Dynamic Stability of Cables Subjected to an Axial Periodic Force

Kazuo TAKAHASHI and Hiroyuki ICHINOSE

Department of Civil Engineering, Faculty of Engineering, Nagasaki University, Nagasaki

Dynamic Stability of a flat sag cable subjected to an axial periodic force is investigated. The equation of motion of the cable is solved by the Galerkin method. Unstable regions are presented first for various sag-to-span ratios and ratios of wave speeds. Amplitudes of unstable motions are obtained using the nonlinear cable theory.

I. INTRODUCTION

Dynamic stability problem of structures subjected to a periodic load has been studied by many researchers.[1,2] Recently, Kovacs[3] pointed out that dynamic instability of a cable employed as a tension member of a cable stayed bridge or a guy tower may occur under the axial periodic load due to bending vibration of the tower or the girder of bridges. He treated dynamic instability of the string which has no sag and discussed the control of this unstable vibration by using a vibration absorber. However, Kovacs neglected the sag of the cable which influences the dynamic properties of the cable. Takahashi and Konishi[4] analyzed unstable out-of-plane vibrations of cables under inplane forcing by the multiple degrees-of-freedom approach.

In this paper, theoretical solutions are reported for the dynamic stability of a flat-sag cable under an axial sinusoidally time-varying load. This problem is solved by using the harmonic balance method described first by Bolotin[1] and lately extended by the author.[5]

After presenting the problem in the eigenvalue form, numerical results are presented for dynamic unstable regions of the cable with various sag-to-span ratios and ratio of wave speeds, and second for amplitudes of unstable motions by using time response analysis based upon nonlinear theory of cables.

II. EQUATION OF MOTION

If a uniform flat-sag suspended cable anchored on the supports at the same level as shown in Fig. 1 is given a longitudinal time-varying load

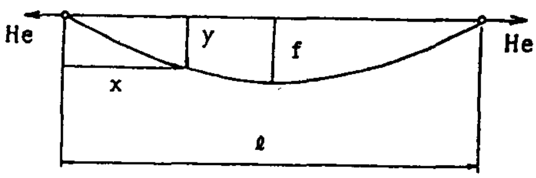


Fig. 1. Geometry of the cable.

$H_t \cos\Omega t$, the equation of motion[6] describing the vertical component is reduced to

$$L(w)=m\frac{\partial^2 w}{\partial t^2}+\left(\frac{8f}{\ell^2}\right)^2\frac{EA}{L_\epsilon}\int w dx-(H_* +H_t \cos\Omega t)\frac{\partial^2 w}{\partial x^2}=0 \tag{1}$$

where, w is the vertical deflection of the cable, x is the co-ordinate in the horizontal direction, t is the time, ℓ is the span length between the supports, m is mass of the cable per unit length, E is Young's modulus, A is the cross-sectional area, $L_\epsilon=\ell(1+8f^2/\ell^2)$, H_* is the horizontal component of the static cable tension due to the own weight of the cable per unit length, H_t is the amplitude of the horizontal component of cable tension due to the support excitation, and Ω is the radian frequency of the applied load.

The second term $(8f/\ell)^2\int w dx$ shows the restoring force due to the additional horizontal component of cable tension. This term is zero in the case of the anti-symmetric vibration because $\int w dx$ is zero. Eq. (1) is reduced to that of the string. Then, dynamic stability of the anti-symmetric vibration of the cable with flat-sag is estimated by using that of the string which has no sag.[2]

III. METHOD OF SOLUTION

The solution of Eq. (1) is assumed to be the form

$$w=\ell\Sigma T_i(t)W_i(x) \tag{2}$$

where $T_i(t)$ is an unknown function of the time and $W_i(x)$ is the space variable satisfying the geometric boundary conditions for the associated linear problem defined as[6]

$$W_i=1-\tan\frac{\pi\omega_i}{2}\sin\pi\omega_i\xi-\cos\pi\omega_i\xi \tag{3}$$

where $\xi=x/\ell$, $\omega_i=n_i\ell/\pi\sqrt{m/H_*}$ is the nondimensional natural frequency of the i-th symmetric in-plane mode which is obtained by the following transendental equation

