

# Pulsating Torque Reduction of Surface-Mounted Permanent Magnet Synchronous Motor with Unequal Slot Angle

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**Abstract**—In the paper, we introduce the concept of uneven slot arrangement for the reduction of pulsating torque in a surface-mounted permanent magnet synchronous motor. The effect of the arrangement of slots and magnets on torque pulsation is studied numerically. The inequality in the slot arrangement reduces the 12th harmonic pulsating torque and increases the 6th harmonic one. In addition, we shape torque waveforms, based on the obtained results.

**Index Terms**—magnet arc width, pulsating torque reduction, uneven slot arrangement, winding method

## I. INTRODUCTION

The arrangement of slot winding and permanent magnets induces harmonics in the spatial distribution of magnetic flux density in air gap. The higher harmonics cause pulsating torque resulting in vibration and acoustic noise in operation for motors [1], [2]. To reduce higher harmonic components in the distribution of magnetomotive force due to the armature current inducing that of magnetic flux density, various kinds of winding methods have been proposed and carried out in practical applications [3], [4]. Most basic winding methods include the distributed winding in which coils for each phase are divided into some slots. The distributed winding produces smaller harmonics than the concentrated winding. The reduction of harmonics corresponds to the approximation of the superposed distribution to a sinusoidal pattern. The distribution is restricted in shape by the arrangement of isolated slot or the discontinuous distribution of winding. An effective arrangement of slots is required with winding methods for more freely shaping the distribution of magnetomotive force.

Slots are pitched at an equal distance in an armature in general design of rotating machines. The equality of slot pitch allows little flexibility to shape the distribution of magnetomotive force. In other words, an arbitrary arrangement of slots makes it possible to design the distribution more flexibly. In the paper, we introduce the arrangement to locate slots unequally in an armature stator for the reduction of pulsating torque in a surface-mounted permanent magnet synchronous motor with the distributed winding. Also, the unequal slot tooth width can offer useful performance benefits [5].

The reduction of pulsating torque is achieved by the disposition of permanent magnets or excitation current [6]. For three phase surface-mounted permanent magnet synchronous motors, the adjustment of magnet arc

width minimizes the 6th harmonic pulsating torque [6]. Since the result is obtained with even arrangement of slots, the applicability to uneven slot arrangement is not clarified. This paper investigates the adjustment of magnet arc width to reduce pulsating torque with uneven slot arrangement and reveals the combination of the slot pitch and the magnet arc width for minimizing pulsating torque.

## II. UNEVEN SLOT ARRANGEMENT

Numerous ways emerge for unequally locating armature slots without regulations. The reduction of pulsating torque and the motor operation impose regulations on the arrangement of slots. We adapt a slot arrangement which is symmetry with respect to pole, phase, and rotational direction for pulsating torque reduction and motor operation in both rotational direction. The polar symmetry corresponds to a rotational symmetry by an electrical angle of  $180^\circ$  and the phase symmetry by an electrical angle of  $360^\circ/m$ , where  $m$  denotes the number of phases. The symmetric arrangement with respect to rotational direction is carried out by a symmetry with respect to the middle position of slots per pole per phase. For the slots arranged in this way, coils are wound in single layer.

Figure 1 shows the cross section of a surface-mounted permanent magnet synchronous motor with 2-pole, 3-phase, and 12 slots based on the above regulations. The slot opening ratio is fixed at  $1/2$ . Because of 2 slots per pole per phase, the slot arrangement is determined uniquely by the arc width between the slots. We define the slot angle  $\alpha_s$  as the electrical angle for half of the arc width in Fig. 1 which indicates  $\alpha_s$  by the angle from the middle position of slots per pole per phase to the center line of each slot. For a 3-phase motor with the slot opening ratio, the slot angle is confined to  $7.5^\circ < \alpha_s < 22.5^\circ$ . The slot angle  $\alpha_s = 7.5^\circ$  or  $22.5^\circ$  represents that the neighboring two slots merge into one slot. In order to examine the effect of the arc width of permanent magnets on the pulsating torque, we define the magnet angle  $\alpha_m$  as the electrical angle for half of the magnet arc width in Fig. 1. The domain of the magnet angle is  $0^\circ < \alpha_m < 90^\circ$ . According to the definitions, Fig. 1 shows a motor with  $\alpha_s = 17^\circ$  and  $\alpha_m = 77.5^\circ$ .

