

鋼アーチ橋の安定に関する各国の設計基準比較

論文

Comparison between multinational criteria for stability of steel arch bridges

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ABSTRACT Multinational criteria for stability of steel arch bridge were analyzed. Inadequacies of criteria were discussed briefly. According to multinational criteria, the influence of rise to span ratio on in-plane critical flexure load, and rise to span ratio, the ratio γ of the length of the arch part stiffened by lateral braces to the total length of the arch rib and arch rib spacing affecting on the out-of-plane critical flexure load were studied by employing a steel arch bridge. The results show that due to the different assumptions in multinational criteria, the parameters of rise to span ratio, γ -value and arch rib spacing has different influence on in-plane and out-of-plane critical flexure load, respectively.

Keywords: 設計基準比較, 鋼アーチ橋, 面内耐力, 面外耐力

Multinational codes comparison, Steel arch bridge, In-plane critical flexure load, Out-of-plane critical flexure load

1. Introduction

More and more steel arch bridges with modern design and complex architecture have been built in the world due to development of design theory and construction method. Steel arch rib is a structure subjected to compressive force primarily. Comparing with the straight steel members, the stability of arch rib becomes more complex and it is closely associated with the critical flexure load of arch bridges. So far, numerous previous research works on the critical flexure load of steel arch bridges have been carried out by many researchers. The results show that rise to span ratio, residual stress, initial imperfection, line shape and type of arch axis, instability analysis process considering nonlinearity have certain effect on the critical flexure load. Among them, loading procedure and boundary condition are the dominant factors affecting the critical flexure load of the steel arch bridges¹⁻⁴⁾. Therefore, the specifications are drawn up by various countries according to their own practical situation of the design and fabrication. In this paper, the provisions for in-plane and out-of-plane stability of steel arch bridge in Chinese code: *Fundamental code for design on railway bridge and culvert*³⁾, Japanese code: *Specification for Highway Bridges*⁶⁾, American code: *AASHTO LRFD Bridge Design Specifications*⁷⁾

and Eurocode 3: *Design of Steel Structure*⁸⁾ are compared. The results provide a reference for scholars and revision of criteria for steel arch bridges.

2. Outline of provisions in each code

2.1 Chinese code³⁾

2.1.1 In-plane stability

In-plane stability of the arch should be checked under the following assumptions. The arch rib is a member subjected to axial compressive load, the length of member relates to the span of the bridge, and no favorable effects of suspenders and tie bars exist. The effective length is determined by the rise to span ratio and structural style since the critical load of steel arch bridges is significantly affected by these two parameters⁹⁾⁻¹¹⁾.

The critical flexure force N_{cr} of the arch for in-plane buckling is expressed by

$$N_{cr} = \frac{\pi^2 EI_x}{L_0^2} \quad (1)$$

where E is young's modulus of arch rib, I_x is moment of inertia of arch rib, L_0 is the effective length of arch rib which can be obtained by the following equation

$$L_0 = \pi \sqrt{8f / KL} \cdot L \quad (2)$$

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Table 1 Value of K

f/L	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0
Fixed arch	60.7	101.0	115.0	111.0	97.4	83.8	59.1	43.7
Two hinged arch	28.5	45.5	46.5	43.9	38.4	30.5	20.0	14.1
Three hinged arch	22.5	39.6	46.5	43.9	38.4	30.5	20.0	14.1

where L is the span of the bridge, f is the rise of arch, and K is a parameter determined by the structural style and rise to span ratio of arch rib. The value of K is given in Table 1. When the rise to span ratio comes between the values shown in Table 1, it may be calculated through linear interpolation.

