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The Vertical Differentiation Model in the Insurance Market

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Abstract- This article explores the vertical differentiation model in the insurance market. The main results are as follows. First, the equilibrium price differential is not a linear function of the highest quality valuation (accident probability) and the maximum and minimum quality differentials. Second, a high quality insurance firm does not always receive greater equilibrium expected profit, even if its average cost is the same as that of a low-quality insurance firm. Finally, a change in the highest quality valuation has an ambiguous effect on the equilibrium expected profit differential.
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Introduction

It is well known that vertical differentiation is a useful strategy for avoiding fierce price competition (e.g. [7-8]). For this reason, vertical differentiation commonly exists not only in markets for tangible goods but also in markets for intangible goods, such as financial services. For example, [5-6] argue that the level of claims handling procedures is a quality factor in the insurance market. In fact, following an accident, all consumers wish to receive kind and friendly advice and payment of their insurance as soon as possible. Thus, even if the insurance products per se do not have quality factors, consumers can still perceive quality through the claims handling procedure offered by each insurance firm. From this viewpoint, insurance firms compete not only on price (the insurance rate) but also on quality (the level of claims handling procedure).

There are at least two good reasons for exploring the vertical differentiation model in the insurance market. The first is that very few studies including vertical differentiation models focus on the insurance market, though some of the extant literature does concern horizontal differentiation (e.g. [1, 4-5]). [1] The second reason is that consumers can enjoy the services from claims handling procedures only if an accident takes place. This suggests that the probability of an accident can help explain the quality valuation parameter of a vertical differentiation model in the insurance market. It is then easy to deduce that consumers with a high accident probability wish to purchase insurance products from a high quality insurance firm and vice versa. From this perspective, even if the average cost (amount of insurance per claim) is the same across insurance firms, the expected average cost in a high quality insurance firm is higher than in a low quality insurance firm. In other words, the quality valuation parameter affects not only consumer demand for insurance but also the expected average cost of the insurance firms themselves.

The model

Suppose there are two duopolistic risk-neutral insurance firms (A and B) in the market. Without loss of generality, we assume that insurance firm A is a low quality insurance firm and insurance firm B is a high quality insurance firm. Here, $q_i \in [q_{\min}, q_{\max}]$ denotes the quality (level of claims handling procedures) offered by the insurance firm $i \in \{A, B\}$, where $q_{\min} \in [0, q_{\max})$ and $q_{\max} \in (q_{\min}, \infty)$ represent the minimum and maximum levels of quality, respectively. Let π be the accident probability of consumers, assumed to lie on the uniform distribution $\pi \in [0, \bar{\pi}]$, where $\bar{\pi} \in (0,1]$ is the highest accident probability of consumers. Let p_i denote the price (insurance rate) offered by insurance firm i. The utility of each consumer is assumed separable in price and quality as in [2]. The utility function of consumer j purchasing insurance product from insurance firm i is then $u_j = -p_i + \pi_j q_i$. Also, assume that each consumer has to purchase one insurance product from a more desirable insurance firm. Let $\tilde{\pi}$ be the marginal consumer who does not differentiate between the two insurance firms. Using the consumer utility function, the marginal consumer is

$$-p_A + \tilde{\pi} q_A = -p_B + \tilde{\pi} q_B \Rightarrow \tilde{\pi} = \frac{p_B - p_A}{q_B - q_A} \quad (1)$$

The expected profit functions of the two insurance firms, denoted by Π_i, are

$$\Pi_A = \frac{1}{\bar{\pi}}\left(p_A - \int_0^{\tilde{\pi}} csds\right)\tilde{\pi} \quad (2)$$

$$\Pi_B = \frac{1}{\bar{\pi}}\left(p_B - \int_{\tilde{\pi}}^{\bar{\pi}} csds\right)(\bar{\pi} - \tilde{\pi}) \quad (3)$$

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[6] provides pioneering work on vertical differentiation model concerning insurance. However, it does not consider the decisions of insurance firms on price and quality. Furthermore, it implicitly assumes that all consumers have the same quality valuation parameters. One of the studies to build vertical differentiation model in which two insurance firms decide price and quality is [3].

where $c \in (0,\infty)$ represents the average cost unrelated to the level of quality.