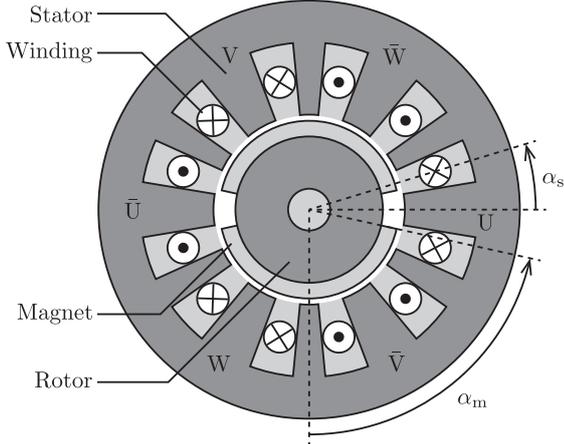


Fig. 1. Cross section of a surface-mounted permanent magnet synchronous motor with uneven slot arrangement.

### III. FINITE ELEMENT ANALYSIS

This section is devoted to a numerical study for the effect of the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$  on torque pulsation. In addition, we show the waveform of torque for some combinations of  $\alpha_s$  and  $\alpha_m$ , based on the obtained result.

#### A. Analyzed Model

Figure 2 shows a 2-dimensional numerical model for a surface-mounted permanent magnet synchronous motor with 2-pole, 3-phase, and 12 slots. We adopt a half model because of the symmetry with variable parameters, namely the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$ . The model in Fig. 2 corresponds to the motor in Fig. 1 with respect to the variable parameters. The detailed specifications of the numerical model are described in Table I.

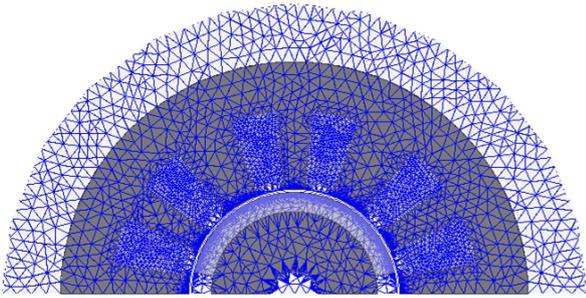


Fig. 2. 2-dimensional numerical model.

#### B. Effect of Arrangement of Slots and Magnets

A numerical study is carried out for the dependence of the pulsating torque on the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$ . The rotor rotates at  $-90^\circ$  with reference to the rotating magnetic field due to the armature current. The torque generated in the motor can be described by

$$T(t) = T_0 + \sum_{k=1}^{\infty} T_k \cos(k\omega t + \theta_k), \quad (1)$$

TABLE I  
SPECIFICATIONS OF THE NUMERICAL MODEL.

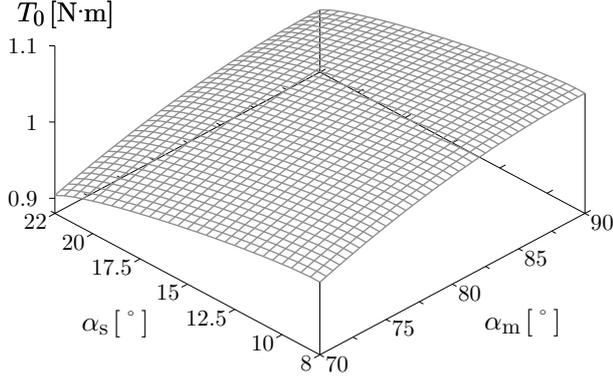
Rotor outer diameter	35 mm
Rotor inner diameter	8 mm
Stator outer diameter	80 mm
Air gap	0.6 mm
Core Length	50 mm
Slot opening ratio	1/2
Width of Stator core back	8.3 mm
Shape of permanent magnet	Ring
Thickness of permanent magnet	3.3 mm
Magnetization pattern	Radial
Material of permanent magnets	NdFe30
Material of cores	Iron
Armature current amplitude	10 A

where  $\omega$  denotes the angular frequency of the armature current,  $T_0$  the constant component or the averaged torque,  $T_k$  and  $\theta_k$  represent the amplitude and the initial phase of the  $k$ -th harmonic component, respectively. In theory, the torque  $T$  is composed of the constant and the  $6i$ -th harmonic components for a positive integer  $i$  because the slot arrangement in the stator has a rotational symmetry by  $60^\circ$  or  $1/6$  rotation. We here focus on the components  $T_0$ ,  $T_6$ , and  $T_{12}$  of the torque.