$$\tan\frac{\pi\omega_i}{2}=\frac{\pi\omega_i}{2}-\frac{1}{\lambda^2}\left(\frac{\pi\omega_i}{2}\right)^3 \tag{4}$$

where $\lambda^2=64\gamma^2k^2/(1+8\gamma^2)$, $\gamma=f/\ell$ is the sag-to-span ratio, $k=\sqrt{EA/H_*}=c_l/c_u$ is the ratio of the wave propagation, $c_u=\sqrt{H_*/m}$ is the speed of transverse wave propagation, $c_l=\sqrt{EA/m}$ is the speed of longitudinal wave propagation and n_i is the natural circular frequency of the cable.

Upon applying a Galerkin method to Eq. (1) by considering orthogonality condition of the vibration mode, the following equation of motion for time variable is obtained as

$$\ddot{T}_i+\omega_i^2\ddot{T}_i+\frac{1}{\pi^2}\overline{H}_t\cos\overline{\omega}\tau\frac{1}{A_{ii}}\Sigma B_{ij}T_j=0 \tag{5}$$

where $\overline{H}_t=H_t/H_*$, $\overline{\omega}=\Omega/n_1{}^o$, $n_1{}^o$ is the first natural circular frequency of the string, and

$$A_{ii}=\int W_i{}^2 d\xi,\ \text{and}\ B_{ij}=\int\frac{dW_i}{d\xi}\frac{dW_j}{d\xi}d\xi$$

Eq. (5) is rewritten by using matrix notations as

$$[I]\{\ddot{T}\}+[E]\{T\}+\overline{H}_t\cos\overline{\omega}\tau[F]\{T\}=\{0\} \tag{6}$$

Here, [I] is the unit matrix, [E] is the diagonal matrix and [F] is the coefficient matrix which defines kinds and widths of the parametric resonances.

Equation (6) is coupled Mathieu equations, for multiple-degree-of-freedom systems, which have been studied by Bolotin,[1] Hsu[7] and Nayfeh and Mook.[2] The solution of Eq. (6) is now sought in the form[1,5]

$$\{T\} = e^{\lambda\tau}\left(\frac{1}{2}\{b_0\} + \Sigma\left(\{a_k\}\sin k\bar{\omega}\tau + \{b_k\}\cos k\bar{\omega}\tau\right)\right) \tag{7}$$

where $\{b_0\}$, $\{a_k\}$ and $\{b_k\}$ are vectors that are independent of the time variable.

Substituting Eq. (7) into Eq. (6) and applying the harmonic balance method yield a set of homogeneous algebraic equations as

$$([M_0] - \lambda[M_1] - \lambda^2[M_2])\{x\} = \{0\} \tag{8}$$

in which $[M_0]$, $[M_1]$ and $[M_2]$ are coefficient matrices of the zeroth (constant), first and second powers of λ respectively, and $\{x\}$ is the column vector consisting of $\{b_0\}$, $\{b_m\}$ and $\{a_m\}$.

The eigen-value λ can be obtained by solving a double sized matrix as an eigenvalue problem.[5] If the eigenvalues of Eq. (7) are distinct, then the necessary and sufficient conditions for stability are that real roots and the real parts of the complex roots should be negative or zero.

The parameters of the cable in the present analysis are the sag-to-span ratio γ and ratio of wave speeds k.

From the information with respect to the coupled Mathieu Eq. (6), the unstable regions are the simple parametric resonance near $\bar{\omega}=2\omega_i/q$ (q=1, 2, ...) and the combination resonance $\bar{\omega}=(\omega_i \pm \omega_j)/q$ in which the positive sign corresponds to the sum type and the negative sign corresponds to the difference type. The principal unstable region is called that for q=1 and the second region that for q=2 and so on. As the signs of e_{ij} and e_{ji} are the same, the combination resonance of the sum type exists only and that of difference type does not exist.