This article sets out the following two-stage vertical differentiation model. In the first stage, both insurance firms simultaneously choose their quality (q_i). After they observe each quality, they simultaneously choose their price (p_i). The subgame perfect equilibrium is a suitable equilibrium concept for this model and is derived though backward induction.[2]

From equations (1), (2) and (3), the first-order conditions in the second stage are

$$\frac{\partial \Pi_A}{\partial p_A} = \frac{3c(p_B - p_A)^2 + 2(p_B - 2p_A)(q_B - q_A)^2}{2\bar{\pi}(q_B - q_A)^3} = 0 \quad (4)$$

$$\frac{\partial \Pi_B}{\partial p_B} = 1 - \frac{3c(p_B - p_A)^2 - 2c\bar{\pi}(p_B - p_A)(q_B - q_A) + (4p_B - 2p_A - c\bar{\pi}^2)(q_B - q_A)^2}{2\bar{\pi}(q_B - q_A)^3} = 0 \quad (5)$$

From equations (4) and (5), the equilibrium prices are derived as follows:[3]

$$p_A = \frac{1}{12c}\left\{4c^2\bar{\pi}^2 + 2c\bar{\pi}(q_B - q_A) + 3(q_B - q_A)^2 + (-q_B + q_A + c\bar{\pi})\sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_B - q_A) + 9(q_B - q_A)^2}\right\} \quad (6)$$

$$p_B = \frac{1}{12c}\left\{4c^2\bar{\pi}^2 + 4c\bar{\pi}(q_B - q_A) - 3(q_B - q_A)^2 + (q_B - q_A + c\bar{\pi})\sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_B - q_A) + 9(q_B - q_A)^2}\right\} \quad (7)$$

To consider the first stage of the model, by substituting equations (6) and (7) into equations (2) and (3), the expected profit functions of the firms can be expressed as

$$\Pi_A = \frac{1}{108}\bar{\pi}\left\{11c\bar{\pi} - 3(q_B - q_A) + 5\sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_B - q_A) + 9(q_B - q_A)^2}\right\} \quad (8)$$

$$\Pi_B = \frac{1}{108}\bar{\pi}\left\{-19c\bar{\pi} + 15(q_B - q_A) + 11\sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_B - q_A) + 9(q_B - q_A)^2}\right\} \quad (9)$$

From equations (8) and (9), the first-order conditions in the first stage are

$$\frac{\partial \Pi_A}{\partial q_A} = \frac{1}{108}\bar{\pi}\left[3 - \frac{15\{c\bar{\pi} + 3(q_B - q_A)\}}{\sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_B - q_A) + 9(q_B - q_A)^2}}\right] < 0 \quad (10)$$

$$\frac{\partial \Pi_B}{\partial q_B} = \frac{1}{108}\bar{\pi}\left[15 + \frac{33\{c\bar{\pi} + 3(q_B - q_A)\}}{\sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_B - q_A) + 9(q_B - q_A)^2}}\right] > 0 \quad (11)$$

From equations (10) and (11), $q_A = q_{\min}$ and $q_B = q_{\max}$ are the equilibrium qualities.