2.1.2 Out-of-plane stability

Out-of-plane stability shall be checked if the width of arch ring or arch rib spacing is smaller than $L/20$ with regard to plate arch and ribbed arch, respectively. The arch rib can be supposed as a Vierendeel truss with the length of arch axis approximately in checking, and the truss is subjected to the longitudinal force N on the quarter point of arch rib. N is determined by the following equation

$$N = H / \cos \phi_m \quad (3)$$

where H is the horizontal thrust, ϕ_m is the horizontal angle of arch axis on quarter point. The critical force N_{cr} shall be calculated by

$$N_{cr} = \alpha_0 \frac{\pi^2 EI}{S^2} \quad (4)$$

where I is the geometrical moment of inertia of two chords around the common axis (central longitudinal axis of the bridge). S is the length of arch axis. α_0 shall be calculated as follows

$$\alpha_0 = \frac{1}{1 + \frac{\pi^2 EI}{S^2} \left(\frac{ab}{12EI_b} + \frac{a}{26EI_y} \cdot \frac{1}{1-\mu} \right)} \quad (5)$$

where a is the panel length, b is the arch rib spacing, I_b is the geometric moment of inertia of transverse brace around the vertical direction, and I_y is the out-of-plane geometric moment of inertia of arch rib. The μ -value can be obtained by equation (6).

$$\mu = \frac{N_{cr} a^2}{2\pi^2 EI_y} \quad (6)$$

The arch rib spacing, panel length, moment of inertia of lateral bracing and arch rib around vertical axis, arch axial length and geometric moment of inertia around bridge axis are considered to estimate the critical load of steel arch bridges. However, the influence of material nonlinearity, transverse initial imperfection and lateral displacement has not been taken into account.

2.2 Japanese code⁶⁾

2.2.1 In-plane buckling of arch

Against in-plane buckling, the arch can be designed as a member subjected to axial force and concurrent bending moment. Stability against buckling may be deemed to be satisfied when the check is made in accordance with the following equations

$$\frac{\sigma_c}{\sigma_{caz}} + \frac{\sigma_{bey}}{\sigma_{bagy} \left(1 - \frac{\sigma_c}{\sigma_{eay}}\right)} + \frac{\sigma_{bcz}}{\sigma_{bao} \left(1 - \frac{\sigma_c}{\sigma_{caz}}\right)} \leq 1 \quad (7)$$

$$\sigma_c + \frac{\sigma_{bey}}{\left(1 - \frac{\sigma_c}{\sigma_{eay}}\right)} + \frac{\sigma_{bcz}}{\left(1 - \frac{\sigma_c}{\sigma_{caz}}\right)} \leq \sigma_{cal} \quad (8)$$

where σ_c is the compressive stress due to the axial force acting on the section to be checked, σ_{bey} , σ_{bcz} are the bending compressive stress due to the bending moment acting about the strong and weak axes, respectively, σ_{caz} is the allowable axial compressive stress about the weak axis, σ_{bagy} is the allowable bending compressive stress about the strong axis that does not consider local buckling, σ_{bao} is the upper limit of allowable bending compressive stress that does not consider local buckling, σ_{cal} is allowable stress of edge-supported, stiffened plates with respect to local buckling, σ_{eay} and σ_{caz} are allowable Euler buckling stress about the strong and weak axes, respectively. These values are given by equations (9) and (10).

$$\sigma_{eay} = 1,200,000 / (l / r_y)^2 \quad (9)$$

$$\sigma_{caz} = 1,200,000 / (l / r_z)^2 \quad (10)$$

Where l is the effective buckling length, r_y , r_z are the radii of gyration of area about the strong and weak axes, respectively.

2.2.2 Out-of-plane buckling of arch

Because the entire structural system of a steel arch bridge having a narrow width and a long span which could lead to buckling laterally in the outward direction of the arch plane, rules and regulations of verification have been established. When the arch axis forms a symmetrical parabola within the vertical

plane, and the lateral bracing and sway bracing are installed in accordance with the provisions, the verification of the out-of-plane buckling of the arch may be performed through the following equation,

$$H / A_g \leq 0.85\sigma_{ca} \quad (11)$$

where H is the horizontal component of the axial force acting on the members of one side arch by the loading shown in Fig. 1, A_g is mean value of the gross cross-sectional area of the members of one side arch, σ_{ca} is the allowable axial compressive stress at the $L/4$ point of the member of one side arch. In this regard the effective buckling length and radius of gyration shall be calculated as follows.

$$l = \varphi\beta_z L, \quad r = \sqrt{\left(I_z + A_g \left(\frac{b}{2} \right)^2 \right) / A_g} \quad (12)$$

