

Applicability of Equal Energy Assumption to the Out-of-Plane Response of Steel Arch Bridges

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ABSTRACT Static pushover analysis, linear and non-linear dynamic response analyses were carried out for six steel arch bridge models having different Arch Rise/Span Length ratios or arch rib distances. Based on the results of these analyses, the applicability of equal energy assumption in out-of-plane direction was examined. Although safety side estimation was achieved by the assumption, the results were too conservative in many cases. For the applicability of the assumption some tendencies were found and correction functions were established to improve the accuracy based on these tendencies.

Keywords: *Seismic Design, Equal Energy Assumption, Steel Arch Bridges, Pushover Analysis, Dynamic Response Analysis.*

1. INTRODUCTION

Japanese seismic design code for highway bridges¹⁾ specifies the Ductility Design Method, which is based on static analysis considering the material and geometrical non-linearity, as the design method against severe earthquakes such as the Great Kanto Earthquake and the Hyogo-Ken Nanbu Earthquake. The method employs equal energy assumption²⁾ for the maximum response estimation. However, the application of this method is limited because the applicability of equal energy assumption is not clear to some types of structures such as steel portal frame bridge piers and deck type steel arch bridges. For these structures time taking and costly dynamic response analysis is required in the seismic design.

There are some papers regarding the applicability of the equal energy assumption to steel bridges. Usami et al.³⁾ examined the applicability of equal energy and equal displacement assumptions based on the results of

pseudo-dynamic tests of cantilever columns of steel bridge piers. In this study, fairly good estimation of non-linear response is achieved by using the equal energy assumption, while the response estimated by the equal displacement assumption is much smaller than the test results. Nakajima et al.⁴⁾ investigated the applicability of equal energy assumption to the seismic design of steel portal frames. The paper states that the assumption can be used as a safety side estimation of the maximum non-linear response, but the estimated maximum displacement can be much larger than the one obtained by elasto-plastic dynamic response analysis. Nakamura et al.⁵⁾ suggested some correction functions to improve the estimation accuracy for steel portal frames. Additionally, a static analysis method to predict the maximum non-linear response of steel portal frame bridge piers was presented by the authors.

In this paper, the applicability of the equal energy

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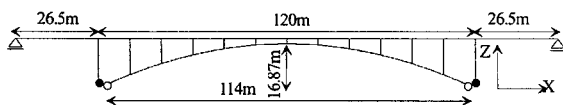
assumption to the out-of-plane inelastic response prediction of deck type steel arch bridges is numerically evaluated for 6 models generated by setting the Arch Rise/Span Length ratio and the distance between the arch ribs as the main structural parameters. Applicability of the assumption is discussed and correction functions to improve the accuracy of the assumption are suggested.

2. OUTLINE OF ANALYSIS

2.1 Analyzed Models and Input Ground Motions

Six steel arch bridge models were studied by MARC⁶⁾ non-linear finite element analysis software. Model 1, shown in Figure 1 is used as the template model for the generation of Model 2, 3, 4 only by changing the arch rise, and Model 5, 6 only by changing the distance between the two arch ribs. Model 1, 2, 3 and 4 constitutes the pattern demonstrating the effect of Arch Rise/Span Length ratio, whereas Model 1, 5 and 6 demonstrates the effect of the distance between the arch ribs on the applicability of equal energy assumption. The models were generated by using JSP-15W⁷⁾ preliminary design software for steel arch bridges. Structural parameters of all models are shown in Table.1.