Ⅳ. NUMERICAL RESULTS

(1) Free Vibrations

Free vibrations with symmetric modes of the cable anchored on the supports at the same level are shown in Fig. 2. These results were obtained by Irvine and Caughey,[6] and Yamaguchi and Ito.[9]

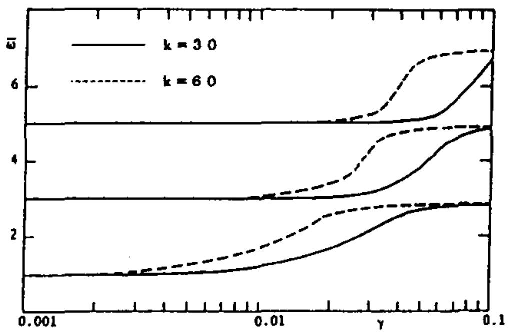


Fig. 2. Nondimensional frequency ω vs sag-to span ratio γ for cables with k=30 and 60.

The effect of the sag-to-span ratio is considerable: that is, a modal crossover occurs from one symmetric vibration to the next for a certain sag-to-span ratio and then the frequency increases. The sag-to-span ratio where modal crossover occurs changes dependently of the order of vibration and ratio of wave speeds. These results give influence upon widths of unstable regions.

(2) Unstable Regions

Figure 3-7 shows unstable regions of the cables with various sag-to-span ratios γ and ratio of the wave speeds k, respectively. In these figures, the ordinate $\bar{H}_t$ denotes the amplitude of the periodic cable tension normalized to the corresponding initial axial tension, which $\bar{\omega}$ is the excitation frequency normalized to the lowest natural frequency. Further, the cross-hatched portions represent the regions of various types of instability. The narrow regions of instability with ω less than 0.2 when $\bar{H}_t$ =0.5 are omitted in the figures.

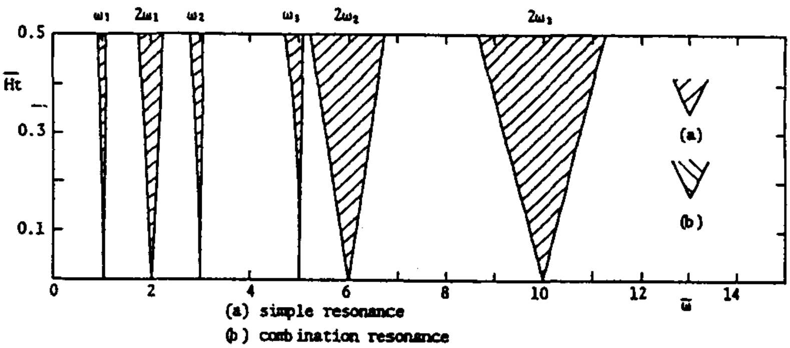


Fig. 3. Unstable regions of a cable with k=30 and γ=0.001 under symmetric forcing.

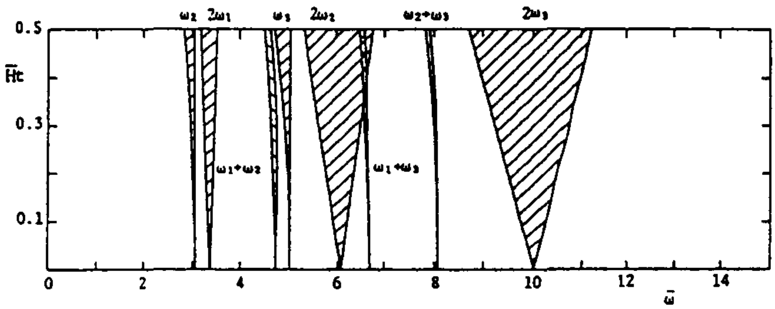


Fig. 4. Unstable regions of a cable with k=30 and γ=0.02 under symmetric forcing.