Where I_z is mean value of the geometrical moment of inertia around the vertical axis of the members of one side arch, b is the spacing of arch axis line, β_z is the values as shown in the Table 2. When the value of f/L comes between the values in Table 2, it may be interpolated in a linear manner. Values of φ are specified as follows: for through stiffened arch $\varphi = 1 - 0.35k$, for upper-deck stiffened arch $\varphi = 1 + 0.45k$, and for midheight-deck stiffened arch $\varphi = 1$. k is the ratio of the load that hanger or shoring shares to the total load in the loading state of Fig. 1. In this regard, when the arch and stiffening girder in the upper-deck stiffened arch aren't rigidly linked together at an arch crown, the value of k shall be 1.

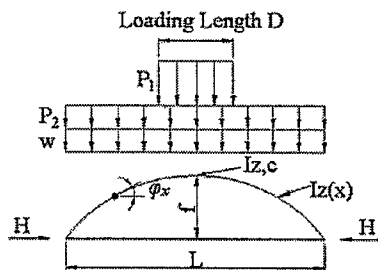


Figure 1 Loading State to Be Used for Verifying Out-of-Plane Buckling

Table 2 Value of β_z

Section	Rise ratio f/L				
	0.05	0.10	0.20	0.30	0.40
$I_z = \text{constant}$	0.50	0.54	0.65	0.82	1.07
$I_z = I_{zc} / \cos\phi_x$	0.50	0.52	0.59	0.71	0.86

The integral lateral stiffness is considered by employing transverse radius of gyration in the

Japanese code, but the influence of stiffness and location of lateral braces, and the ratio γ of the length of the arch part stiffened by lateral braces to the total length of the arch rib haven't been considered. Therefore, the out-of-plane critical flexure load calculated according to Japanese code is not conservative when the γ -value is small¹²⁾⁻¹³⁾.

2.3 American code⁷⁾

2.3.1 In-plane buckling of arch

In lieu of a rigorous analysis, the effective length for buckling may be estimated as the product of the arch half span length and the factor β specified in Table 3. The critical flexure load is specified as following

$$N_{cr} = \frac{\pi^2 EI_x}{(\beta L / 2)^2} \quad (13)$$

where L is the span of the arch rib, EI_x is the in-plane flexure stiffness of the arch rib.

Table 3 Value of β

Rise to span ratio	3-Hinged Arch	2-Hinged Arch	Fixed Arch
0.1~0.2	1.16	1.04	0.70
0.2~0.3	1.13	1.10	0.70
0.3~0.4	1.16	1.16	0.72

2.3.2 Out-of-plane buckling of arch

In the specification of AASHTO, there is no provision for calculating out-of-plane critical flexure load. Steel arch ribs are designed as a steel member subjected to combined axial compression and flexure. The axial compressive load P_u , and concurrent moments M_{ux} and M_{uy} , calculated for the factored loadings by elastic analytical procedures shall satisfy the following relationship

If $P_u / P_r < 0.2$, then

$$\frac{P_u}{2.0P_r} + \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \right) \leq 1.0 \quad (14)$$

If $P_u / P_r \geq 0.2$, then

$$\frac{P_u}{P_r} + \frac{8.0}{9.0} \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \right) \leq 1.0 \quad (15)$$

where P_r is the factored compressive resistance, M_{rx} and M_{ry} are the factored flexure resistance about the x-axis and y-axis taken equal to ϕ_f times the nominal flexural resistance about x-axis and y-axis, respectively, M_{ux} and M_{uy} are the factored flexural moment about the x-axis and y-axis, respectively, ϕ_f is the resistance factor for flexure.