The ground motions used in dynamic response



	D_x	D_y	D_z	Θ_x	Θ_y	Θ_z
\triangle	Free	Fixed	Fixed	Free	Free	Free
\circ	Fixed	Fixed	Fixed	Free	Free	Free
\bullet	Fixed	Fixed	Fixed	Fixed	Free	Fixed

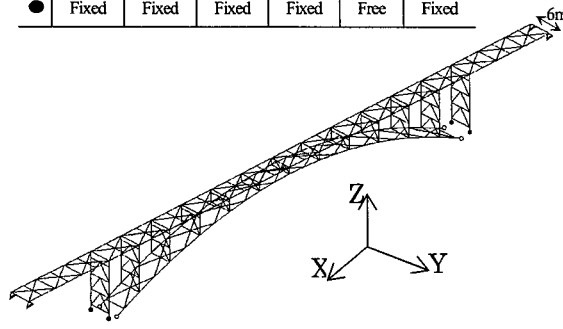


Fig. 1: Model 1

analysis are spectral fitted to the response spectra specified in JRA Code¹⁾. Basically three Level-2, Type-2 ground motions for ground condition 1 are used for the dynamic response analysis in out-of-plane direction whose names, duration time and maximum accelerations are summarized in Table 2. Additionally these ground motions are amplified by 1.5, 2 and 5 respectively (Also by 1.2 and 1.7 for JMA Kobe OBS. N-S ground motion) to obtain the sufficiently inelastic response.

Table 1: Analyzed models

Model No.	Span Length (m)	Arch Rise (m)	Arch Rise / Span Length	Width (m)
Model 1	114	16.87	0.15	6.0
Model 2	114	22.80	0.20	6.0
Model 3	114	34.20	0.30	6.0
Model 4	114	45.60	0.40	6.0
Model 5	114	16.87	0.15	9.5
Model 6	114	16.87	0.15	13

Table 2: Input ground motions

Name	Duration Time (sec)	Max. Acc. (gal)
1995 JMA Kobe OBS N-S (Le2.t211)	30	812
1995 JMA Kobe OBS E-W (Le2.t212)	30	766
1995 HEPC Inagawa N-S (Le2.t213)	30	780

2.2 Analysis Considerations

Fiber model is employed in order to consider the material non-linearity. Lumped mass approach is used for all models. The stiffness of concrete slab on the stiffening girders is not considered in the analysis, but its mass is taken into account. The stress-strain

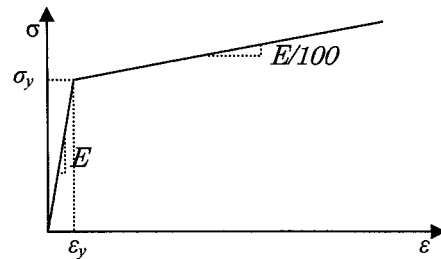


Fig.2: Stress-strain relationship

relationship of the material is considered as bi-linear where the slope of plastic portion was taken as 0.01 of elastic portion, as seen in Figure.2. Kinematic hardening rule is employed.

Principle free vibration mode shapes and frequencies, which are two symmetric and one asymmetric side sway modes are shown in Figure 3. The first symmetric free vibration mode was found to have the largest contribution to the structural response. Damping effect is considered as Rayleigh damping of Equation (1).

$$C = \alpha M + \beta K \quad (1)$$

where, **C**: Rayleigh damping matrix, **M**: Mass matrix, **K**: Stiffness matrix. The mass matrix multiplier α and stiffness matrix multiplier β is obtained by equation (2).

$$\alpha = 4\pi \cdot f_1 \cdot f_2 \cdot \frac{f_1 \cdot h_2 - f_2 \cdot h_1}{f_1^2 - f_2^2} \quad (2)$$

$$\beta = \frac{f_1 \cdot h_1 - f_2 \cdot h_2}{\pi \cdot (f_1^2 - f_2^2)}$$

Here, f_1 and f_2 are the first and second symmetric principle side sway mode frequencies shown in Fig. 3. h_1 and h_2 are the modal damping ratios of these modes which are both assumed as 0.03.