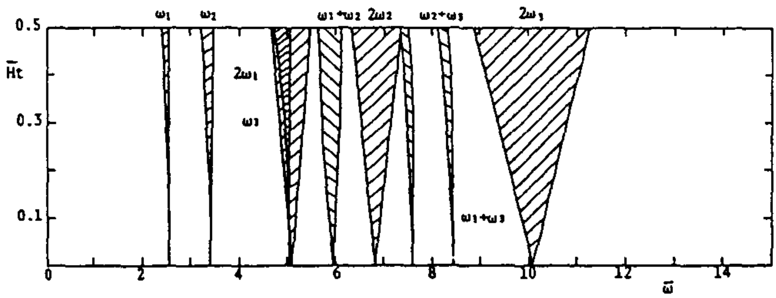


Fig. 5. Unstable regions of a cable with k=30 and γ=0. 04 under symmetric forcing.

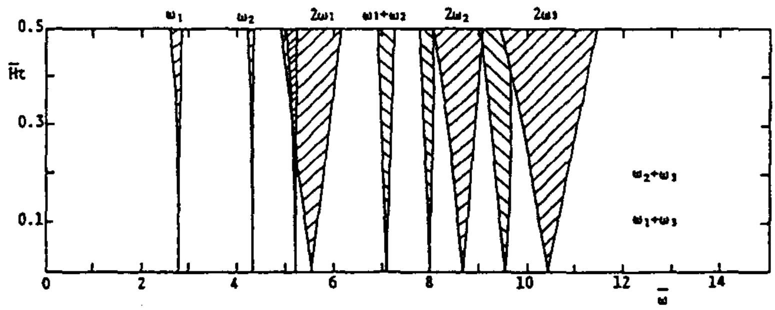


Fig. 6. Unstable regions of a cable with k=30 and γ=0. 06 under symmetric forcing.

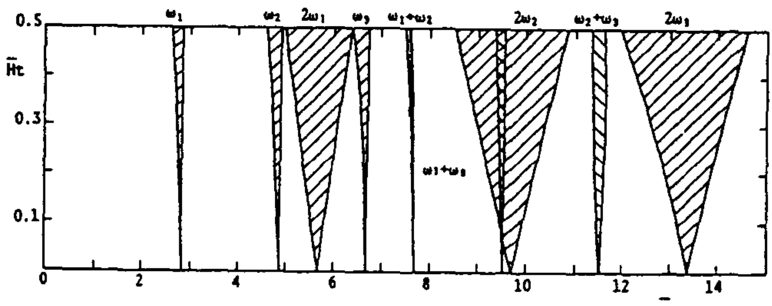


Fig. 7. Unstable regions of a cable with k=30 and γ=0. 01 under symmetric forcing.

Normalized natural frequencies ω_i (i=1, 2, 3) for each case are given in Table 1.

Table 1. Nondimensional frequency ω_i of cables.

k	30			60		
γ\i	1	2	3	1	2	3
0. 001	1. 0024	3. 0001	5. 0000	1. 0094	3. 0004	5. 0001
0. 002	1. 0094	3. 0004	5. 0001	1. 0371	3. 0014	5. 0003
0. 01	1. 2120	3. 0093	5. 0019	1. 6793	3. 0447	5. 0083
0. 02	1. 6781	3. 0445	5. 0083	2. 5566	3. 4096	5. 0462
0. 03	2. 1830	3. 1421	5. 0210	2. 7779	4. 3545	5. 2353
0. 04	2. 5518	3. 4023	5. 0455	2. 8216	4. 7851	6. 0484
0. 05	2. 7133	3. 8542	5. 0967	2. 8374	4. 8627	6. 7290
0. 06	2. 7754	4. 3273	5. 2220	2. 8450	4. 8841	6. 8627
0. 08	2. 8198	4. 7724	5. 9739	2. 8520	4. 9013	6. 9122
0. 1	2. 8358	4. 8545	6. 6810	2. 8550	4. 9077	6. 9251

k: ratio of wave speeds, γ: sag-to-span ratio and i: mode number.

Wide unstable regions of simple parametric resonances in the vicinity of $2\omega_i$ are obtained and the width of unstable regions become broad with increase of order of vibration. The second unstable regions of the simple resonance such as ω_i are also obtained and widths of these unstable regions are narrower than those of the principal unstable regions such as $2\omega_i$. The widths of unstable regions of the combination resonances are narrow in the case of the string. However, The width of the combination resonance becomes of larger width with increase in the sag-to-span ratio. Kovacs[3] discussed the principal unstable region of the first mode of the string i. e., $2\omega_1$ shown in Fig. 3.