2.4 Eurocode 3⁸⁾

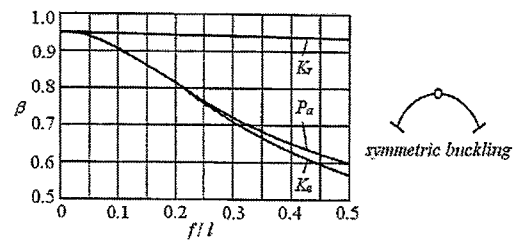
2.4.1 In-plane critical buckling load

The fixed and two-hinged arches always buckle into an in-plane asymmetrical mode in which the arch sways sideways with the crown moving horizontally and becoming a point of contraflexure. The arch has a mode shape from support to crown similar to that of fixed-hinged column¹⁴⁾. Therefore, the in-plane critical flexure load in Eurocode 3 is provided as that of the end-loaded, fixed-hinged column whose length is equal to the arc length of the arch from support to crown.

The critical buckling force N_{cr} in the arch for in-plane buckling is expressed by

$$N_{cr} = (\pi / \beta S)^2 EI_x \quad (16)$$

where N_{cr} relates to the force at the supports, S is the half length of the arch, EI_x is the in-plane flexure stiffness of the arch, β is the buckling length factor. For arches with rigid supports, buckling factors β are given in the Fig. 2. For arches with a tension tie and hangers, buckling factors β are given in Fig. 3. In which P_a means parabolic form, K_c means chain form, K_r means circular form. For P_a and K_c the loading is vertical.



(d) Single Hinged Arch

Figure 2 Buckling Length Factor β for Arches

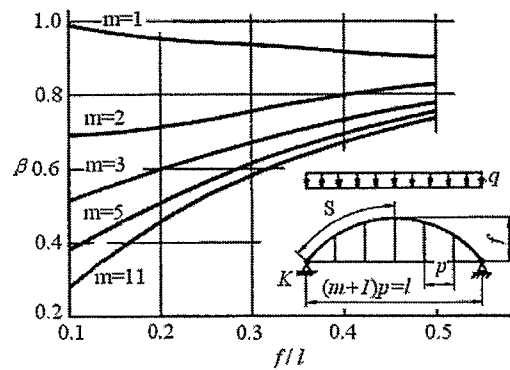
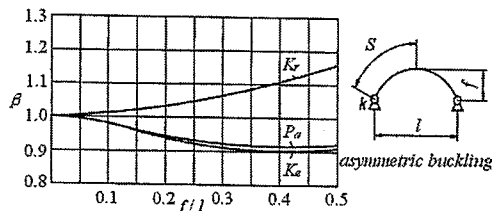
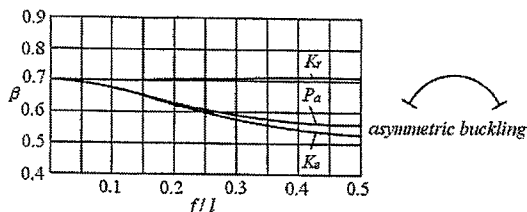


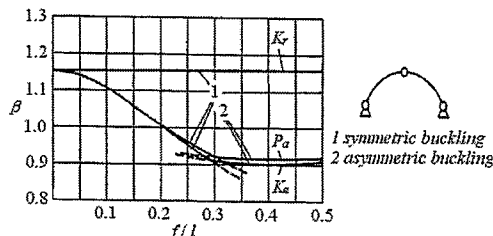
Figure 3 Buckling Factor β



(a) Two Hinged Arch



(b) Fixed Arch



(c) Three Hinged Arch

2.4.2 Out-of-plane buckling flexure load

The out-of-plane critical flexure load of free standing arches provided in Eurocode 3 is also obtained by simplifying the arch as a column subjected to axial compressive force. However, the length of column is equal to the projection length of the arch since the fundamental mode of out-of-plane buckling is the symmetric first mode between two springings³⁾. For free standing steel arch bridge, the effective-length factor depends on the rise to span ratio, geometric moment of inertia of arch rib and load transmission from deck to arch rib.

The critical buckling force in free standing arches for out-of-plane buckling is expressed by

$$N_{cr} = (\pi / \beta l)^2 EI_y \quad (17)$$

where l is the projection length of the arch, EI_y is the out-of-plane flexure stiffness of the arch.