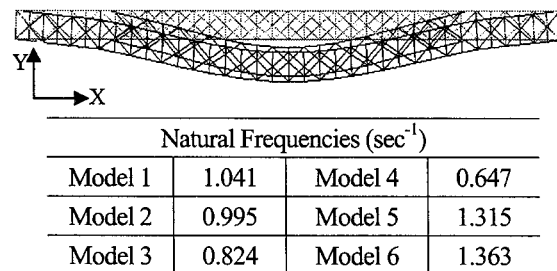
Newmark's β method ($\beta=1/4$) is employed to solve the equation of motion in the dynamic response analysis.

3. APPLICABILITY OF EQUAL ENERGY ASSUMPTION

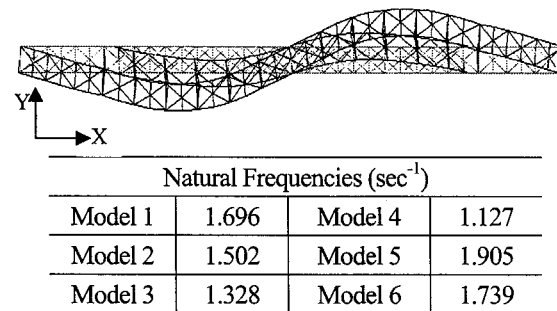
3.1 Examination Procedure

The applicability of equal energy assumption to steel arch bridges in out-of- plane direction is examined by comparing the estimated maximum inelastic response with that of non-linear dynamic response analysis result. The examination procedure is described below.

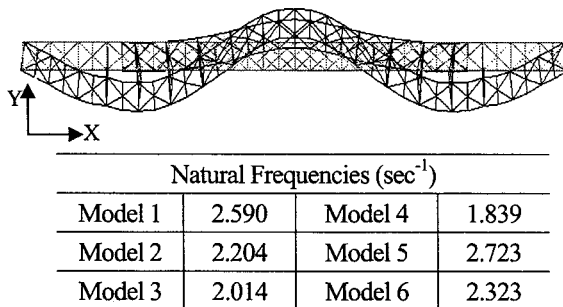
- 1) Free vibration analysis is carried out to get the natural frequencies and mode shapes.
- 2) Elasto-plastic finite displacement pushover analysis of each model is performed in order to obtain the force-displacement relation curve. A force pattern which is directly proportional to the first symmetric side sway mode shape and the lumped masses at each point is applied to the structure.
- 3) Maximum elastic response displacement and the corresponding force are obtained by elastic dynamic response analysis of the model and the maximum strain energy stored in the structure is calculated by using these two values.
- 4) Maximum inelastic response displacement δ_{SP} is estimated by applying the equal energy assumption to the force-displacement curve in 2) and the maximum strain energy in 3) (See Figure 4).
- 5) Inelastic finite displacement dynamic response analysis is conducted to get the maximum inelastic response displacement δ_{DP} .



(a) First symmetric side sway mode



(b) Asymmetric side sway mode



(c) Second symmetric side sway mode

Fig.3: Principle mode shapes and frequencies

6) The estimated maximum response displacement (δ_{SP}) and the calculated one (δ_{DP}) are compared in order to evaluate the accuracy of the assumption. Above procedure is carried out for the deck center node where the maximum response displacements are observed in all cases.

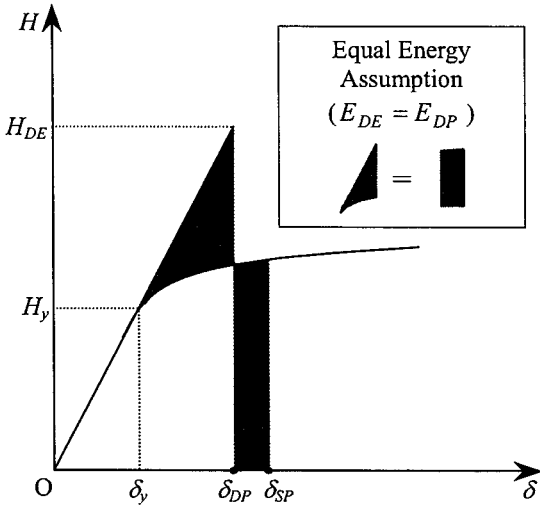


Fig.4: Equal energy assumption

3.2 Relationship between accuracy of the estimation and some parameters.

The natural frequency and the structural parameters such as Arch Rise/Span Length ratio and the distance between the two arch ribs can be considered to have an influence on the applicability of the equal energy assumption. The relationship between these parameters and δ_{SP}/δ_{DP} , which is the basic factor expressing the accuracy of the estimation, is examined.