Figure 8 and 9 summarizes the relation between the frequency ratio ω when $\bar{H}_t$ =0. 5 and the sag-to-span ratio γ for ratios of wave speeds k=30 and 60. The width of the principal unstable region of the simple resonance changes with the increase of the sag-to-span ratio.

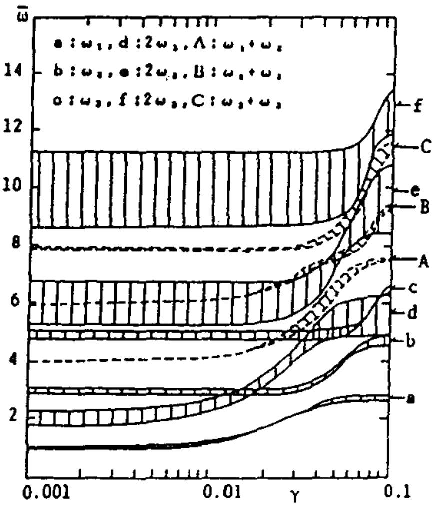


Fig. 8. Variations of unstable regions with the sag-to-span ratio: $\bar{H}_t$ =0. 5 and k=60.

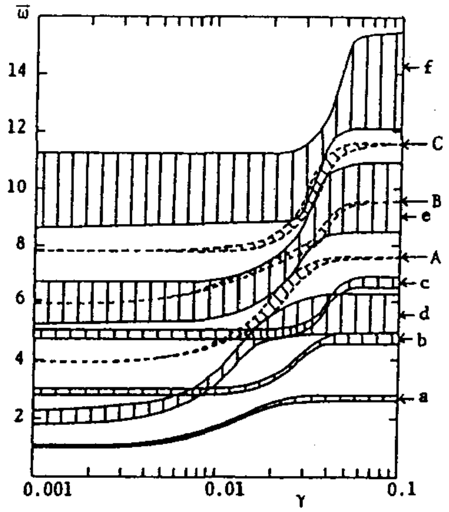


Fig. 9. Variations of unstable regions with the sag-to-span ratio: $\overline{H}_t$ =0.5 and k=30.

Its width at first becomes narrow near the sag-to-span ratio where the modal cross-over is producing, next becomes wider again and finally approaches to the next unstable region ($2\omega_1 \Rightarrow 2\omega_2$, $2\omega_2 \Rightarrow 2\omega_3$) for more greater sag-to-span ratio where modal crossover is finished. Width of the combination resonance becomes wide when natural frequencies of the unstable motions are closed to each other, that is, $\omega_1+\omega_2$ ($\omega_1 \Rightarrow \omega_2$) and $\omega_2+\omega_3$ ($\omega_2 \Rightarrow \omega_3$).

Unstable regions of the anti-symmetric vibration are shown in Fig. 10. This result is efficient for all sag-to-span ratios under the assumption of the flat sag cable. Unstable regions of simple parametric resonances are obtained only because the anti-symmetric vibrations are not affected by the sag-to-span ratio where a parabolic profile is valid.

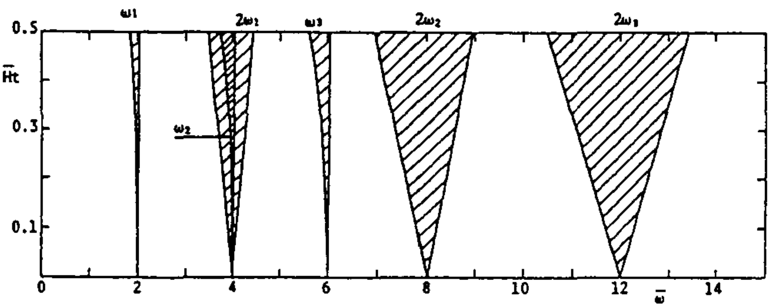


Fig. 10. Unstable regions of a cable with k=30 under anti-symmetric forcing.