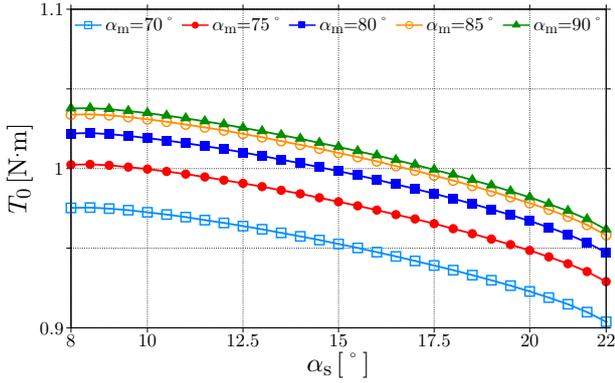
Figure 3(a) shows the dependence of the averaged torque  $T_0$  on the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$ . The torque  $T_0$  is a monotonically decreasing function of the slot angle  $\alpha_s$  in Fig. 3(b). The relationship is confirmed by the distribution factor. As the slot angle  $\alpha_s$  decreases, the slot arrangement converges to the concentrated winding generating the largest factor or torque. Thus the decrement of the slot angle  $\alpha_s$  corresponds to the increment of the averaged torque  $T_0$ . Fig. 3(c) shows that the averaged torque  $T_6$  increases as the magnet angle  $\alpha_m$ . The result is consistent with the increase of the magnetic energy with the magnet angle  $\alpha_m$ .

The 6th harmonic component or the fundamental one  $T_6$  of the torque arises for the combination of the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$  as shown in Fig. 4. The points in Fig. 4(a) indicate the slot angle  $\alpha_s$  with local minimum torque at each of the magnet angles  $\alpha_m$ . The pulsating torque  $T_6$  is minimized at  $\alpha_s \approx 15^\circ$  for  $\alpha_m \approx 75^\circ$ . The equality in the slot arrangement suppresses the 6th harmonic torque. In particular, the combination of  $\alpha_s \approx 15^\circ$  and  $\alpha_m \approx 74.5^\circ$  generates little 6th harmonic torque in Fig. 4(c). The minimizing design is consistent with the research [6].

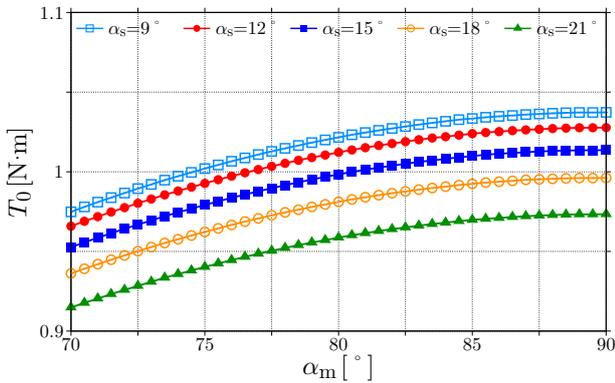
Figure 5 shows the 12th harmonic component  $T_{12}$  of the torque for the variable design parameters  $\alpha_s$  and  $\alpha_m$ . The points in Fig. 5(a) correspond to the magnet angle  $\alpha_m$  with local minimum torque at each of the slot angles  $\alpha_s$ . The even slot arrangement maximizes the pulsating torque  $T_{12}$ . At  $\alpha_s \approx 8^\circ$  and  $22^\circ$ , little torque  $T_{12}$  arises. The structure with  $\alpha_s = 7.5^\circ$  or  $22.5^\circ$  represents the stator with 6 slots because the neighboring two slots merge into one slot. Based on the winding factor indicating non-existence of even harmonics, little torque  $T_{12}$  appears with a 6 slots stator. Therefore the pulsating torque  $T_{12}$  is governed by the distribution of the permeance due to the number of slots. Fig. 5(c) reveals the



(a) Torque for the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$ .



(b) Torque for the slot angle  $\alpha_s$  at the fixed magnet angles  $\alpha_m$ .



(c) Torque for the magnet angle  $\alpha_m$  at the fixed slot angles  $\alpha_s$ .

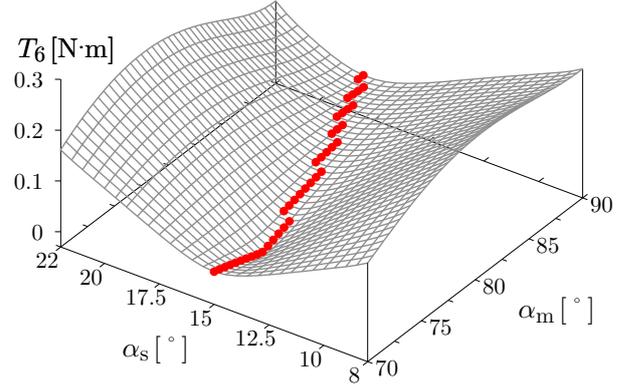
Fig. 3. Averaged torque  $T_0$ .

minimization of the 12th harmonic torque  $T_{12}$  is achieved at  $\alpha_m \approx 77.5^\circ$ .