Figure 5 illustrates the relationship between δ_{SP}/δ_{DP} and 1st symmetric side sway mode frequency which has the most contribution to the whole structural response. Any correlation between δ_{SP}/δ_{DP} and natural frequencies could not be found, suggesting that the natural frequency of the structure has no apparent effect on the accuracy of the estimation. But it can be seen in the figure that all values of δ_{SP}/δ_{DP} are greater than 1.0. This means that the equal energy assumption results in safe side estimation. But in many cases the estimated results are much larger than the responses calculated by inelastic dynamic response analysis causing the accuracy of the estimation to be quite low.

The relationship between δ_{SP}/δ_{DP} and ductility

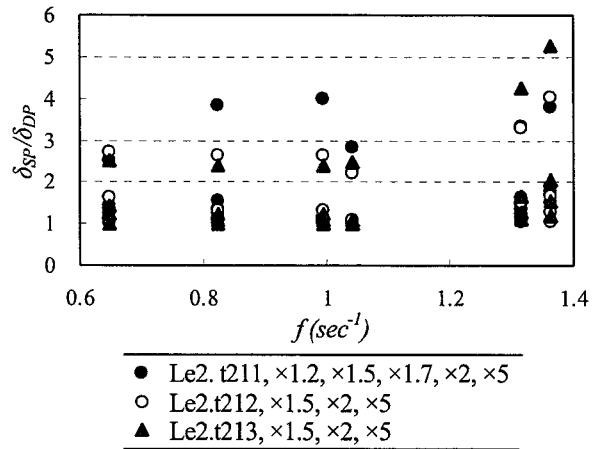
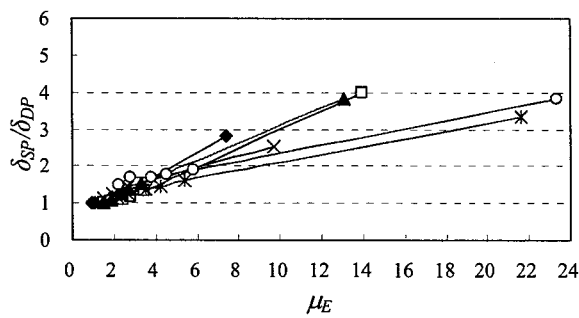


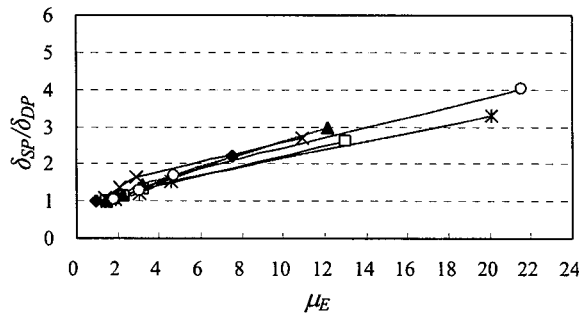
Fig.5: δ_{SP}/δ_{DP} – natural frequency relationship

factor $\mu_E (= \delta_{SP}/\delta_y, \delta_y$: yield displacement) is illustrated in Figure 6 for different ground motion groups. The results residing on the right side represent the more intensive ground motions for each ground motion group. Here the ductility factors may seem to be too large to be practical for any design procedure. But it should be noted that the μ_E is not the real ductility ratio $\mu_R (= \delta_{DP}/\delta_y)$, and containing the error of the estimation, which becomes more than 300% in some cases.