V. NONLINEAR TIME RESPONSE

The amplitude of the unstable region becomes infinite under the assumption of the small deflection theory. However, the amplitude is bounded because of the stretching of the axial tension of the cable. From this fact, the amplitude of the unstable motion must be estimated by the nonlinear theory of the cable. The basic equations for large amplitude free vibrations of a cable subjected to an axial periodic load can be written as[4]

$$L(w)=\frac{\partial^2 w}{\partial t^2}+\frac{8f}{\ell^2}\frac{EA}{L_\varepsilon}\left(\frac{8f}{\ell^2}\int w dx+\frac{1}{2}\int\left(\frac{\partial w}{\partial x}\right)^2 dx\right)-(H_0+H_t\cos\Omega t)\frac{\partial^2 w}{\partial x^2}$$
$$\frac{EA}{L_\varepsilon}\left(\frac{8f}{\ell^2}\int w dx+\frac{1}{2}\int\left(\frac{\partial w}{\partial x}\right)^2 dx\right)\frac{\partial^2 w}{\partial x^2}=0 \qquad (9)$$

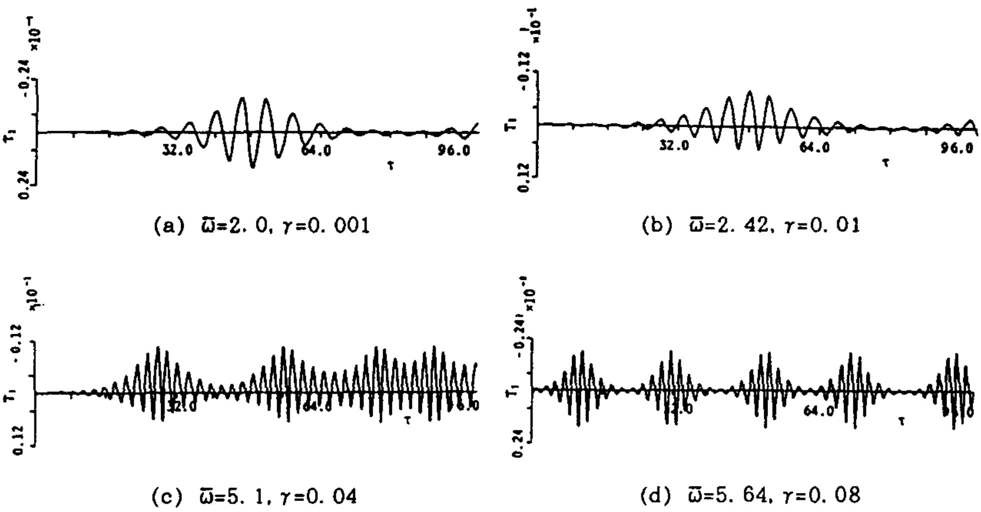
Assuming the same solution as Eq. (2) and applying the Galerkin method, we obtain the following differential equation for the time variable

$$\ddot{T}_i+\omega_i{}^2\ddot{T}_i+\frac{1}{\pi^2}\overline{H}_t\cos\overline{\omega}\tau\frac{1}{A_{ii}}\Sigma B_{ij}T_j+D_i\,\{4\gamma\Sigma\Sigma T_p T_q B_{pq} C_i+8\gamma\Sigma T_q C_q \Sigma T_p C_{pi}$$
$$+\frac{1}{2}\Sigma\Sigma T_q T_r B_{qr}\Sigma T_p B_{pi}\}=0 \qquad (10)$$

where $D_i=\frac{1}{1+8\gamma^2}\frac{k^2}{\pi^2}\frac{1}{A_{ii}}$, $C_j=\int W_j dx$

Time variable T_i is integrated numerically by using the Runge-Kutta-Gill method. The purpose of the present analysis is to determine the amplitude of the unstable motion which occurs under the assumptions of the small deflection theory. Therefore, the initial conditions for time variables are $T_i=T_j=0.0$ and $T_i=T_j=0.001$ to satisfy the small amplitude vibration.