For out-of-plane buckling of free standing arches, the buckling factors may be taken as $\beta = \beta_1 \beta_2$, where β_1 and β_2 is given in Tables 4 and 5, respectively. For out-of-plane buckling of free standing circular arches with radial loading, the buckling factor β may be obtained by equation (18). Where r is the radius of the circle, α is the section angle of the arch $0 < \alpha < \pi$, $K = EI_z / GI_z$.

Table 4 β_1 -Values

f/l	0.05	0.10	0.20	0.30	0.40
$I_z = \text{constant}$	0.50	0.52	0.65	0.82	1.07
$I_z \text{ varies}$					
$I_z(\alpha_B) = I_{z,0} / \cos \alpha_B$	0.50	0.52	0.59	0.71	0.86

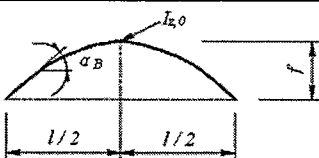
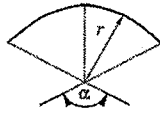


Table 5 β_2 -Values

Loading	β_2	Comments
Conservative (The deck is fixed to the top of the arch)	1	 \dot{q} : total load q_H : load part transmitted by hangers q_{SI} : load part transmitted by posts
by hangers	$1 - 0.35 q_H / q$	
by posts	$1 - 0.45 q_{SI} / q$	

$$\beta = \pi \alpha \frac{\sqrt{\pi^2 + \alpha^2 K}}{l(\pi^2 - \alpha^2)} \quad (18)$$

The out-of-plane buckling of arches with wind bracing and portals may be verified by a stability check of the end portals. The buckling length factor β may be taken from Fig. 5 in using the geometry in Fig. 4. The value h_r in Fig. 4 may be taken as the mean of all lengths $h_H / \sin \alpha_k$ of the hangers.

The influence of location and stiffness of end portals, transversal stiffness of arch rib are considered in the Eurocode 3, which reflects elements of integral lateral rigidity of arch rib, whereas leaving the influence of arrangement style of lateral brace, transversal initial imperfection and lateral displacement out of consideration³⁾.

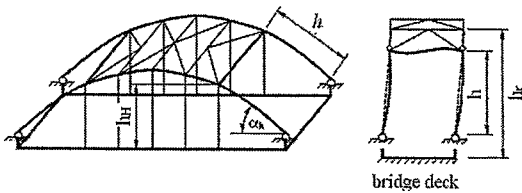
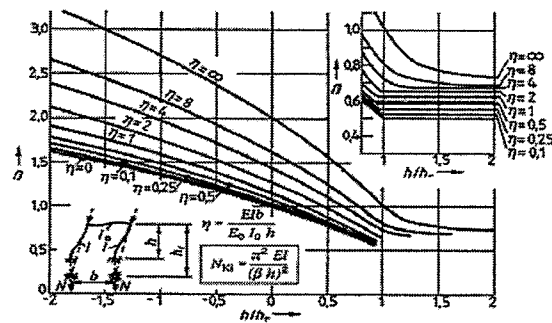
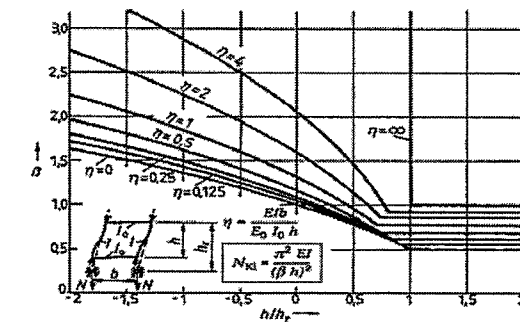