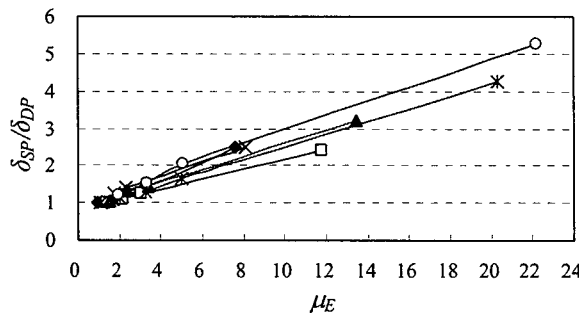
$\delta_{SP}/\delta_{DP} - \mu_E$ relationship in Figure 6 points out that the accuracy of the estimation decreases by the increase in ductility factor. This trend is almost the same for different models and ground motions although there are some irregularities. These irregularities are caused mainly by the results of Model 5 and Model 6 for the ground motions amplified by 5. This divergence is especially more apparent for Le2.t211 ground motion. Merely, these results could be excluded as the real ductility ratios (δ_{DP}/δ_y) for these ground motions ranging from 5 to 6 are too large for the practical seismic design. At the same time, the results for model 6 for Le2.t211 ground motion group also seem to diverge from the general tendency. Le2.t211 is the most severe ground motion among the three ground motions as shown in Table 2, and Model 6 is the model that has the largest distance between its arch ribs having the total deck width of 13 meters carrying four-lane traffic. Model 6 was generated from the template model (Model 1) by only changing the cross-sections of the arch ribs, columns and the stiffening girder keeping the cross sections of lateral bracings unchanged. This



(a) Le2.t211, $\times 1.2, \times 1.5, \times 1.7, \times 2, \times 5$



(b) Le2.t212, $\times 1.5, \times 2, \times 5$



(c) Le2.t213, $\times 1.5, \times 2, \times 5$

◆ Model 1	× Model 4
□ Model 2	✱ Model 5
▲ Model 3	○ Model 6

Fig.6: $\delta_{SP}/\delta_{DP} - \mu_E$ relationship

caused Model 6 to have relatively slender members that goes plastic at the early stage of the analysis especially for Le2.t211 ground motion. This is thought to be the reason why the estimation accuracy for this ground motion is comparatively low even for the moderate ductility factors. On the other hand the results for other ground motion groups do not have such a sharp divergence from the general tendency. The JRA code¹⁾ recommends using at least three ground motions per analysis, and taking an average of them to evaluate the response. From this point of view the

average of response displacements for three ground motion groups was calculated and the $\delta_{SP}/\delta_{DP} - \mu_E$ relationship for the average response displacements are shown in Figure 7. The tendency for different models is almost the same since the error coming from Le2.t211 diminishes to a certain level with the contribution of other ground motions.

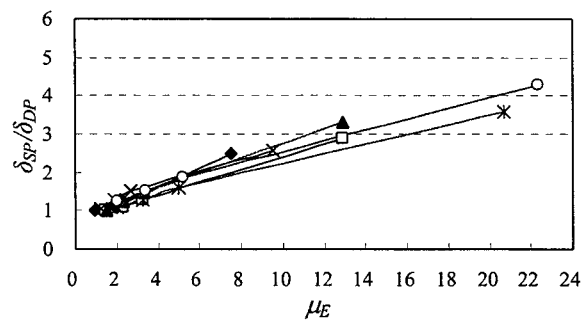
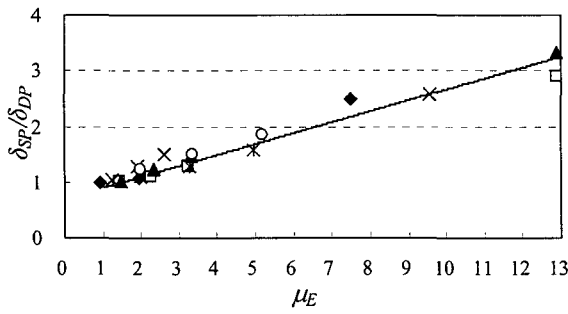