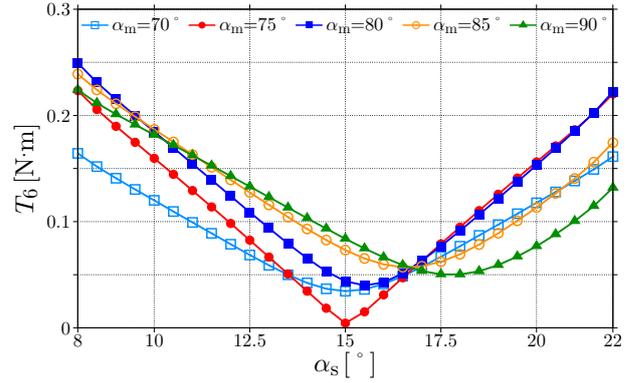
Figures 4(b) and 5(b) reveal the different effects of the slot angle  $\alpha_s$  on the pulsating torques  $T_6$  and  $T_{12}$  from each other. The equality in the slot arrangement reduces the 6th harmonic torque  $T_6$  and the inequality the 12th harmonic torque  $T_{12}$ . These results imply that it is difficult to minimize both the components simultaneously with the single layer distributed winding for 2 slots per pole per phase.

### C. Shaped Torque

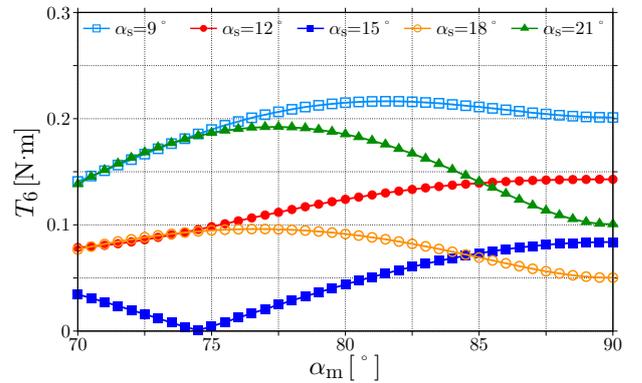
Figure 6 shows the waveforms of shaped torque based on the numerical results with the conventional one at



(a) Torque for the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$ .



(b) Torque for the slot angle  $\alpha_s$  at the fixed magnet angles  $\alpha_m$ .



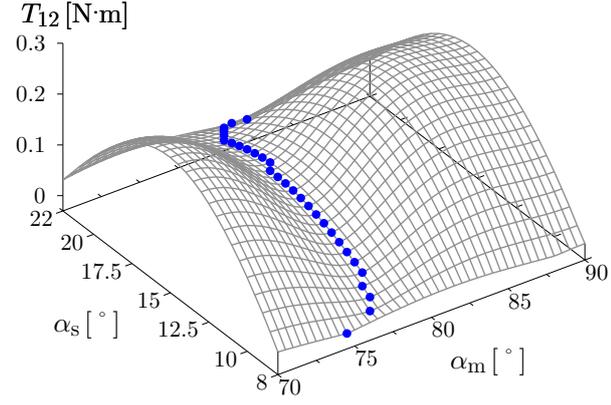
(c) Torque for the magnet angle  $\alpha_m$  at the fixed slot angles  $\alpha_s$ .

Fig. 4. 6th harmonic pulsating torque  $T_6$ .

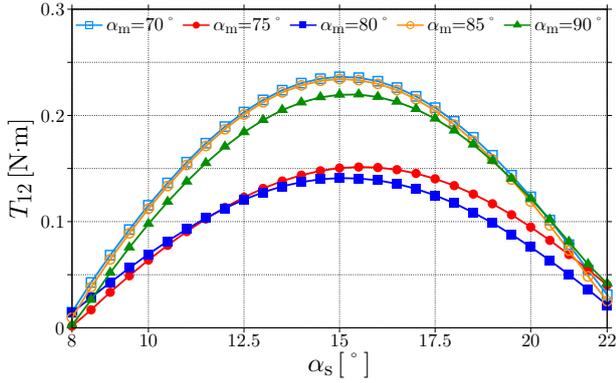
$\alpha_s = 15^\circ$  and  $\alpha_m = 90^\circ$ . For the torque in Fig. 6(a), the minimization of the 6th harmonic component is achieved by adjusting the slot angle and the magnet angle, namely  $\alpha_s = 15^\circ$  and  $\alpha_m = 74.5^\circ$ . The combination of  $\alpha_s = 8^\circ$  and  $\alpha_m = 74.5^\circ$  induces little pulsating torque of the 12th order in Fig. 6(b). Fig. 6(c) shows both the 6th and 12th harmonic pulsating torques are suppressed about 10% of the averaged torque.