Nonlinear time responses of the cable with various sag-to-span ratios for the simple parametric resonance $2\omega_1$ are shown in Fig. 11. Amplitudes are bounded due to the nonlinear terms effect which is caused by axial force due to stretching of the cable. Beating can be seen in the nonlinear parametric dynamic system. The effect of quadratic nonlinear terms which is caused by the sag of the cable is found in the particular sag-to-span ratio as can be seen in Fig. 11.

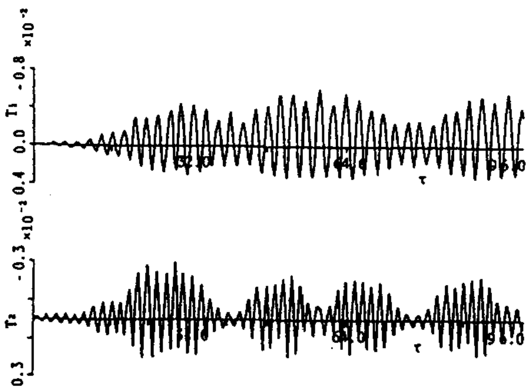


(a) $\overline{\omega}$=2.0, γ=0.001 (b) $\overline{\omega}$=2.42, γ=0.01

(c) $\overline{\omega}$=5.1, γ=0.04 (d) $\overline{\omega}$=5.64, γ=0.08

Fig. 11. Wave forms of the simple parametric resonance $2\omega_1$: $\overline{H}_t$ =0.5 and k=30.

Nonlinear time responses for the combination resonance $\omega_1+\omega_2$ are shown in Fig. 12. Amplitudes of the combination resonance are smaller than these of the simple resonance.



$\bar{\omega}=5.9,\ \gamma=0.04$

Fig. 12. Wave forms of the combination resonance $\omega_1+\omega_2$: $\bar{H}_t=0.5$ and k=30.

Ⅵ. CONCLUSIONS

Dynamic stability of cables subjected to an axial periodic load has been presented in this paper. The results of numerical examples lead to the following conclusions.

(1) Unstable regions of the cable consist of the simple parametric resonance with one mode and the combination resonance of sum type with two modes.

(2) Unstable regions of the simple parametric resonances are wider than those of the combination resonances.

(3) Width of the unstable regions changes dependently of the sag-to-span ratio of the cable. Unstable regions of the simple parametric resonances become narrow where modal crossover is producing and become wide again to approach the unstable region of the next mode.

(4) Unstable regions of the combination resonance become wide near the sag-to-span ratio where modal crossover of the lower mode is finished and that of the higher mode does not change yet.

(5) Amplitudes of unstable motion of the cable are bounded due to nonlinearity of cable.

REFERENCES

1) Bolotin, V. V., The Dynamic Stability of Elastic Systems. Holden-Day, Inc., San Fransisco (1964).

2) Nayfeh, A. H., and Mook, D. T., Nonlinear Oscillations. New York: John Wiley & Sons (1979).

3) Kovacs, I., Leonhardt, A., und Partner, G., Bautechnik, 59, H10. Zur Frage der Seilschwingugen und der Seildampfung (1982).

4) Takahashi, K., and Konishi, Y., Nonlinear vibrations of cables in three dimensions, Part Ⅱ. Journal of Sound and Vibration 118 (1988), pp. 85-97.

5) Takahashi, K., An approach to investigate the instability of the multiple-degree-of-freedom parametric dynamic systems, Journal of Sound and Vibration

78(1981), pp. 519-529.
6) Irvine, H. M., Cable Structures, The MIT Press(1981), pp. 87-99.
7) Hsu, C. S., On the parametric excitation of a dynamic system having multiple-degrees of freedom. Journal of Applied Mechanics 30(1963), pp. 367-372.
8) Irvine, H. M., and Caughey, T. K., The linear theory of free vibrations of a suspended cable. Proceedings of the Royal Society, London 341(1974), pp. 299-315.
9) Yamaguchi, H., and Ito, M., Linear theory of free vibrations of an inclined cable in three dimensions. Proceedings of Japan Society of Civil Engineers 286(1979), pp. 29-38.