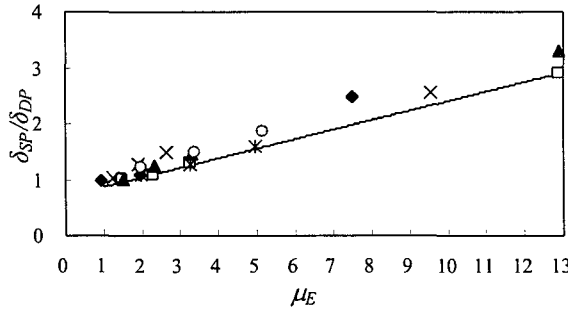
Fig.7: $\delta_{SP}/\delta_{DP} - \mu_E$ relationship for the average response displacements

3.3 Approximation of $\delta_{SP}/\delta_{DP} - \mu_E$ relationship

As illustrated in Figure 6 and Figure 7 δ_{SP}/δ_{DP} values are gathered almost in the same positions, having the same decreasing tendency in estimation accuracy with the increase in ductility factor μ_E regardless of ground motions and model types as it is stated before. This suggests that the estimation accuracy is not affected by the ground motion type for the considered ground condition (ground condition 1 in this study) and the structural parameters which are the Arch Rise/Span Length ratio and the distance between the arch ribs. With this finding it could be possible to approximate the $\delta_{SP}/\delta_{DP} - \mu_E$ relationship with a single function that represents the general tendency which is valid for different ground motions and parameters. This approximation was carried out by considering only the average response displacement results of the three different ground motion groups as recommended by JRA code¹⁾. Average and lower bound values of δ_{SP}/δ_{DP} were approximated by lines as shown in Figure 8. The average approximation is the optimum line between δ_{SP}/δ_{DP} values as shown in equation (3), whereas the lower bound approximation is the bottom boundary line of $\delta_{SP}/\delta_{DP} - \mu_E$ relationship as shown in equation (4).



(a) Average approximation



(b) Lower bound approximation

◆ Model 1	× Model 4
□ Model 2	✖ Model 5
▲ Model 3	○ Model 6

Fig.8: Approximation by line

Average Approximation

$$\frac{\delta_{SP}}{\delta_{DP}} = 0.1958\mu_E + 0.7063, \quad (3)$$

Lower Bound Approximation

$$\frac{\delta_{SP}}{\delta_{DP}} = 0.1700\mu_E + 0.7050, \quad (4)$$

3.4 Correction functions for equal energy assumption

Even though the equal energy assumption resulted in safe side estimation, the estimation accuracy is quite low in many cases as it is illustrated before. However, since the $\delta_{SP}/\delta_{DP} - \mu_E$ relationship can be approximated by a single line which is valid for all ground motions and models considered in this study, it could be possible to improve the accuracy of the estimation by establishing some correction functions based on these approximations. By using this principle, correction function $f(\mu_E)$ is proposed for both average estimation and lower bound estimation. Lower bound

estimation is the safe side estimation where the predicted maximum inelastic response is always equal to or greater than the actual inelastic response (δ_{DP}). These functions are presented in equation (5).

Average Estimation

$$f(\mu_E) = 1/(0.1958\mu_E + 0.7063), \quad (0 < f(\mu_E) \leq 1)$$

Lower Bound Estimation

$$f(\mu_E) = 1/(0.1700\mu_E + 0.7050), \quad (0 < f(\mu_E) \leq 1) \quad (5)$$

Corrected ductility factor μ_D is obtained by multiplying the above correction functions $f(\mu_E)$ with the ductility factor μ_E . No correction is necessary if $f(\mu_E)$ becomes more than 1.0. Corrected value of estimated maximum inelastic response δ_{SP}' is obtained by equation (6), which is simply multiplying the corrected ductility factor μ_D with the yield displacement.

$$\delta_{SP}' = \mu_D \times f(\mu_E) \times \delta_y \quad (6)$$

The corrected values of the estimated ductility factor calculated from the average response displacements for three ground motions are plotted in Figure 9 with the values without correction, versus the real ductility factor (μ_R). It can be seen that the accuracy of the estimation is significantly improved.