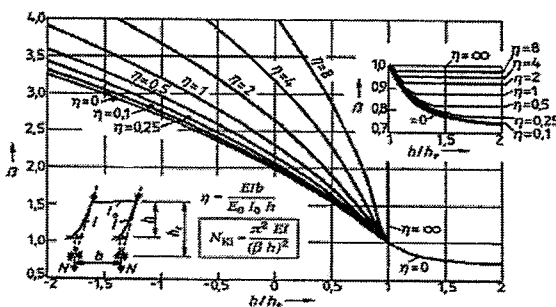
Figure 4 Buckling of Portals for Arches



(b) Arch Springing Fixed



(c) Arch Springing With Wind Brace
Figure 5 Buckling Length Factors β



(a) Arch Springing Hinged

3. Numerical Comparison between Multinational Criteria

3.1 Outline of example

Parameters affecting the in-plane and out-of-plane critical flexure load calculated according to multinational codes will be studied below by using the uniform steel arch bridge in references 13) shown in Fig. 6. The main span of the bridge is 150m long with rise to span ratio of 0.15. The arch rib spacing b was chosen as 20m, 10m, 5m respectively, and

coefficient γ , which is the ratio of the length of the arch part stiffened by lateral braces to the total length of the arch rib, was taken as 0.864, 0.733 and 0.48 based on statistical analysis of existing steel arch bridges. The yield stress of steel σ_y is assumed to be 235MPa. The dimensions of arch rib and lateral brace are given in Table 6.

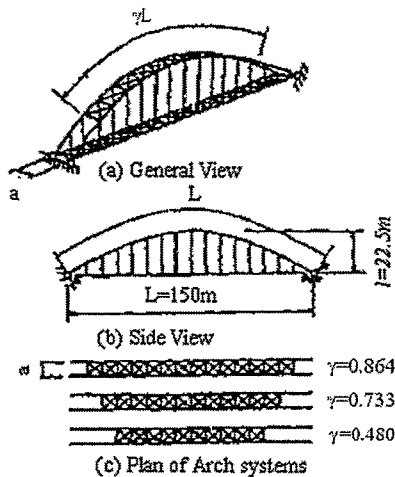


Figure 6 Model of Steel Arch Bridge

Table 6 Dimensions of Model (Units: mm)

Size Member	Dimensions (mm)			
	Height	Width	Flange thickness	Web thickness
Arch rib	2880	480	12	6
Lateral brace	400	400	21	6.5

3.2 In-plane critical flexure load

Since the steel with yield stress of 235MPa in the references [13] are seldom adopted in current steel arch bridges, it will be replaced by material of SM490Y specified in Japanese industrial standard in this paper. The influence of rise to span ratio on in-plane critical flexure loads according to multinational codes with different structural style are demonstrated in Fig. 7. It shows that the in-plane critical flexure loads decrease along with the increase of rise to span ratio in Chinese code and Eurocode, while the rise to span ratio has a slight influence on the critical load in the American code. The critical flexure loads of fixed arch bridge in Chinese and American code are larger than those of two-hinged and three-hinged bridges. Moreover, critical flexure loads of two-hinged and three-hinged arch bridge are similar. In Eurocode 3, only Fig. 3 is specified for the arch bridge with a tension tie and hangers. Therefore, only one line is shown in Fig. 7.

The in-plane stability of the arch is checked as a member subjected to axial compressive load in

multinational codes, but the effective buckling length of arch rib specified in each code is different. This difference results in the difference of the in-plane critical flexure load. With the same rise to span ratio, the maximum critical flexure load adopted by calculating with respect to Eurocode, followed by American code and Chinese code comes minimum.

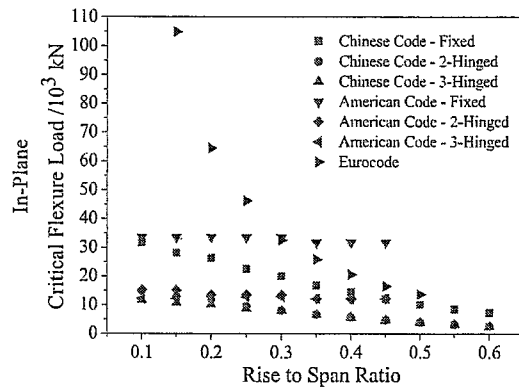


Figure 7 Relationship between In-Plane Critical Flexure Load and Rise to Span Ratio

3.3 Out-of-plane critical flexure load

The influence of γ -value on out-of-plane critical flexure load with different arch rib spacing and rise to span ratio is illustrated in Figs. 8 and 9, respectively. The figures show that the out-of-plane critical flexure load according to Chinese code and Eurocode increase with the increase of γ -value, and γ -value has a greater effect on the results of Eurocode. The critical flexure load in terms of Japanese code doesn't change with the values of γ . Moreover, according to Chinese and European code, critical flexure load decreases with the increase of arch rib spacing and rise to span ratio at the same γ -value. Arch rib spacing and rise to span ratio have a certain level of influence on the critical flexure load with constant γ -value.