Implications

In order to illustrate properly the implications of our model, we first provide the results of the seminal vertical differentiation model where the quality valuation parameter is unrelated to the expected average cost that introduced in [9]. That is, the profit functions are $\hat{\Pi}_A = (1/\bar{\pi})(\hat{p}_A - c)\hat{\pi}$ and $\hat{\Pi}_B = (1/\bar{\pi})(\hat{p}_B - c)(\bar{\pi} - \hat{\pi})$, where $\hat{\pi} = (\hat{p}_B - \hat{p}_A)/(\hat{q}_B - \hat{q}_A)$. In the same manner, the following equilibrium values are derived: $\hat{p}_A = c + (1/3)\bar{\pi}(q_{\max} - q_{\min})$, $\hat{p}_B = c + (2/3)\bar{\pi}(q_{\max} - q_{\min})$, $\hat{q}_A = q_{\min}$, $\hat{q}_B = q_{\max}$, $\hat{\Pi}_A = (1/9)\bar{\pi}(q_{\max} - q_{\min})$, $\hat{\Pi}_B = (4/9)\bar{\pi}(q_{\max} - q_{\min})$.

Using equations (6) and (7), the equilibrium price differential in our model is

$$p_B - p_A = \frac{1}{6c}(q_{\max} - q_{\min})\left\{c\bar{\pi} - 3(q_{\max} - q_{\min}) + \sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_{\max} - q_{\min}) + 9(q_{\max} - q_{\min})^2}\right\} \quad (12)$$

In contrast, the equilibrium price differential in the seminal vertical differentiation model is

$$\hat{p}_B - \hat{p}_A = \frac{1}{3}\bar{\pi}(q_{\max} - q_{\min}) \quad (13)$$

From equations (12) and (13), we know that an increase in $\bar{\pi}$ and $(q_{\max} - q_{\min})$ expands the price differential in both models. However, unlike equation (13), equation (12) is not a linear function of $\bar{\pi}$ and $(q_{\max} - q_{\min})$, because the change in $\bar{\pi}$ and $(q_{\max} - q_{\min})$ changes the expected average cost through the change in the marginal consumer.

Further, from equations (8) and (9), the equilibrium expected profit differential in our model is

$$\Pi_B - \Pi_A = \frac{1}{18}\bar{\pi}\left\{-5c\bar{\pi} + 3(q_{\max} - q_{\min}) + \sqrt{7c^2\bar{\pi}^2 + 6c\bar{\pi}(q_{\max} - q_{\min}) + 9(q_{\max} - q_{\min})^2}\right\} \quad (14)$$

In contrast, the equilibrium profit differential in the seminal vertical differentiation model is

$$\hat{\Pi}_B - \hat{\Pi}_A = \frac{1}{3}\bar{\pi}(q_{\max} - q_{\min}) \quad (15)$$

[2] That model structure is normal to analyze vertical differentiated market. For example, [7] sets out three-stage vertical differentiation model that includes entry stage before choosing quality and price.

[3] There are two solutions for equations (4) and (5). However, only this solution satisfies the second-order conditions.

When using equation (14), which insurance firm receives more equilibrium expected profit is ambiguous, even if the average cost is the same in our model; in contrast, using equation (15), the high quality insurance firm always receives more profit in the seminal vertical differentiation model. The reason is that the high quality insurance firm has to accept consumers with a high accident probability, and so its expected average cost becomes higher. When $\overline{\pi}$ is higher and $(q_{\max} - q_{\min})$ is lower, average expected cost is higher than profit through high quality supplying, and a "low quality advantage" appears in the insurance market. Also, an increase in $(q_{\max} - q_{\min})$ expands the equilibrium (expected) profit differential in both models. In contrast, a change in $\overline{\pi}$ has an ambiguous effect on the equilibrium expected profit differential in our model, while $\overline{\pi}$ always expands the equilibrium profit differential in the seminal vertical differentiation model, because an increase in $\overline{\pi}$ increases not only profits through more differentiation but also losses through the increase in expected average cost.

Conclusion

This article exposed the vertical differentiation model in the insurance market. The main results are as follows. First, the equilibrium price differential is not a linear function of the highest quality valuation (accident probability) and the maximum and minimum quality differentials. Second, a high quality insurance firm does not always receive greater equilibrium expected profit, even if its average cost is the same as that of a low quality insurance firm. Finally, a change in the highest quality valuation has an ambiguous effect on the equilibrium expected profit differential.

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