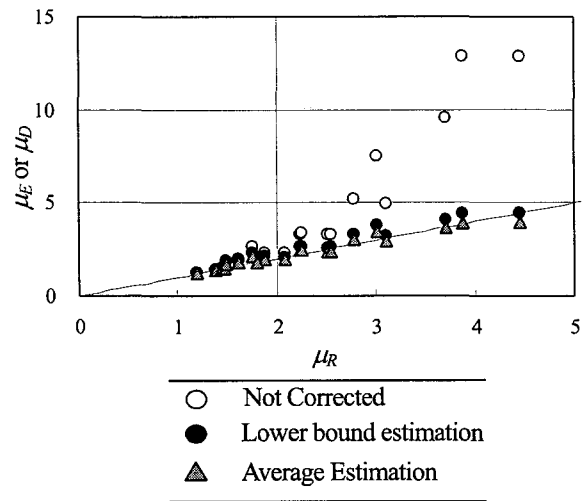


Fig.9: Correction results for the average response displacements

The established correction functions are also applied to the results of individual ground motion groups as shown in Figure 10. Although the correction

functions are generated only by considering the average response values as stated in design specifications it can be seen that the estimation accuracy is also improved for each of the ground motion groups. The lower bound estimation is not plotted since it is meaningful only for design procedure in which the average of three ground motion response displacements should be taken.