Here are some crucial reasons for results: the Chinese code supposes the arch rib as a Vierendeel truss with the length of arch axis, and the values of a and S , b in Equation (5) increase with the increase of rise to span ratio and arch rib spacing, respectively, moreover, value of a decreases with the increase of γ -value. According to the Eurocode, the height h of end portals decreases along with the increase of γ , then the height of equivalent frame structure will decrease, which lead to the increase of out-of-plane critical flexure load, besides, increase of arch rib spacing and rise to span ratio cause increase of η value and decrease of h/h_0 value. There is no parameter relating to the stiffness, location and influence of lateral bracing in equation of Japanese code. Therefore, the critical flexure load according to

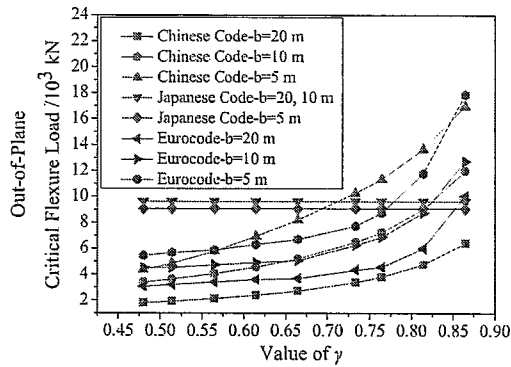


Figure 8 Relationship between Out-of-Plane Critical Flexure Load and Coefficient of γ

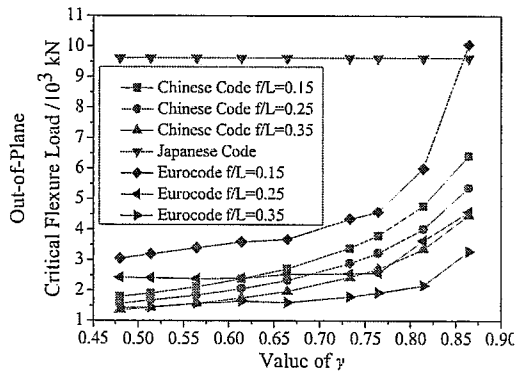


Figure 9 Relationship between Out-of-Plane Critical Flexure Load and Coefficient of γ

Japanese code is independent of γ -value.

Influence of arch rib spacing on out-of-plane critical flexure load with different γ -value and rise to span ratio is illustrated in Figs. 10 and 11, respectively. Figures show that critical flexure load according to Chinese code comes to a peak with the arch rib spacing of approximately 5m. The critical flexure load in accordance with Japanese code decreases with the decrease of arch rib spacing, and tendency becomes more pronounced when distance is smaller than 5m. The critical flexure load in line with Eurocode increases with the decrease of arch rib spacing. Furthermore, according to Chinese and European codes, critical flexure load increases with the increase of γ -value and decrease of rise to span ratio at the same arch rib spacing. Critical flexure load in Japanese code also increases with decrease of rise to span ratio, but variation is small.

There are several major reasons for these results. The Chinese code and Eurocode are based on the energy method. Lateral braces with the same section become more tubbiness as its length becomes small. Consequently, the deformation of arch rib is more intensively restrained, and thus the critical flexure

load increases with the increase of integral deformation energy of two arch ribs. In contrast, the integral lateral stiffness is considered in the calculation of transverse radius of gyration in the Japanese code, and larger arch rib spacing results in the greater integral lateral rigidity of arch rib. Therefore, the critical flexure load in line with Japanese code increases with the increase of arch rib spacing. The reasons for critical flexure load change with γ -value and rise to span ratio at constant arch rib spacing are the same as preceding text.

