confined to the asymmetric case, that is  $q^B > q^A$ . It is easy to calculate that

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<sup>13</sup> In contrast, if  $q_{\min} \geq \bar{q}$ , there is no incentive to change the quality level from  $\hat{q} = q_{\min}$ . Thus, it has an uninteresting symmetric equilibrium  $\{q_{\min}, q_{\min}\}$ .

$$\frac{\partial E(\Pi^A)}{\partial q^A} = -\frac{1}{9}N_C - aq^{A*} < 0 \Rightarrow q^{A*} = q_{\min}, \quad (D6)$$

$$\frac{\partial E(\Pi^B)}{\partial q^B} = \frac{4}{9}N_C - aq^{B*} = 0 \Rightarrow q^{B*} = \frac{4N_C}{9a} = \bar{q}. \quad (D7)$$

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