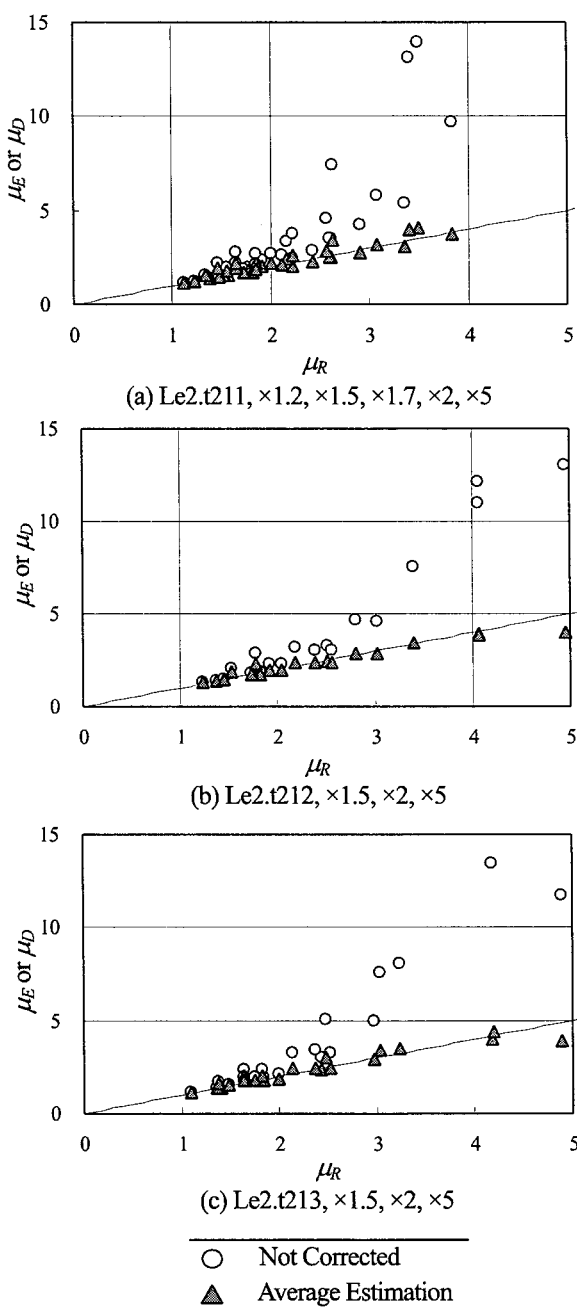


Fig.10: Correction results for individual ground motion groups

3.5 Validity of the correction functions

Figure 11 represents the relationship between the calculated (δ_{DP}) and the estimated (δ_{SP}') maximum responses. Lower bound estimation is plotted only for the average response displacements. Average estimation is plotted for both the average response displacements and individual ground motion results. All of the lower bound estimation results are conservative side, and its estimation error is less than 20% except a few cases. Fairly good results are obtained in the average estimation for average response displacements. Their error mostly ranges from -10% to 10%. For the individual ground motion, the average estimation with the error ranging from -20% to 20% is obtained with the exception of few cases. Therefore, it could be concluded that the proposed correction functions are valid for the maximum inelastic response estimation of steel arch bridges in out-of-plane directions.

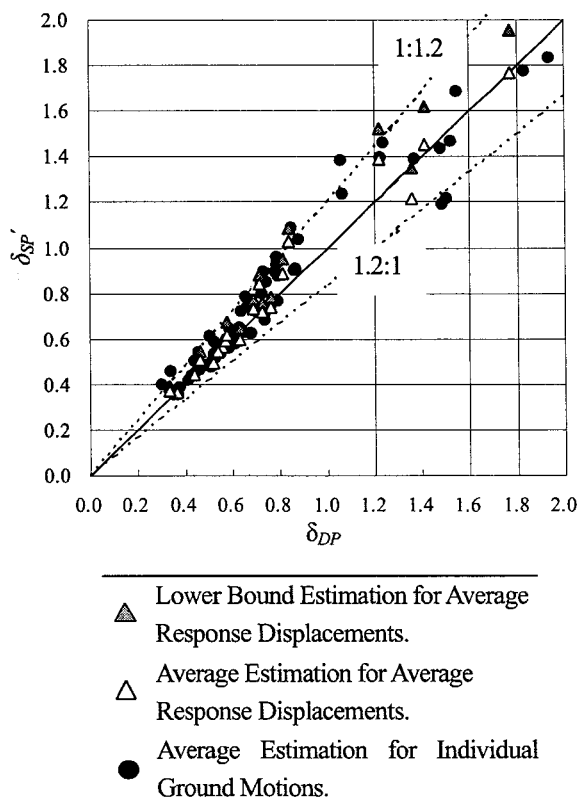


Fig.11: Estimation results by proposed correction functions

4. CONCLUDING REMARKS

Static pushover analysis, linear and non-linear dynamic response analyses of 6 steel arch bridges are carried out. The applicability of the equal energy assumption for the structure is examined based on the results of these analyses, and correction functions are proposed to improve the estimation accuracy of the maximum response displacement. Main findings in this study can be summarized as follows.

- 1) The predicted maximum inelastic response displacement based on the equal energy assumption is conservative for the structure studied in this paper. But too conservative results may be obtained in many cases.
- 2) It is found that the structural parameters considered in this study which are the Arch Rise/Span Length ratio and the distance between the arch ribs do not have any significant influence on the applicability of equal energy assumption.
- 3) The prediction accuracy can be improved by using proposed correction functions.

In this study maximum elastic response to predict the maximum inelastic response by equal energy assumption is obtained by dynamic response analysis. If the elastic maximum response could be obtained by using response spectra, it could be possible to achieve the estimation of maximum inelastic response displacement without dynamic response analysis. On the basis of this concept, development of a static-analysis-based prediction method of maximum inelastic seismic response of steel arch bridges will be tried in the future work. Also the scope of the study will be broadened to the in-plane response estimation of the

structure by considering more ground conditions.

5. REFERENCES

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