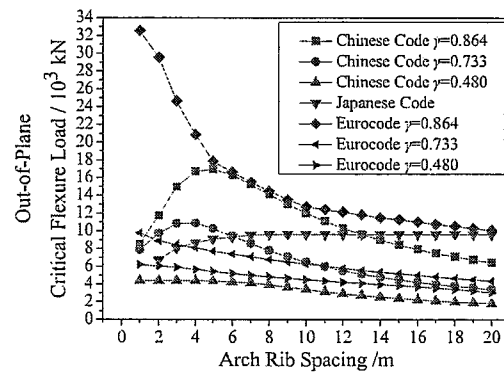


Figure 10 Relationship between Out-of-Plane Critical Flexure Load and Arch Rib Spacing

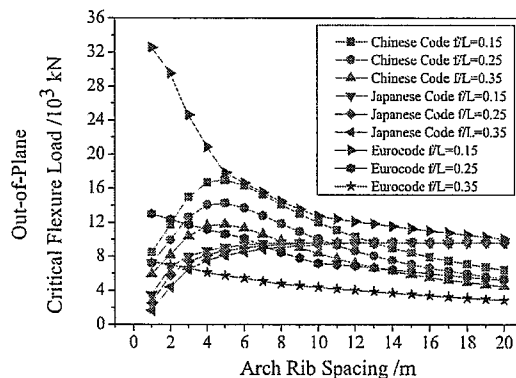


Figure 11 Relationship between Out-of-Plane Critical Flexure Load and Arch Rib Spacing

Influence of rise to span ratio on out-of-plane critical flexure load with different γ -value and arch rib spacing is illustrated in Figs. 12 and 13, respectively. The figures show that critical flexure load decreases with the increase of rise to span ratio and decrease of γ -value at the same rise to span ratio according to Chinese code and Eurocode. Moreover, according to Japanese code, arch rib spacing and γ -value have small and no influence on critical flexure load with the same rise to span ratio, respectively. Reasons are the same as preceding text.

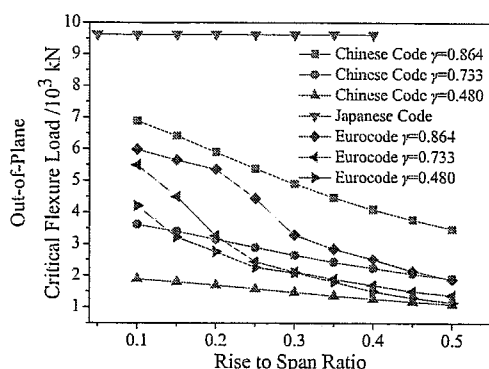


Figure 12 Relationship between Out-of-Plane Critical Flexure Load and Rise to Span Ratio

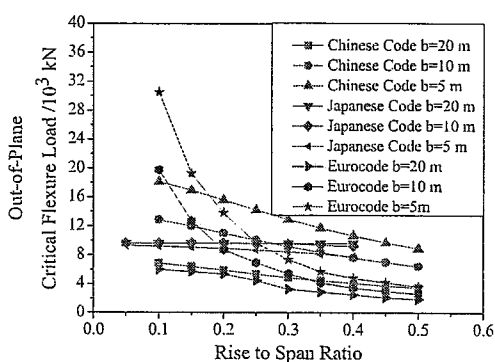


Figure 13 Relationship between Out-of-Plane Critical Flexure Load and Rise to Span Ratio

4. Conclusions

The results of comparison of the critical flexure load between multinational codes for steel arch bridges can be summarized as follows.

- (1) The in-plane critical flexure load decreases along with the increase of rise to span ratio according to Chinese code and Eurocode, while it doesn't change with the rise to span ratio by the American code.
- (2) The out-of-plane critical flexure load according to Chinese code comes to a peak with the arch rib spacing of approximately 5m. The larger arch rib spacing gives the larger critical flexure load in Japanese code, while it gives smaller critical load in Eurocode. The increase of γ -value and decrease of rise to span ratio result in the increase of out-of-plane critical flexure load according to Chinese code and Eurocode, while in terms of Japanese code it doesn't change with the values of γ , and slightly affected by rise to span ratio.

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