## IV. CONCLUSION

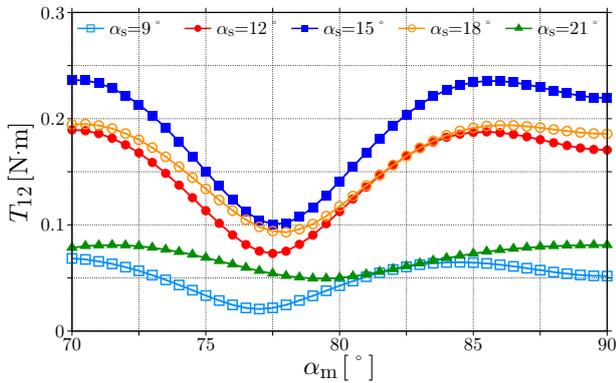
In the paper, we introduce the concept of uneven slot arrangement for the reduction of pulsating torque in a surface-mounted permanent magnet synchronous motor. The effect of the arrangement of slots and magnets on



(a) Torque for the slot angle  $\alpha_s$  and the magnet angle  $\alpha_m$ .



(b) Torque for the slot angle  $\alpha_s$  at the fixed magnet angles  $\alpha_m$ .



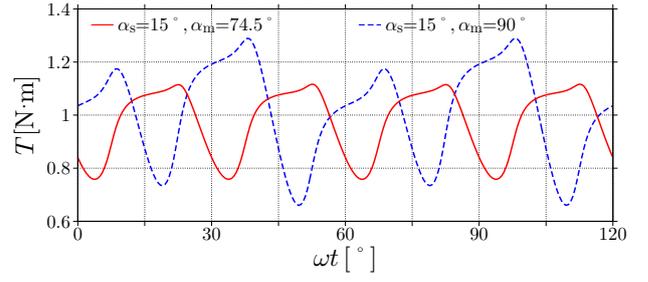
(c) Torque for the magnet angle  $\alpha_m$  at the fixed slot angles  $\alpha_s$ .

Fig. 5. 12th harmonic pulsating torque  $T_{12}$ .

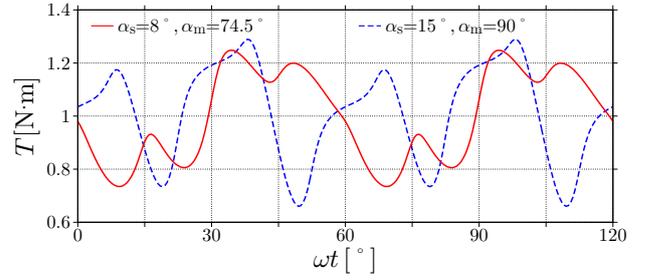
torque pulsation is studied numerically based on the finite element method. The inequality in the slot arrangement reduces the 12th harmonic pulsating torque and increases the 6th harmonic one. The uneven slot arrangement has a potential to adjust torque pulsation. This implies that we can shape torque waveform by adopting uneven slot arrangement with other design techniques.

#### ACKNOWLEDGMENT

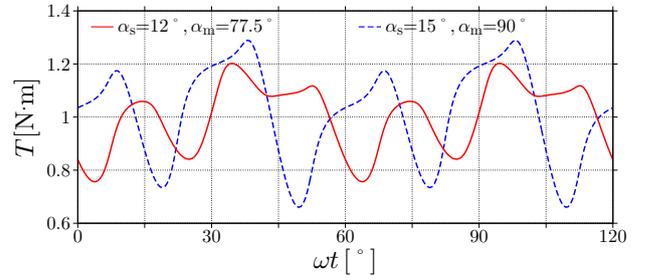
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(a) Torque at  $\alpha_s = 15^\circ$  and  $\alpha_m = 74.5^\circ$ .



(b) Torque at  $\alpha_s = 8^\circ$  and  $\alpha_m = 74.5^\circ$ .



(c) Torque at  $\alpha_s = 12^\circ$  and  $\alpha_m = 77.5^\circ$ .

Fig. 6. Waveforms of shaped torque.

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